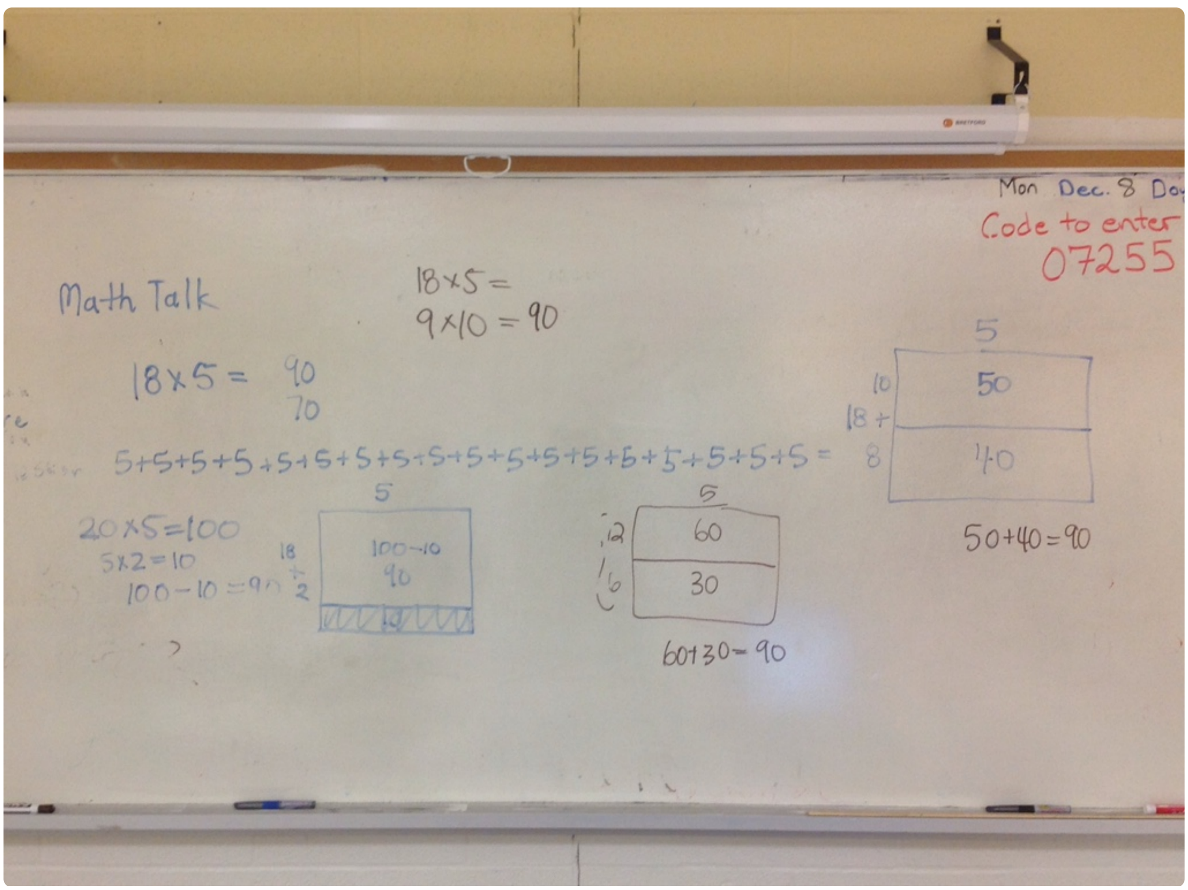
Learning Goal
We are learning how to compose and decompose shapes, like rectangles, squares, parallelograms and triangles, to discover that shapes with perimeters of different lengths can have the same area.

Minds On
Number talk
In your head, calculate 18x5, give a silent, close to you, thumbs up when you have the answer



The Area Stays the Same

Materials:

- 5-by-8-inch index cards, one per group
- centimeter grid paper
- newsprint

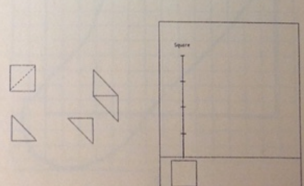
Directions:

Cut a square that measures 5 centimeters on each side from the index card. Use the centimeter grid paper to help you. Use this shape to make other different shapes. You can do this by cutting the square on the diagonal into two triangles and putting them together in various ways or by cutting the square in other ways and arranging the pieces together.

