Ontario

Number Sense and Numeration, Grades 4 to 6

Volume 4
Division

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6

Ontario Education excellence for all

2006

Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.

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CONTENTS

INTRODUCTION

Number Sense and Numeration, Grades 4 to 6 is a practical guide, in six volumes, that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Number Sense and Numeration strand of *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. This guide provides teachers with practical applications of the principles and theories behind good instruction that are elaborated on in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

The guide comprises the following volumes:

- Volume 1: The Big Ideas
- Volume 2: Addition and Subtraction
- Volume 3: Multiplication
- Volume 4: Division
- Volume 5: Fractions
- Volume 6: Decimal Numbers

The present volume – Volume 4: Division – provides:

- a discussion of mathematical models and instructional strategies that support student understanding of division;
- sample learning activities dealing with division for Grades 4, 5, and 6.

A glossary that provides definitions of mathematical and pedagogical terms used throughout the six volumes of the guide is included in Volume 1: The Big Ideas. Each volume also contains a comprehensive list of references for the guide.

The content of all six volumes of the guide is supported by “eLearning modules” that are available at www.eworkshop.on.ca. The instructional activities in the eLearning modules that relate to particular topics covered in this guide are identified at the end of each of the learning activities (see pp. 44, 55, and 68).

Relating Mathematics Topics to the Big Ideas

> *The development of mathematical knowledge is a gradual process. A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the "big ideas" of mathematics – that is, the interrelated concepts that form a framework for learning mathematics in a coherent way.*
>
> (*The Ontario Curriculum, Grades 1–8: Mathematics, 2005*, p. 4)

In planning mathematics instruction, teachers generally develop learning opportunities related to curriculum topics, such as fractions and division. It is also important that teachers design learning opportunities to help students understand the big ideas that underlie important mathematical concepts. The big ideas in Number Sense and Numeration for Grades 4 to 6 are:

- quantity
- operational sense
- relationships
- representation
- proportional reasoning

Each of the big ideas is discussed in detail in Volume 1 of this guide.

When instruction focuses on big ideas, students make connections within and between topics, and learn that mathematics is an integrated whole, rather than a compilation of unrelated topics. For example, in a lesson about division, students can learn about the relationship between multiplication and division, thereby deepening their understanding of the big idea of operational sense.

The learning activities in this guide do not address all topics in the Number Sense and Numeration strand, nor do they deal with all concepts and skills outlined in the curriculum expectations for Grades 4 to 6. They do, however, provide models of learning activities that focus on important curriculum topics and that foster understanding of the big ideas in Number Sense and Numeration. Teachers can use these models in developing other learning activities.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills. The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The learning activities described in this guide demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful for students.

The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

Problem Solving: Each of the learning activities is structured around a problem or inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;
- work collaboratively and talk about mathematics;
- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;
- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

Reasoning and Proving: The learning activities described in this guide provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The learning activities include questions teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

Reflecting: Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

Selecting Tools and Computational Strategies: Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this guide provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and represent and communicate mathematical ideas at their own level of understanding.

Connecting: The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections among concrete, pictorial, and symbolic mathematical representations. Advice on helping students connect

procedural knowledge and conceptual understanding is also provided. The problem-solving experiences in many of the learning activities allow students to connect mathematics to real-life situations and meaningful contexts.

Representing: The learning activities provide opportunities for students to represent mathematical ideas using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students' own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

Communicating: Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

Addressing the Needs of Junior Learners

Every day, teachers make many decisions about instruction in their classrooms. To make informed decisions about teaching mathematics, teachers need to have an understanding of the big ideas in mathematics, the mathematical concepts and skills outlined in the curriculum document, effective instructional approaches, and the characteristics and needs of learners.

The following table outlines general characteristics of junior learners, and describes some of the implications of these characteristics for teaching mathematics to students in Grades 4, 5, and 6.

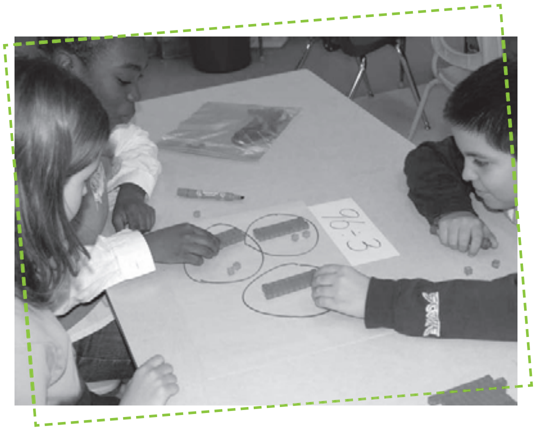
Characteristics of Junior Learners and Implications for Instruction		
Area of Development	**Characteristics of Junior Learners**	**Implications for Teaching Mathematics**
Intellectual development	Generally, students in the junior grades: • prefer active learning experiences that allow them to interact with their peers; • are curious about the world around them; • are at a concrete operational stage of development, and are often not ready to think abstractly; • enjoy and understand the subtleties of humour.	The mathematics program should provide: • learning experiences that allow students to actively explore and construct mathematical ideas; • learning situations that involve the use of concrete materials; • opportunities for students to see that mathematics is practical and important in their daily lives; • enjoyable activities that stimulate curiosity and interest; • tasks that challenge students to reason and think deeply about mathematical ideas.
Physical development	Generally, students in the junior grades: • experience a growth spurt before puberty (usually at age 9–10 for girls, at age 10–11 for boys); • are concerned about body image; • are active and energetic; • display wide variations in physical development and maturity.	The mathematics program should provide: • opportunities for physical movement and hands-on learning; • a classroom that is safe and physically appealing.
Psychological development	Generally, students in the junior grades: • are less reliant on praise but still respond well to positive feedback; • accept greater responsibility for their actions and work; • are influenced by their peer groups.	The mathematics program should provide: • ongoing feedback on students' learning and progress; • an environment in which students can take risks without fear of ridicule; • opportunities for students to accept responsibility for their work; • a classroom climate that supports diversity and encourages all members to work cooperatively.
Social development	Generally, students in the junior grades: • are less egocentric, yet require individual attention; • can be volatile and changeable in regard to friendship, yet want to be part of a social group; • can be talkative; • are more tentative and unsure of themselves; • mature socially at different rates.	The mathematics program should provide: • opportunities to work with others in a variety of groupings (pairs, small groups, large group); • opportunities to discuss mathematical ideas; • clear expectations of what is acceptable social behaviour; • learning activities that involve all students regardless of ability. *(continued)*

Characteristics of Junior Learners and Implications for Instruction		
Area of Development	**Characteristics of Junior Learners**	**Implications for Teaching Mathematics**
Moral and ethical development	Generally, students in the junior grades: • develop a strong sense of justice and fairness; • experiment with challenging the norm and ask "why" questions; • begin to consider others' points of view.	The mathematics program should provide: • learning experiences that provide equitable opportunities for participation by all students; • an environment in which all ideas are valued; • opportunities for students to share their own ideas and evaluate the ideas of others.