Trace around the different shapes you make, cutting each from the index card. You will need at least 5 shapes, including the original 5-centimeter square. All of your shapes will have the same area.

On the newsprint, draw a line segment equal in length to the perimeter of each shape. (You can use the actual shapes and trace around each edge.) Label each line segment with the shape.

How do the perimeters compare? Record and make a statement about your findings.



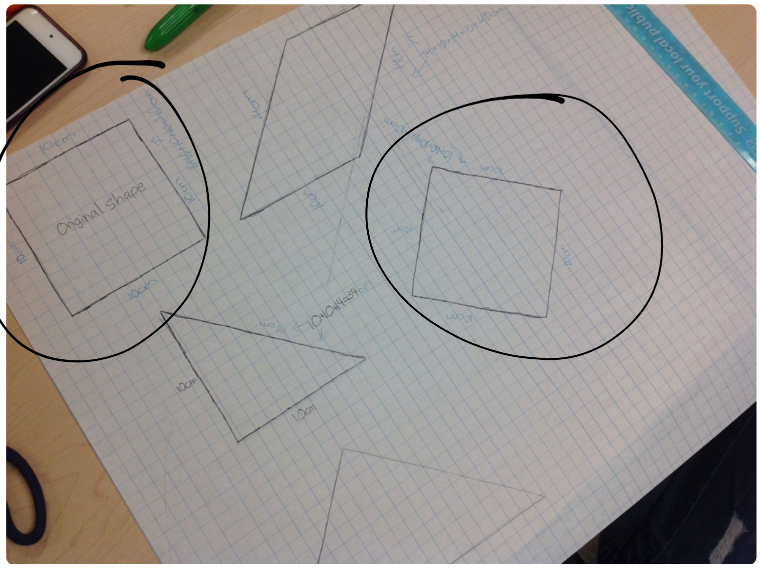
Adapted from About *Teaching Mathematics: A K-8 Resource*, by Marilyn Burns
PRIME Summer Institute 2010

Working on it

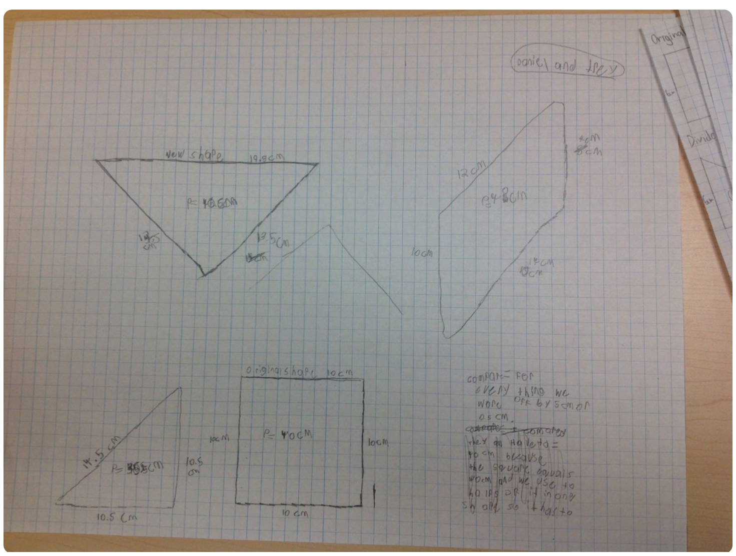
We are differentiating and giving some students 8x6, 10x10

Cut out your shape on the centimetre grid paper. Cut your shape in half, on the diagonal, to make other different common shapes. Trace your shapes onto a big piece of grid paper, including your original shape, and label the sides with units. How do the perimeters compare?

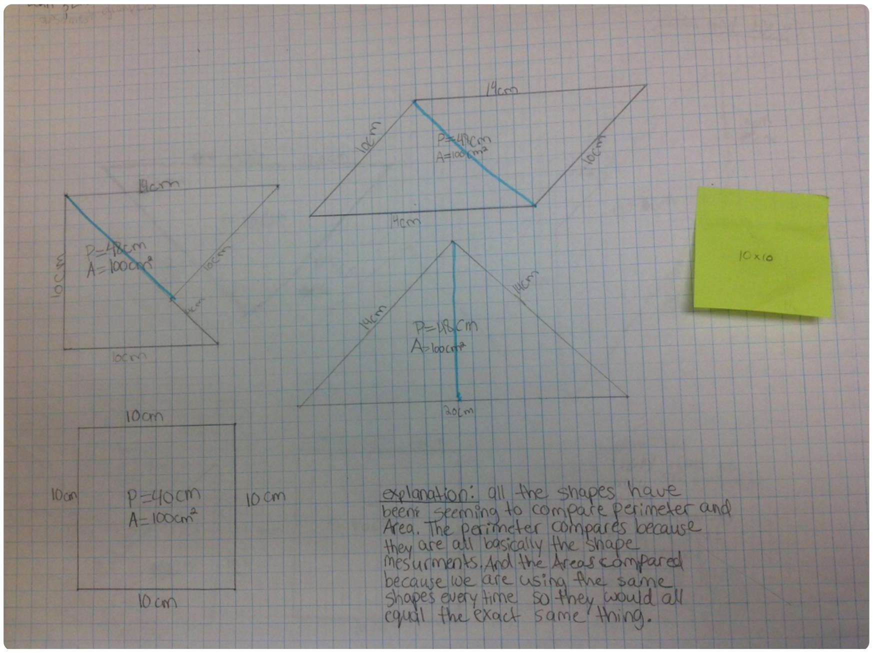
-then prompt to compare the areas, once done



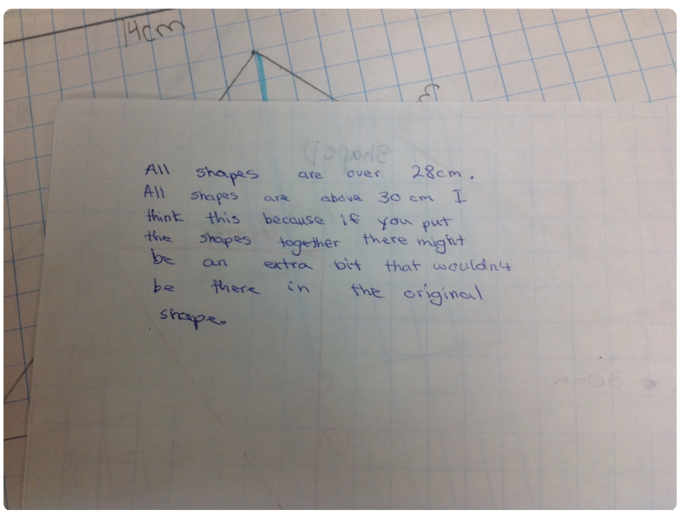
Are these shapes different?



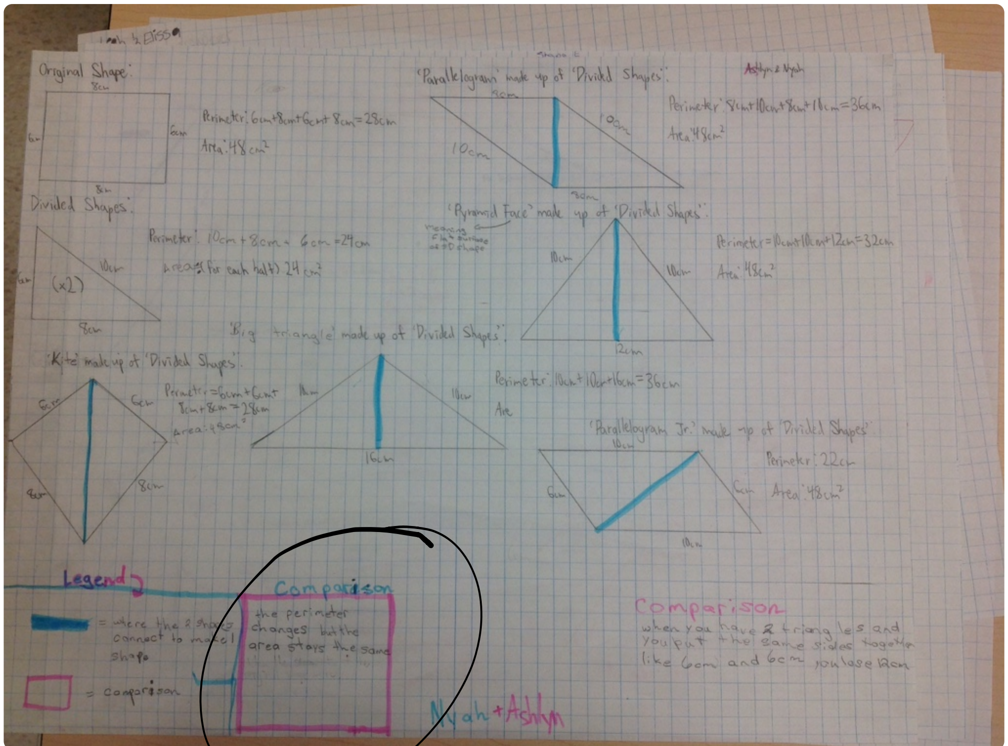
Believe that the perimeters must be the same, even changed their measurements to make it work