(Adapted, with permission, from *Making Math Happen in the Junior Grades*. Elementary Teachers' Federation of Ontario, 2004.)

LEARNING ABOUT DIVISION IN THE JUNIOR GRADES

Introduction



Students' understanding of division concepts and strategies is developed through meaningful and purposeful problem-solving activities. Solving a variety of division problems and discussing various strategies and methods helps students to recognize the processes involved in division, and allows them to make connections between division and addition, subtraction, and multiplication.

PRIOR LEARNING

Initial experiences with division in the primary grades often involve sharing objects equally. For example, students might be asked to show how 4 children could share 12 boxes of raisins fairly. Using 12 counters to represent the boxes, students might divide the counters into 4 groups while counting out, "One, two, three, four, one, two, three, four, . . ." until all the "boxes" have been distributed.

Students in the primary grades also apply their understanding of addition, subtraction, and multiplication to solve division problems. Consider the following problem.

> *"Chad has 28 dog treats. If he gives Rover 4 dog treats each day, for how many days will Rover get treats?"*

Using addition: Students might repeatedly add 4 until they get to 28, and then count the number of times they added 4. Students often use drawings to help them keep track of the number of repeated additions they make.

$$\boxed{4}\ \boxed{4}\ \boxed{4}\ \boxed{4}\ \boxed{4}\ \boxed{4}\ \boxed{4}$$

$$4 + 4 + 4 + 4 + 4 + 4 + 4 = 28$$

Using subtraction: Students might start with 28 counters and remove them in groups of 4. Later, students make connections to repeated subtraction (e.g., repeatedly subtracting 4 from 28 until they get to 0, and then counting the number of times 4 was subtracted).

Using multiplication: Students might use their knowledge of multiplication. For example, "Rover gets 4 treats each day. Since $4 \times 7 = 28$, Rover will get treats for 7 days."

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES

In the junior grades, instruction should focus on developing students' understanding of division concepts and meaningful computational strategies, rather than on having students memorize the steps in algorithms.

Development of division concepts and computational strategies should be rooted in meaningful experiences that allow students to model multiplicative relationships (i.e., represent a quantity as a combination of equal groups), and encourage them to develop and apply a variety of strategies.

Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to division, listed in the following table.

Curriculum Expectations Related to Division, Grades 4, 5, and 6

By the end of Grade 4, students will:	By the end of Grade 5, students will:	By the end of Grade 6, students will:
Overall Expectation • solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies. ***Specific Expectations*** • multiply to 9×9 and divide to $81 \div 9$, using a variety of mental strategies; • multiply whole numbers by 10, 100, and 1000, and divide whole numbers by 10 and 100 using mental strategies; • divide two-digit whole numbers by one-digit whole numbers, using a variety of tools and student-generated algorithms.	***Overall Expectation*** • solve problems involving the multiplication and division of multidigit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies. ***Specific Expectations*** • divide three-digit whole numbers by one-digit whole numbers, using concrete materials, estimation, student-generated algorithms, and standard algorithms; • multiply decimal numbers by 10, 100, 1000, and 10 000, and divide decimal numbers by 10 and 100, using mental strategies; • use estimation when solving problems involving the addition, subtraction, multiplication, and division of whole numbers, to help judge the reasonableness of a solution.	***Overall Expectation*** • solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies. ***Specific Expectations*** • use a variety of mental strategies to solve addition, subtraction, multiplication, and division problems involving whole numbers; • solve problems involving the multiplication and division of whole numbers (four-digit by two-digit), using a variety of tools and strategies; • multiply and divide decimal numbers to tenths by whole numbers, using concrete materials, estimation, algorithms, and calculators; • multiply and divide decimal numbers by 10, 100, 1000, and 10 000 using mental strategies.

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005)

The following sections explain content knowledge related to division concepts in the junior grades, and provide instructional strategies that help students develop an understanding of division. Teachers can facilitate this understanding by helping students to:

- interpret division situations;
- relate multiplication and division;
- use models to represent division;
- learn basic division facts;
- consider the meaning of remainders;
- develop a variety of computational strategies;
- develop estimation strategies for division.

Interpreting Division Situations

In the junior grades, students need to encounter problems that explore both partitive division and quotative division.

In partitive division (also called distribution or sharing division), the whole amount and the number of groups are known, but the number of items in each group is unknown.

Examples:

- Daria has 42 bite-sized granola snacks to share equally with her 6 friends. How many snacks does each friend get?
- 168 DVDs are packaged into 8 boxes. How many DVDs are there in each box?
- Zeljko's father bought a new TV for $660. He is paying it off monthly for one year. How much does he pay each month?

In quotative division (also called measurement division), the whole amount and the number of items in each group are known, but the number of groups is unknown.

Examples:

- Thomas is packaging 72 ears of corn into bags. If each bag contains 6 ears of corn, how many bags does Thomas need?
- Anik's class wants to raise $1100 for the Red Cross. Each month they collect $125 through fundraising. How many months will it take to raise $1100?
  (*Note:* In this problem, students need to deal with the remainder. For example, students might conclude that more money will need to be raised one month or that an extra month of fundraising will be needed.)
- The hardware store sells light bulbs in large boxes of 24. The last order was for 432 light bulbs. How many large boxes of light bulbs were ordered?

Students require experiences in interpreting both types of problems and in applying appropriate problem-solving strategies. It is not necessary, though, for students to identify or define these problem types.

Relating Multiplication and Division

Multiplication and division are inverse operations: multiplication involves combining groups of equal size to create a whole, whereas division involves separating the whole into equal groups. In problem-solving situations, students can be asked to determine the total number of items in the whole (multiplication), the number of items in each group (partitive division), or the number of groups (quotative division).

Students should experience problems such as the following, which allow them to see the connections between multiplication and division.

> *"Samuel needs to equally distribute 168 cans of soup to 8 shelters in the city. How many cans will each shelter get?"*

> *"The cans come in cases of 8. How many cases will Samuel need in order to have 168 cans of soup?"*

Although both problems seem to be division problems, students might solve the second one using multiplication – by recognizing that 20 cases would provide 160 cans ($20 \times 8 = 160$), and that an additional case would provide another 8 cans ($1 \times 8 = 8$), therefore determining that 21 cases would provide 168 cans. With this strategy, students, in essence, decompose 168 into (20×8) (1×8), and then add $20 + 1 = 21$.