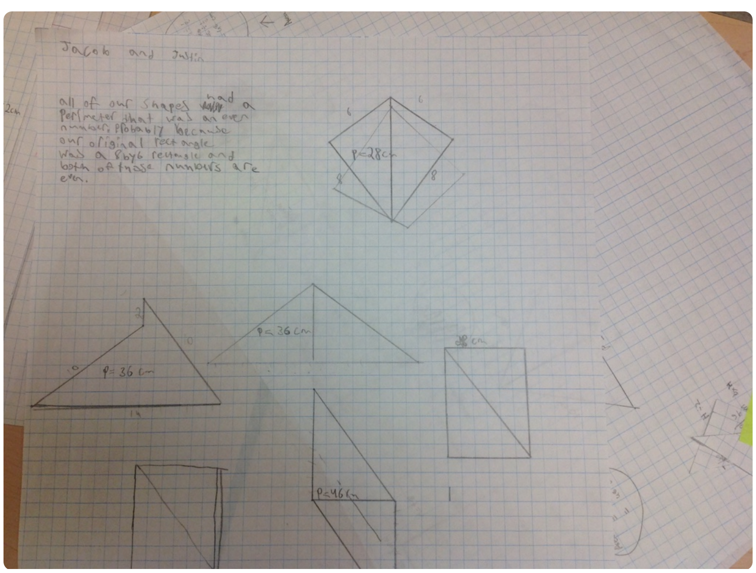
Areas are the exact same thing, what about the perimeters?



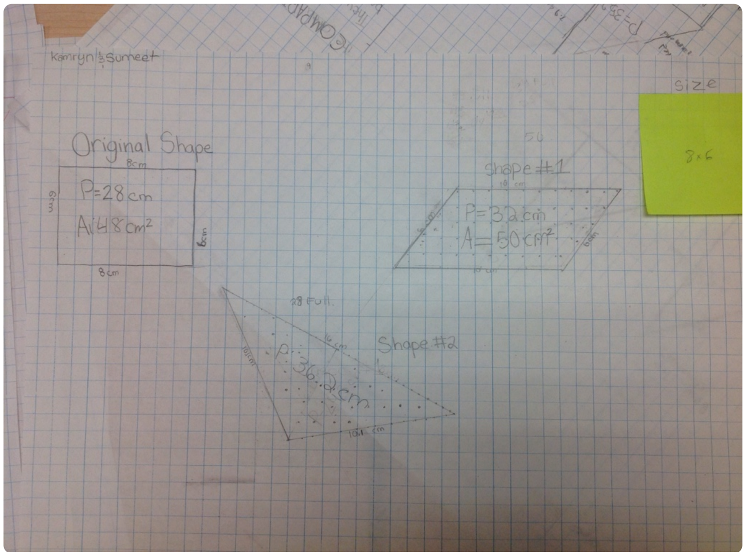
Where does the extra bit come from?



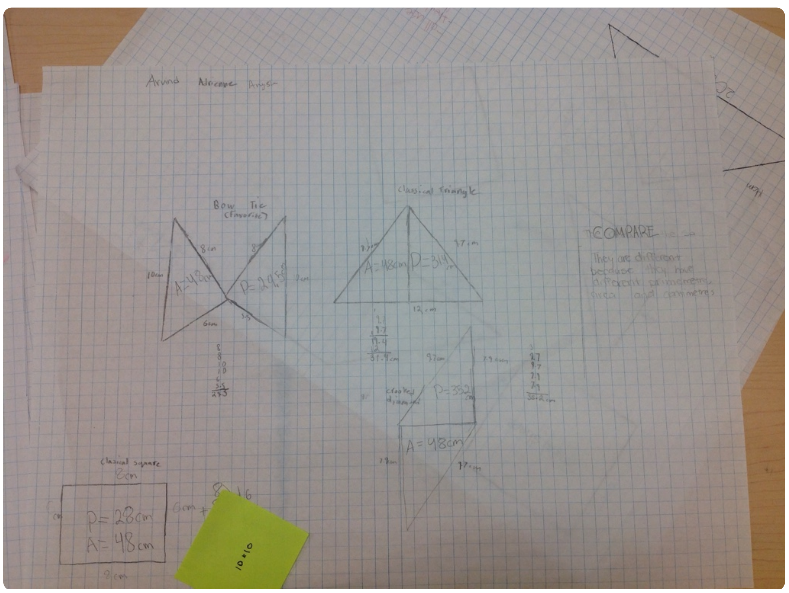
! Why is that?