Providing opportunities to solve related problems helps students develop an understanding of the part-whole relationships inherent in multiplication and division situations, and enables them to use multiplication and division interchangeably, depending on the problem situation.

Using Models to Represent Division

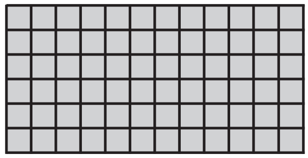
Models are concrete and pictorial representations of mathematical ideas. It is important that students have opportunities to represent division using models that they devise themselves (e.g., using counters to solve a problem involving fair sharing; drawing a diagram to represent a quotative division situation).

Students also need to develop an understanding of conventional mathematical models for division, such as arrays and open arrays. Because array models are also useful for representing multiplication, they help students to recognize the relationships between the two operations. Consider the following problem.

> *"In preparation for their concert in the gym, a class is arranging 72 chairs in rows of 12. How many rows will there be?"*

To solve this problem, students might arrange square tiles in an array, by creating rows of 12, and discover that there are 6 rows. The array, as a model of a mathematical situation, provides

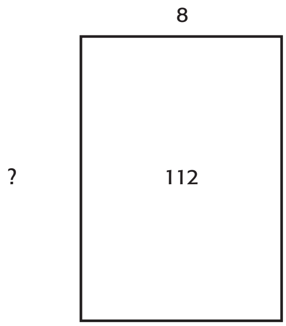
a representation of $72 \div 12 = 6$. It helps students to visualize how the factors of 12 and 6 can be combined to create a whole of 72.



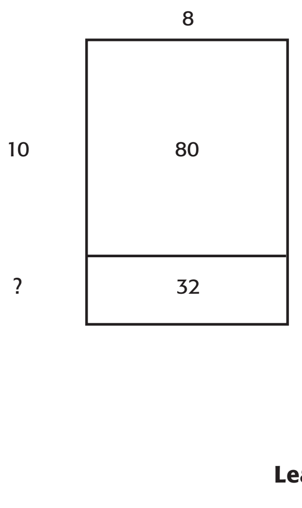
Teachers can also use open arrays to help students represent division situations where it is impractical to create an array in which every square or item within the array is indicated. Consider this problem.

> *"The organizing committee for a play day needs to organize 112 students into teams of 8. How many teams will there be?"*

Students can represent the problem using an open array.



The open array may not represent how students visualize the problem (i.e., how students will be organized into teams), and it does not provide an apparent solution to $112 \div 8$. The open array does, however, provide a tool with which students can reason their way to a solution. Students might realize that 10 teams of 8 would include 80 students but that another 32 students (the difference between 112 and 80) also need to be organized into teams of 8. By splitting the array into sections to show that 112 can be decomposed into 80 and 32, students can re-create the problem in another way.



The parts in the open array help students to determine the solution. Since $32 \div 8 = 4$ (although many students will likely think "$4 \times 8 = 32$"), students can determine that the number of teams will be $10 + 4$, or 14.

Initially, students use mathematical models, such as open arrays, to represent problem situations and their own mathematical thinking. With experience, students can also learn to use models as powerful tools to think with (Fosnot & Dolk, 2001). Appendix 4–1: Using Mathematical Models to Represent Division provides guidance to teachers on how they can help students use models as representations of mathematical situations, as representations of mathematical thinking, and as tools for learning.

Learning Basic Division Facts

A knowledge of basic division facts supports students in understanding division concepts and in carrying out mental computations and paper-and-pencil calculations. Because multiplication and division are related operations, students often use multiplication facts to answer corresponding division facts (e.g., $4 \times 6 = 24$, so $24 \div 4 = 6$).

The use of models and thinking strategies helps students to develop knowledge of basic facts in a meaningful way. Chapter 10 in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006* (Volume 5) provides practical ideas on ways to help students learn basic division facts.

Considering the Meaning of Remainders

The following problem was administered to a stratified sample of 45 000 students nationwide on a National Assessment of Educational Progress secondary mathematics exam.

> *"An army bus holds 36 soldiers. If 1128 soldiers are being bussed to their training site, how many buses are needed?"*

Seventy percent of the students completed the division computation correctly. However, in response to the question "How many buses are needed?", 29 percent of students answered "31 remainder 12"; 18 percent answered "31"; 23 percent answered "32", the correct response (Schoenfeld, 1987).

The preceding example illustrates the impact that a mathematics program focusing on learning algorithms can have on students' ability to interpret mathematical problems and their solutions. The example also highlights the importance of considering the meaning of remainders in division situations.

In a problem-solving approach to teaching and learning mathematics, students must consider the meaning of remainders within the context of the problem. Consider this problem.

"There are 11 players on a soccer team. 139 students signed up for an intramural soccer league. How many teams will there be?"

In solving the problem, students discover that there are 12 teams, and 7 extra players. The solution requires students to consider what can be done with the 7 additional players. Some students might distribute these players to 7 teams, whereas others might suggest smaller teams.

The following problem, which involves the same numbers as in the preceding situation but with a different context, requires students to think differently about the remainder.

"11 classmates purchased a painting for their teacher, who was moving to a new school. If the painting cost $139, how much did each classmate contribute for the gift?"

In this problem, students discover that each classmate contributes $12 but that the classmates are still short $7. Students would have to come up with a fair way to account for the shortfall.

Students can deal with remainders in division problems in several ways:

- The remainder can be discarded.

  "Alexandrea cuts 1 m of string into 30 cm pieces. How many pieces can she make?" (3 pieces, and the remaining 10 cm is discarded)

- The remainder can be partitioned into fractional pieces and distributed equally.

  "If 4 people share 5 loaves of bread, how much does each person get?" (1 and 1/4 loaves)

- The remainder can remain a quantity.

  "Six children share 125 beads. How many beads will each child get?" (20 beads, with 5 beads left over)

- The remainder can force the answer to the next highest whole number.

  "Josiah needs to package 80 cans of soup in boxes. Each box holds 12 cans. How many boxes does he need?" (7 boxes, but one box will not be full)

- The quotient can be rounded to the nearest whole number for an approximate answer.

  "Tara and her two brothers were given $25 to spend on dinner. About how much money does each person have to spend?" (about $8)

Presenting division problems in a variety of meaningful contexts encourages students to think about remainders and determine appropriate strategies for dealing with them.

Developing a Variety of Computational Strategies

Developing effective computational strategies for solving division problems is a goal of instruction in the junior grades. However, a premature introduction to a standard division algorithm does little to promote student understanding of the operation or of the meaning behind computational procedures. In classrooms where rote memorization of algorithmic steps is emphasized, student often make computational errors without understanding why they

are doing so. The following example illustrates an error made by a student who does not understand the division processes represented in an algorithm:

$$\begin{array}{r}
81 \quad R\,7 \\
9\,\overline{)\,716} \\
\underline{72} \\
16 \\
\underline{9} \\
7
\end{array}$$

The student constructs the algorithm in his own mind as, "Come as close to the number as you can, then subtract." Recalling multiplication facts, he knows that 9×8 is 72 (a product that is very close to 71) and subsequently subtracts incorrectly.

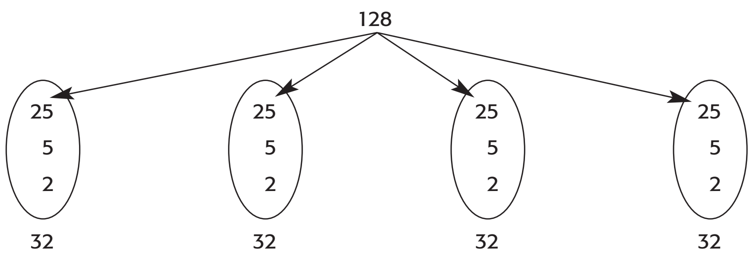
EARLY STRATEGIES FOR PARTITIVE DIVISION PROBLEMS

Students are able to solve division problems long before they are taught procedures for doing so. When students are presented with problems in meaningful contexts, they rely on strategies that they already understand to work towards a solution. In the primary grades, students often solve partitive division problems by dealing out or distributing concrete objects one by one. When students use this strategy to divide larger numbers, they realize that dealing out objects one by one can be cumbersome, and that it is difficult to represent large numbers using concrete materials.

In the junior grades, students learn to employ more sophisticated methods of fair sharing as they develop a greater understanding of ways in which numbers can be decomposed.

> *"Jamie's grandmother brought home 128 shells from her beach vacation. She wants to divide the shells equally among her 4 grandchildren. How many shells will each grandchild receive?"*

To solve this problem, students might first think of 128 as $100 + 28$. They realize that 100 is four 25's and begin by allocating 25 to each of 4 groups. Students might then distribute the remaining 28 by first allocating 5 and then 2 to each group, or they might recognize that 28 is a multiple of 4 ($4 \times 7 = 28$) and allocate 7 to each group. After distributing 128 equally to 4 groups, students solve the problem by recognizing that each grandchild will receive 32 shells. The following illustration shows how students might represent their strategy.



The strategy of decomposing the dividend into parts (e.g., decomposing 128 into $100 + 28$) and then dividing each part by the divisor is an application of the **distributive property**. According to the distributive property, division expressions, such as $128 \div 4$, can be split into smaller parts, for example, $(100 \div 4) + (28 \div 4)$. The sum of the **partial quotients** $(25 + 7)$ provides the answer to the division expression.

EARLY STRATEGIES FOR QUOTATIVE DIVISION PROBLEMS

Division is often referred to as "repeated subtraction" (e.g., $24 \div 6$ is the same as $24 - 6 - 6 - 6 - 6$). Although this interpretation of division is correct, students in the early stages of learning division strategies often use *repeated addition* to solve quotative problems. For many students, it makes more sense to start at zero and add up to the dividend.

> *"144 baseballs are placed in trays for storage. Each tray holds 24 balls. How many trays are needed?"*

To solve this problem, students might repeatedly add 24 until they get to 144, and then count the number of times they added 24 to determine the number of groups of 24, as shown at right.

$$\begin{array}{r} 24 \\ \underline{+\ 24} \\ 48 \\ \underline{+\ 24} \\ 72 \\ \underline{+\ 24} \\ 96 \\ \underline{+\ 24} \\ 120 \\ \underline{+\ 24} \\ 144 \end{array}$$

Students might also use repeated subtraction in a similar way. Beginning with 144, they continually subtract 24 until they get to 0, and then count the number of times they subtracted 24.

Students demonstrate a growing understanding of multiplicative relationships when they realize that they can add or subtract "chunks" (groups of groups), rather than adding or subtracting one group at a time.

> *"The library just received 56 new books. The librarian wants to create take-home book packs with 4 books in each pack. How many packs can he make?"*

Two methods, both involving "chunking", are illustrated in the following strategies. In the first example (on the left), a familiar fact, 5×4, is used to determine that 5 packs can be created with 20 books, and therefore 10 packs can be created with 40 books. Another fact, 2×4, is used to determine that there are 4 packs for the remaining 16 books. In the second example (on the right), the same multiplication facts help to determine quantities that can be subtracted from 56.

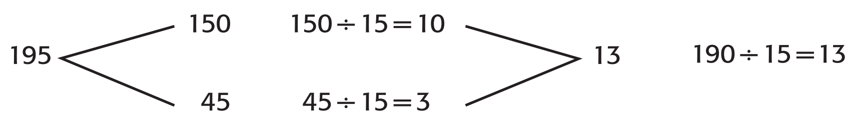
$$\begin{array}{rl} 20 & \text{(5 packs)} \\ 20 & \text{(5 packs)} \\ 8 & \text{(2 packs)} \\ 8 & \text{(2 packs)} \\ \multicolumn{2}{l}{20 + 20 + 8 + 8 = 56} \\ \multicolumn{2}{l}{\text{56 books} \rightarrow \text{14 packs}} \end{array}
\qquad\qquad
\begin{array}{rl} 56 & \\ \underline{-\ 20} & \text{(5 packs)} \\ 36 & \\ \underline{-\ 20} & \text{(5 packs)} \\ 16 & \\ \underline{-\ \ 8} & \text{(2 packs)} \\ 8 & \\ \underline{-\ \ 8} & \text{(2 packs)} \\ 0 & \\ & \text{14 packs} \end{array}$$

It is important to note that both methods make use of the distributive property. In the first example, 56 is decomposed into $(5 \times 4) + (5 \times 4) + (2 \times 4) + (2 \times 4)$. In the second example, the number of 4's is found by decomposing $56 \div 4$ into $(20 \div 4) + (20 \div 4) + (8 \div 4) + (8 \div 4)$. Providing opportunities for students to explore informal division strategies (which are often based on the distributive property) prepares students for understanding more formal methods and algorithms.