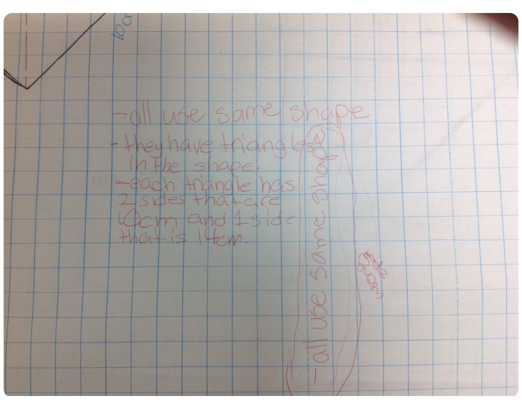
Is this always true?
Can you make a rectangle or parallelogram with an odd number perimeter, using whole numbers?



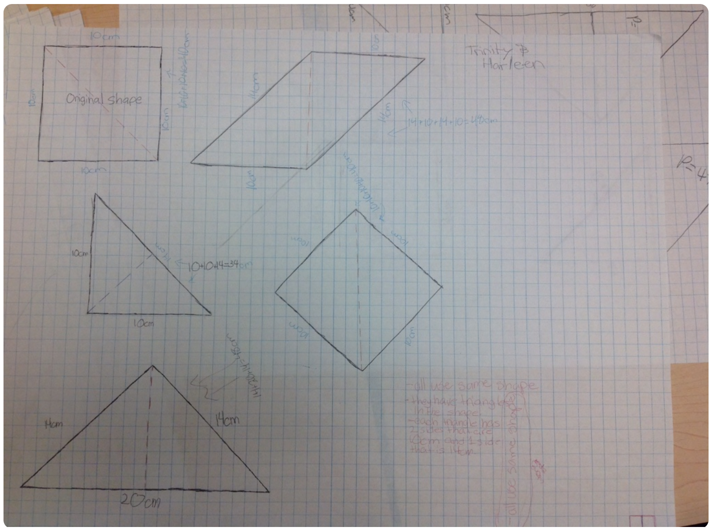
Can you have two different areas?



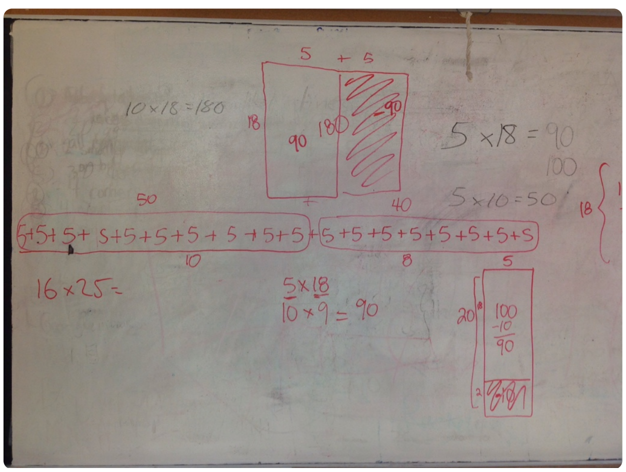
Same areas, different perimeters

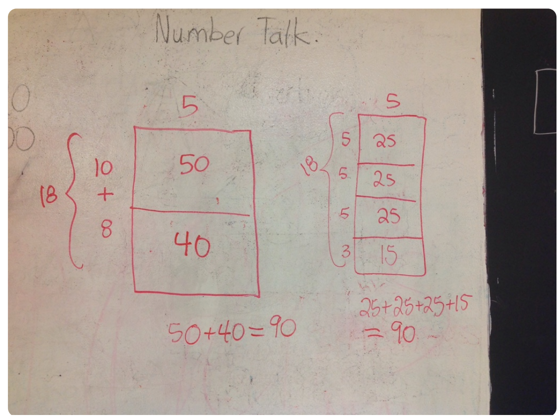


The rectangles are the same but they change when put in different orders

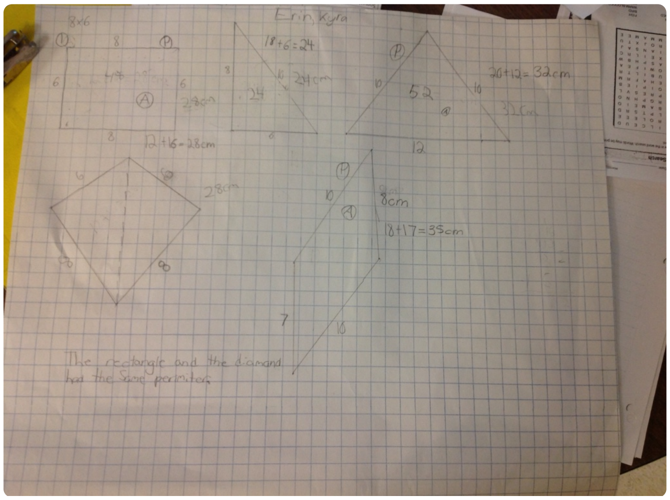


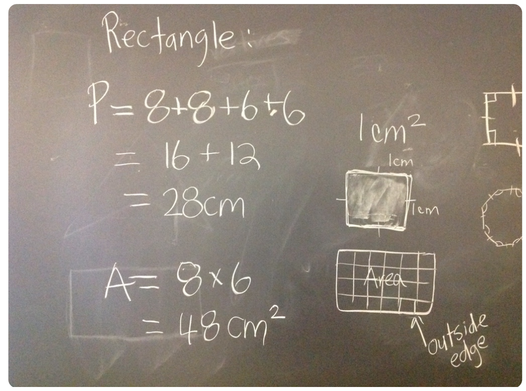
Other class' number talk





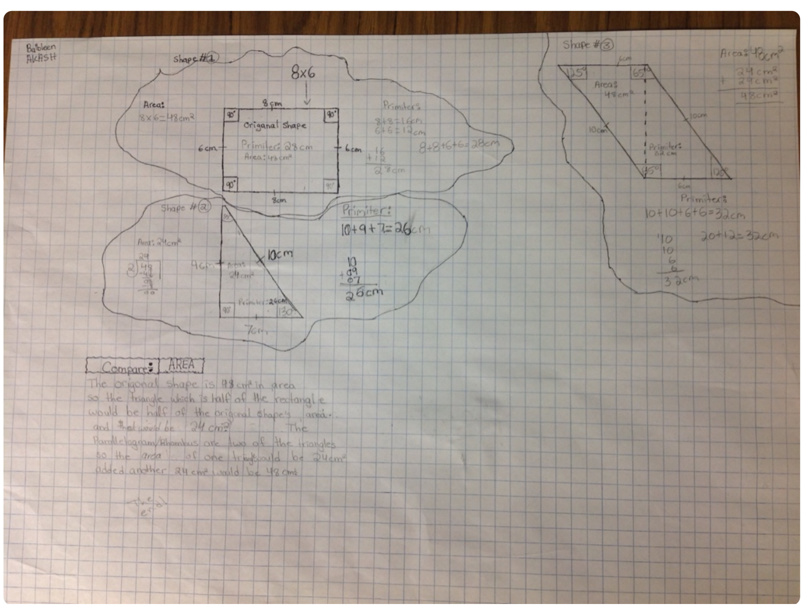
Other class' debrief





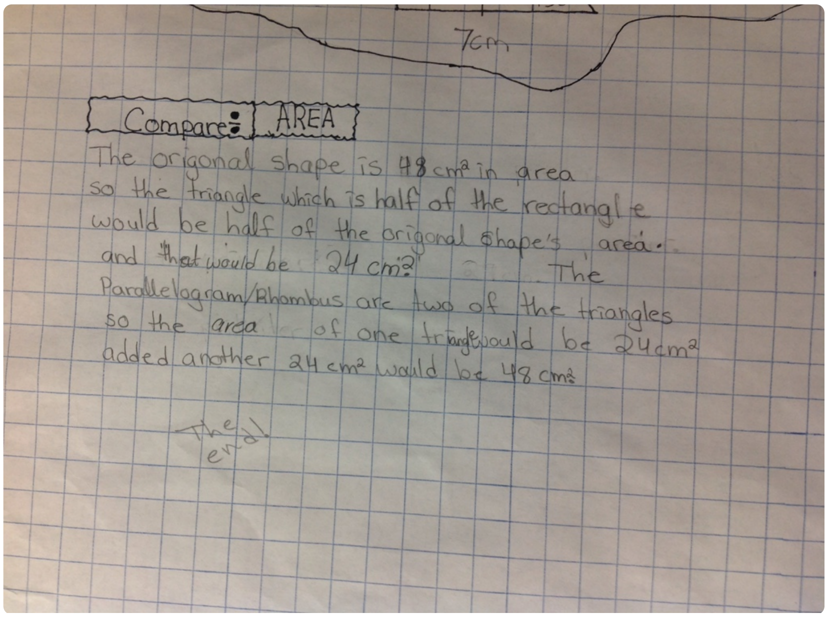
Making thinking visible, how did you calculate the perimeter? The area? Are the units the same? Does it matter?

The rectangle and the diamond/rhombus have the same perimeter, do you agree or disagree? Explain your thinking...
-turn and talk to your partner



Are all of your perimeters the same, or different? Explain your thinking
-some will be the same, but others will be different, because the diagonal is longer than a side

What about your areas? Do the shapes have different areas or the same? Why or why not?



Do you agree or disagree?

But they have different perimeters, why is that?

Big idea...shapes with the same area can have different perimeters

Where to next?
-more concrete, hands on investigation looking at, for example, a rectangle made of 24 tiles, what are all of the possible perimeters? The smallest? The largest? What do you notice about the area?
-link this investigation to a number talk where the teacher continues to model the open array

Then..Making Math Meaningful...(Small), p. 444, 445...

ACTIVITY 17.15

Draw a rectangle with not much area but a lot of perimeter.

Students can then look for relationships in the table among the length, width, and area, which will lead to the formula, *Area* = *length* × *width*. They might also notice that they can determine the length by dividing the area by the width, or the width by dividing the area by the length.