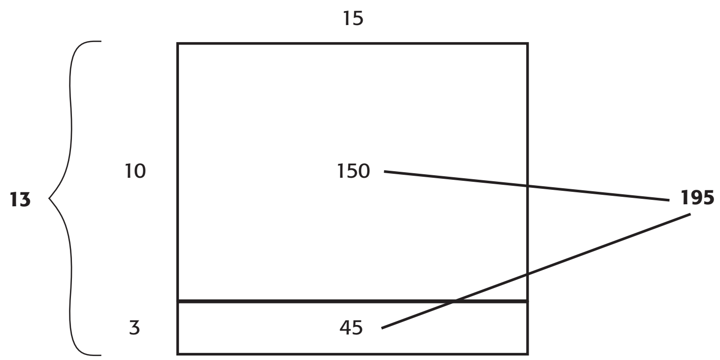
DEVELOPING AN UNDERSTANDING OF THE DISTRIBUTIVE PROPERTY

The distributive property is the basis for a variety of division strategies, including the standard algorithm. An understanding of how the property can be applied in division allows students to develop flexible and meaningful strategies, and helps bring meaning to the steps involved in algorithms.

Consider the division expression $195 \div 15$. When instruction focuses on the algorithmic steps, students are taught to figure out how many times 15 "goes into" 19, despite the fact that 19 is really 190. A deeper understanding of the distributive property allows students to rework the problem into friendly numbers: 190 can be decomposed into $150 + 45$, and each part can be divided by 15.



Students can use an open array to model the strategy.



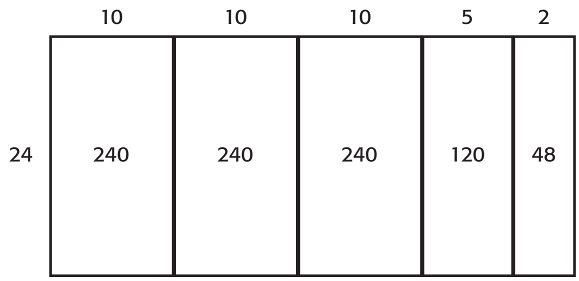
There is significant flexibility in using the distributive property to solve division problems. For example, the preceding division expression could have been calculated by decomposing 195 into 75 and 120, then dividing $75 \div 15$ and $120 \div 15$, and then adding the partial quotients $(5 + 8)$. However, strategies that use the distributive property are most effective when division expressions can be broken into friendly numbers and are easy to compute. For example, $150 \div 15$ and $45 \div 15$ are generally easier to compute mentally than $75 \div 15$ and $120 \div 15$ are.

Students learn that facts involving 10 × and 100 × are helpful when using the distributive property. To solve 889 ÷ 24, for example, students might take a “stepped” approach to decomposing 889 into groups of 24.

$$
\begin{array}{r}
24 \times 10 = 240 \\
24 \times 10 = 240 \\
24 \times 10 = 240 \\
24 \times 5 = 120 \\
24 \times \underline{2} = \underline{\ \ 48} \\
37 \quad 888
\end{array}
$$

Students calculate that 37 groups of 24 is 888, and therefore the solution is 889 ÷ 24 = 37 R1.

The strategy can be illustrated by using an open array.



When division involves large numbers, informal strategies make it difficult for students to keep track of numerical operations. In these situations, algorithms become useful to help students record and keep track of the multiple steps and operations in division.

DEVELOPING AN UNDERSTANDING OF FLEXIBLE DIVISION ALGORITHMS

Flexible division algorithms, like the standard algorithm, are based on the distributive property. With flexible algorithms, however, students use known multiplication facts to decompose the dividend into friendly “pieces”, and repeatedly subtract those parts from the whole until no multiples of the divisor are left. Students keep track of the pieces as they are “removed”, which is illustrated in the two examples below.

$$
\begin{array}{r|r}
17\,\big)\ \ 387 & \\
\underline{-\ 170} & 10 \\
217 & \\
\underline{-\ 170} & 10 \\
47 & \\
\underline{-\ 34} & \underline{2} \\
13 & 22
\end{array}
$$

387 ÷ 17 = 22 R13

$$
\begin{array}{r|r}
26\,\big)\ \ 5562 & \\
\underline{-\ 2600} & 100 \\
2962 & \\
\underline{-\ 2600} & 100 \\
362 & \\
\underline{-\ 260} & 10 \\
102 & \\
\underline{-\ 52} & 2 \\
50 & \\
\underline{-\ 26} & \underline{1} \\
24 & 213
\end{array}
$$

5562 ÷ 26 = 213 R24

A student who is using a flexible algorithm to solve the first example, $387 \div 17$, might reason as follows:

> *"I need to divide 387 into groups of 17. How many groups can I make? I know I can get at least 10 groups. That's 170, and if I remove that, I have 217 left. Another 10 groups would leave me with 47. I can get 2 groups from that, so I can take off another 34. That leaves me with 13, which isn't enough for another group. So altogether, I made $10 + 10 + 2 = 22$ groups, and have 13 left."*

As students become more comfortable multiplying and dividing by multiples of 10, they learn to compute using fewer partial quotients in the algorithm, as illustrated below:

$$
\begin{array}{r|r}
17\,\overline{)\;387} & \\
-\,340 & 20 \\
\hline
47 & \\
-\,34 & 2 \\
\hline
13 & 22
\end{array}
$$

$$387 \div 17 = 22 \text{ R}13$$

DEVELOPING AN UNDERSTANDING OF THE STANDARD DIVISION ALGORITHM

> *Historically the algorithms (standardized steps for calculation) were created to be used for efficiency by a small group of "human calculators" when calculators were not yet invented. They were not designed to support the sense making that is now expected from students.*
>
> (*Teaching and Learning Mathematics in Grades 4 to 6 in Ontario, 2004*, p. 12)

Although the standard division algorithm provides an efficient computational method, the steps in the algorithm can be very confusing for students if they have not had opportunities to solve division problems using their own strategies and methods.

Working with flexible division algorithms can prepare students for understanding the standard algorithm. A version of the flexible division algorithm involves stacking the quotients above the algorithm (rather than down the side, as demonstrated in the above example). The following example shows how the parts in the flexible algorithm can be connected to the recording method used in the standard algorithm.

$$
\begin{array}{r}
1 \\
25 \\
100 \\
100 \\
4\,\overline{)\,904} \\
-\,400 \\
\hline
504 \\
-\,400 \\
\hline
104 \\
-\,100 \\
\hline
4 \\
-\,4 \\
\hline
0
\end{array}
\qquad\qquad
\begin{array}{r}
226 \\
4\,\overline{)\,904} \\
-\,8 \\
\hline
10 \\
-\,8 \\
\hline
24 \\
-\,24 \\
\hline
0
\end{array}
$$

Developing Estimation Strategies for Division

Students need to develop effective estimation strategies, and they also need to be aware of when one strategy is more appropriate than another. It is important for students to consider the context of a problem before selecting an estimation strategy. Students should also decide beforehand how accurate their estimation needs to be. Consider the following problem.

> *"Ms. Wu's class is putting cans in boxes for the annual canned-food drive. They have 188 cans and put approximately 20 cans in a box. About how many boxes do they need?"*

In this problem situation, it is useful to use an estimation strategy that results in enough boxes to package all the cans (e.g., round 188 to 200 and divide by 20 to get 10 boxes).

The following table outlines different estimation strategies for division. It is important to note that the word "rounding" is used loosely – it does not refer to any set of rules or procedures for rounding numbers (e.g., look to the number on the right; is it greater than 5? . . .).

Strategy	Example
Rounding the dividend and/or divisor to the nearest multiple of 10, 100, 1000, . . .	$442 \div 50$ is about $450 \div 50 = 9$ $785 \div 71$ is about $800 \div 80 = 10$
Finding friendly numbers	$318 \div 23$ is about $325 \div 25 = 13$
Rounding the dividend up or down and adjusting the divisor accordingly	$237 \div 11$ is about $240 \div 12 = 20$ $237 \div 11$ is about $230 \div 10 = 23$
Using front-end estimation (Note that this strategy is less accurate with division than with addition and subtraction.)	$453 \div 27$ is about $400 \div 20 = 20$ (actual answer is 16 R21)
Finding a range (by rounding both numbers down, then up)	$565 \div 24$ is about $500 \div 20 = 25$ $565 \div 24$ is about $600 \div 30 = 20$ The quotient is between 20 and 25.

A Summary of General Instructional Strategies

Students in the junior grades benefit from the following instructional strategies:

- experiencing a variety of division problems, including partitive and quotative problems;
- using concrete and pictorial models to represent mathematical situations, to represent mathematical thinking, and to use as tools for new learning;
- solving division problems that serve different instructional purposes (e.g., to introduce new concepts, to learn a particular strategy, to consolidate ideas);
- providing opportunities to develop and practise mental computation and estimation strategies;
- providing opportunities to connect division to multiplication through problem solving.

The Grades 4–6 Multiplication and Division module at www.eworkshop.on.ca provides additional information on developing division concepts with students. The module also contains a variety of learning activities and teaching resources.

eworkshop.on.ca

APPENDIX 4-1: USING MATHEMATICAL MODELS TO REPRESENT DIVISION

The Importance of Mathematical Models

Models are concrete and pictorial representations of mathematical ideas, and their use is critical in order for students to make sense of mathematics. At an early age, students use models such as counters to represent objects and tally marks to keep a running count.

Standard mathematical models, such as number lines and arrays, have been developed over time and are useful as “pictures” of generalized ideas. In the junior grades, it is important for teachers to develop students’ understanding of a variety of models so that models can be used as tools for learning.

The development in understanding a mathematical model follows a three-phase continuum:

- **Using a model to represent a mathematical situation:** Students use a model to represent a mathematical problem. The model provides a “picture” of the situation.
- **Using a model to represent student thinking:** After students have discussed a mathematical idea, the teacher presents a model that represents students’ thinking.
- **Using a model as a tool for new learning:** Students have a strong understanding of the model and are able to apply it in new learning situations.

An understanding of mathematical models takes time to develop. A teacher may be able to take his or her class through only the first or second phase of a particular model over the course of a school year. In other cases, students may quickly come to understand how the model can be used to represent mathematical situations, and a teacher may be able to take a model to the third phase with his or her class.

USING A MODEL TO REPRESENT A MATHEMATICAL SITUATION

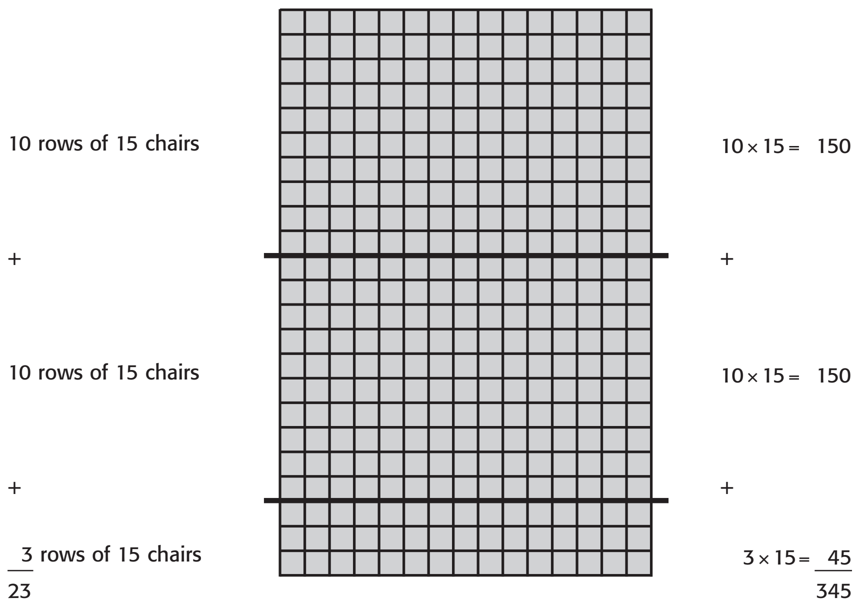
A well-crafted problem can lead students to use a mathematical model that the teacher would like to highlight. The following example illustrates how the use of an array as a model for division might be introduced.

A teacher provides students with the following problem:

> *"Students in the primary division are putting on a concert, and the principal has asked our class to set up chairs in the gym for parents and guests. We have 345 chairs, and the principal wants rows with 15 chairs in each row. How many rows do we need to set up?"*

The teacher has purposefully selected the numbers in the problem: they are friendly (easy to work with) but large enough to prevent quick solutions. They are also too large for students to use counters or other manipulatives, and repeated subtraction or repeated addition would be inefficient strategies. To this point, the class has not been taught any formal algorithms for division by a two-digit number.

The problem also lends itself to the use of an array. Although some students attempt to solve the problem using only numerical calculations, others use drawings to recreate the situation. One student uses grid paper, with each square representing a chair:



The student explains her strategy:

> *"I started drawing rows of 15 chairs. I knew that 10 rows would have 150 chairs, because I know that 15 × 10 = 150. So I drew a line around 10 rows, and wrote 150. Another 10 rows would give me another 150, for a total of 300 chairs. That just left 45 chairs, which is 3 rows of 15. So I know we'd have 10 rows plus 10 rows plus 3 rows, for a total of 23 rows."*

This student used an array to model the rows of chairs. Although not all students used this model, the teacher is able to draw attention to it during the Reflecting and Connecting part of the lesson. The student, having no formal strategy for dividing by two-digit numbers, has used an array to represent 345 chairs in rows of 15, and then has broken the array into parts to determine the total number of rows.