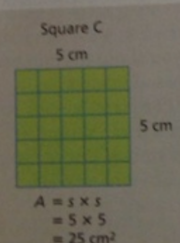
Because the formula for determining the area of a rectangle is usually introduced before any other area formulas, it is a common misconception that the formula *Area* = *length* × *width* can be applied to determine the area of shapes other than rectangles. When students develop the formula in a concrete way using arrays and using relationships, they are less likely to have this misconception.

Area of a Square

The formula for calculating the area of a square,

$$\textit{Area} = \textit{side length} \times \textit{side length}\ (A = s \times s)$$

is a special case of the rectangle formula since *length* × *width* in this case is the same as *side length* × *side length*. Students can create different squares on grid paper and record data about the squares in a table such as that shown. They will soon realize that the area of a square is its side length multiplied by itself.



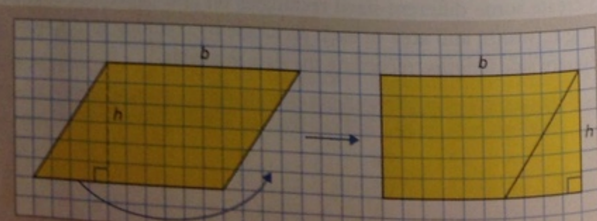
Square	Side length (cm)	Area (cm²)
A	8	64
B	7	49
C	5	25

Area of a Parallelogram

Once students have a solid understanding of the rectangle formula, they are ready to learn that a related formula,

$$\textit{Area} = \textit{base length} \times \textit{height}$$

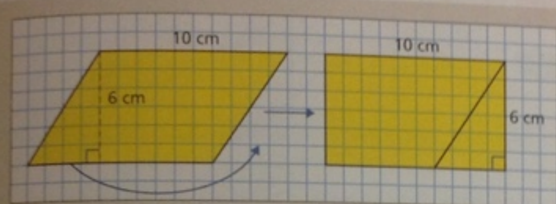
can be used to calculate the area of a parallelogram. To explore this idea, the teacher can demonstrate that it is possible to transform a parallelogram into a rectangle with the same base length and height by cutting a triangle from one side of the parallelogram and moving it to the opposite side, or by cutting to create two trapezoids that can be similarly rearranged.