The student has also used another important idea in division – making groups of tens.

Not all students used the array as a model to represent the mathematical situation, and there is no guarantee that students who did use it can or will apply it to other division problems. It is the teacher's role to help students generalize the use of the array as a model in other division situations.

USING A MODEL TO REPRESENT STUDENT THINKING

Teachers can guide students in recognizing how models can represent mathematical thinking. The following example provides an illustration.

After solving problems in which the class used arrays to represent division situations, a teacher presents the following problem:

> *"My neighbour is a potter and is well known for her unique coffee mugs. She sells them to kitchen stores in sets of 12, in special boxes that protect the mugs during shipping. Yesterday, a store placed an order for 288 mugs. She needs to know how many boxes she needs to ship the mugs to the store."*

The teacher encourages students to use strategies that make sense to them, and suggests that they use concrete materials and diagrams to help them understand and solve the problem.

One student solved the problem mentally, recording the results of his mental calculations on paper as he worked through the problem.

$288 \div 12$ . . . how many groups of 12?

10 groups is 120
20 groups is 240 $\quad 288 - 240 = 48$
$\quad\quad 48 \div 12 = 4$

So, 20 groups + 4 groups is 24 groups

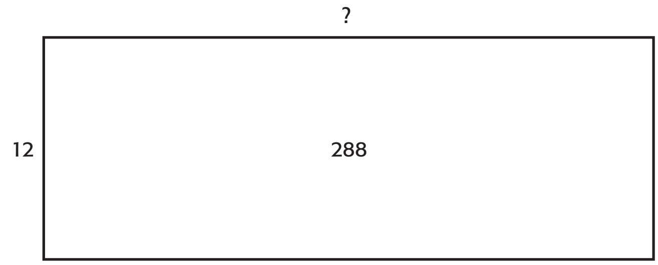
Check:
$$\begin{array}{r} 24 \\ \times\ 12 \\ \hline 48 \\ 240 \\ \hline 288 \end{array}$$

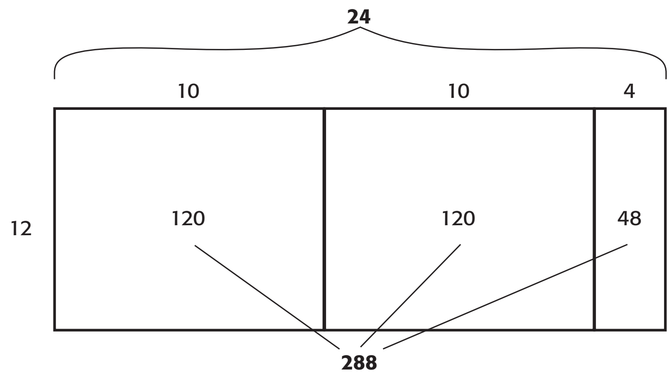
The teacher, wanting to highlight the student's strategy with the class, asks the student to explain his work. The student explains:

> *"I figured out that the problem is finding how many groups of 12 there are in 288. I started thinking about numbers I knew. I knew 10 groups would be 120 mugs, and another 10 is 240. I subtracted 240 from 288 and had 48 left. That's 4 more groups of 12, so in total I had (20 + 4) 24 groups. I checked by multiplying 24 × 12 and got 288."*

Although the student did not use an array model to solve the problem, his teacher presents an open array to the class to help students visualize their classmate's thinking. The solution is represented through a series of diagrams.

> *"If we think about the problem as an array, then the area of the array is 288, and the length of one side is 12. We need to find the length of the other side. You solved the problem by breaking 288 into friendlier numbers: 120 + 120 + 48."*





In this case, the array is used to model a strategy in which partial quotients are determined by using friendly numbers that are multiples of 10. The dividend has been decomposed into numbers that are easier to work with.

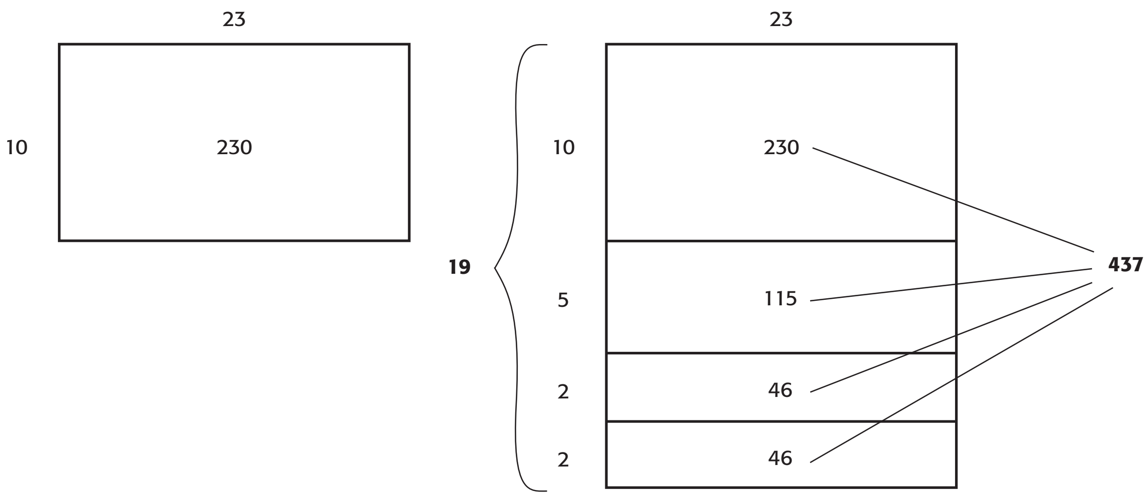
The teacher has provided a visual representation of a student solution that makes the strategy more accessible to other students in the class, and has built upon students' understanding of the array model. With meaningful practice rooted in contextual problems, the open array model can become a useful tool for dividing numbers.

USING A MODEL AS A TOOL FOR NEW LEARNING

To help students generalize the use of an open array as a model for division, and to help them recognize its utility as a tool for learning, teachers need to provide problems that allow students to apply and extend the strategy of partial quotients. A sample problem comes out of a fundraising scenario.