Students can then construct and cut out some other parallelograms and test them to see if they can all be transformed the same way. They may notice that the fact that the opposite sides of a parallelogram are equal is the reason why the triangle that is cut off on one side can always be attached to the other side. It is also important for students to recognize that

- the base that is cut remains the same length because the part removed from one end is added to the other end
- the height is measured at right angles to the base
- a single cut along the height forms both sides of the rectangle

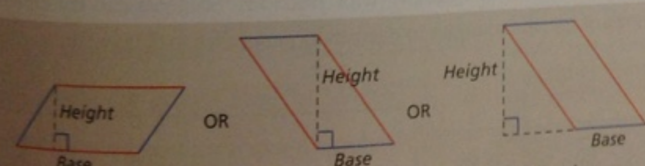
For these reasons, the parts of a parallelogram can always be reassembled to form a rectangle with the same base length and height.



$A = b \times h$
$= 10 \times 6$
$= 60\ \text{cm}^2$

The area of this parallelogram is the same as the area of a rectangle that is 10 cm long and 6 cm high. So the area is 60 cm^2.

Some students will be interested to note that either side length of the parallelogram can be used as the base, and that they can measure the height from either inside or outside the boundaries of the parallelogram. As long as they measure the height from the side they are using as the base length, the formula will produce the correct area.

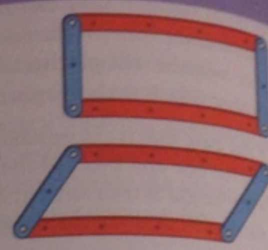


Any parallelogram has 2 possible base lengths and 2 possible heights.

An activity that will help students understand how the base length and height of a parallelogram relate to its area is described in **Activity 17.16** on the next page. This activity indirectly demonstrates the independence of perimeter and area—the perimeter remains the same as the area decreases.

ACTIVITY 17.16

Have students form a rectangle out of Geostrips or cardboard strips hinged together at the four vertices with butterfly clips or pipe cleaners. Each side should be a whole number of centimetres long. Ask them to place the rectangle on grid paper and record a sketch of the shape, the base length, the height, and the area.



Then have students adjust the rectangle to change the height several times to form a series of other parallelograms (with the base anchored on the same location on the grid paper each time). They can record the information about each shape in a table like the one shown here.

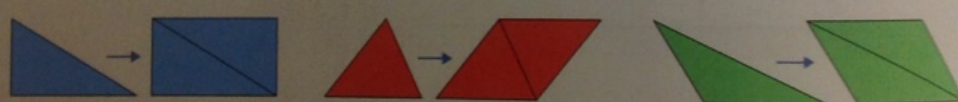
Sketch of shape	Base (cm)	Height (cm)	Area (cm^2)
	8	5	40
	8	3	24

Area of a Triangle

Once students have developed and worked with the parallelogram area formula, the next step is usually the introduction of the formula for determining the area of a triangle:

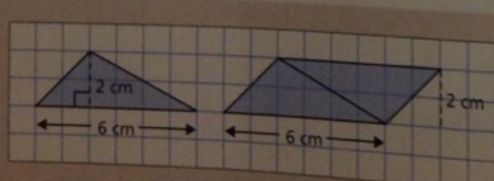
$$\textit{Area} = \textit{base} \times \textit{height} \div 2$$

This formula is derived from the parallelogram formula and works because every triangle, no matter what type of triangle it is, can be shown to be half of a parallelogram with the same base and height.



Any triangle can be shown to be half of a parallelogram with the same base and height.

The triangle formula can also be written as $\textit{Area} = \frac{1}{2} \times \textit{base} \times \textit{height}$ or $\textit{Area} = \frac{\textit{base} \times \textit{height}}{2}$, but it makes sense to use $\textit{Area} = \textit{base} \times \textit{height} \div 2$ until students learn to multiply with fractions. Students can try different examples of specific triangles to eventually generalize to develop the formula.



A = b × h ÷ 2
= 6 × 2 ÷ 2
= 12 ÷ 2
= 6

The area of the parallelogram is 12 cm^2 because 6 x 2 = 12. The triangle is half of the parallelogram, so its area is 6 cm^2.