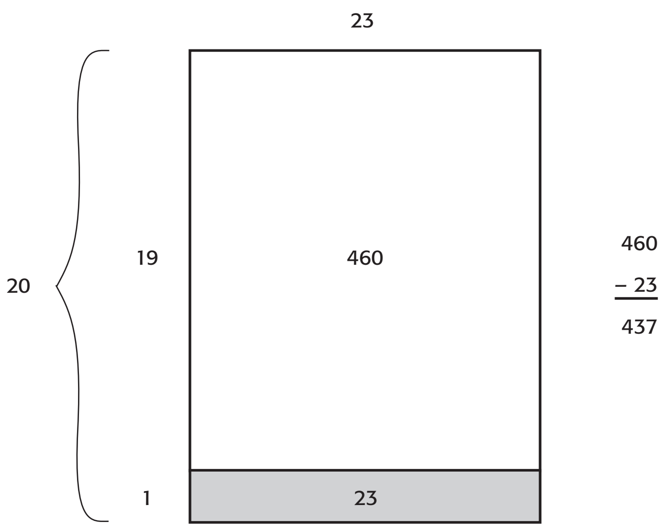
> *"23 students raised $437 for the United Way. If each student brought in the same amount of money, how much did each student raise?"*

The numbers in the problem were chosen to be challenging, but they also allow for various strategies to find a solution. Many students use a strategy that involves determining partial quotients by decomposing 437. To begin, they recognize that $10 \times 23 = 230$ and draw an open array to represent this idea. They continue to multiply 23 by other factors, drawing other sections on their diagram until 437 has been accounted for.



"I kept multiplying 23 by friendly numbers. I started with 10, and got 230. I tried another 10, but that would have given me 460, which is too much. So I timesed by 5, which was easy because it was half of 10. I kept going that way, trying numbers that fit. Each time I tried a new number, I had to take it away from 437 to find out how much was left. It ended up that each student raised $19."

Another student started down a similar path but used the distributive property and subtraction instead of addition.



"I knew that 20 times 23 is 460, which was more than I needed. I subtracted 437 from 460, and found the difference was 23. So that's 1 group of 23 less, or 19 groups. So 437÷23=19."

In both cases, students used an open array as a tool for solving a division problem. The first student used a strategy that replicates a solution modelled by the teacher, but the second

student used the model to come up with a *compensation* strategy. The second student used the open array model as a tool for solving a division problem in a new way.

When developing a model for division, it is practical to assume that not all students will come to understand or use the model with the same degree of effectiveness. Teachers should continue to develop meaningful problems that allow students to use strategies that make sense to them. However, part of the teacher's role is to use models to represent students' ideas so that these models will eventually become thinking tools for students. The ability to generalize a model and use it as a learning tool takes time (possibly years) to develop.

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Learning Activities for Division

Introduction

The following learning activities for Grades 4, 5, and 6 provide teachers with instructional ideas that help students achieve some of the curriculum expectations related to division. The learning activities also support students in developing their understanding of the big ideas outlined in Volume 1: The Big Ideas.

The learning activities do not address all concepts and skills outlined in the curriculum document, nor do they address all the big ideas – one activity cannot fully address all concepts, skills, and big ideas. The learning activities demonstrate how teachers can introduce or extend mathematical concepts; however, students need multiple experiences with these concepts to develop a strong understanding.

Each learning activity is organized as follows:

OVERVIEW: A brief summary of the learning activity is provided.

BIG IDEAS: The big ideas that are addressed in the learning activity are identified. The ways in which the learning activity addresses these big ideas are explained.

CURRICULUM EXPECTATIONS: The curriculum expectations are indicated for each learning activity.

ABOUT THE LEARNING ACTIVITY: This section provides guidance to teachers about the approximate time required for the main part of the learning activity, as well as the materials, math language, instructional groupings, and instructional sequencing for the learning activity.

ABOUT THE MATH: Background information is provided about the mathematical concepts and skills addressed in the learning activity.

GETTING STARTED: This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or activity.

WORKING ON IT: In this part, students work on the mathematical activity, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.

REFLECTING AND CONNECTING: This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.

ADAPTATIONS/EXTENSIONS: These are suggestions for ways to meet the needs of all learners in the classroom.

ASSESSMENT: This section provides guidance for teachers on assessing students' understanding of mathematical concepts.

HOME CONNECTION: This section is addressed to parents or guardians, and includes an activity for students to do at home that is connected to the mathematical focus of the main learning activity.

LEARNING CONNECTIONS: These are suggestions for follow-up activities that either extend the mathematical focus of the learning activity or build on other concepts related to the topic of instruction.

BLACKLINE MASTERS: These pages are referred to and used throughout the learning activities.

Grade 4 Learning Activity
Intramural Dilemmas

OVERVIEW

In this learning activity, students solve problems by dividing groups of students into teams of equal size. The focus in this activity is on solving different kinds of division problems (e.g., partitive, quotative) and on having students use meaningful strategies. Allowing students to develop and apply their own strategies helps them develop an understanding of division situations, and of flexible approaches for solving division problems.

BIG IDEAS

This learning activity focuses on the following big ideas:

Operational sense: Students solve division problems by using strategies that make sense to them. They discover that solutions to division problems sometimes involve a remainder and that the remainder must be dealt with within the context of the problem.

Relationships: By solving division problems, students explore the relationship involving a quantity, the number of groups the quantity can be divided into, and the size of each group. They also explore the relationships between the operations, particularly between division and repeated subtraction.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectation**.

Students will:

- divide two-digit whole numbers by one-digit whole numbers, using a variety of tools (e.g., concrete materials, drawings) and student-generated algorithms.

This specific expectation contributes to the development of the following **overall expectation**.

Students will:

- solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies.

TIME: approximately two 60-minute periods

INSTRUCTIONAL GROUPING: pairs

ABOUT THE LEARNING ACTIVITY

MATERIALS

- a variety of manipulatives (e.g., counters, base ten blocks, square tiles)
- sheets of chart paper or large sheets of newsprint (1 per pair of students)
- markers (a few per pair of students)
- **Div4.BLM1: Intramural Dilemmas** (1 per student)
- **Div4.BLM2: The Remainders Game** (1 per student)

MATH LANGUAGE

- division
- divide
- dividend
- divisor
- quotient
- divisible
- remainder

INSTRUCTIONAL SEQUENCING

This learning activity provides an introductory exploration of strategies for dividing a two-digit number by a one-digit number. Before starting this learning activity, students should have had experiences in dividing quantities into equal-sized groups.

ABOUT THE MATH

Students develop an understanding of division concepts when they solve problems that involve separating a quantity into equal groups. *Partitive division* involves situations in which the quantity is separated into a specified number of equivalent groups. The quotient indicates the number of items in each group.

EXAMPLE OF A PARTITIVE DIVISION PROBLEM

A grocer has 30 apples. He puts the apples into 5 bags so that each bag contains the same number of apples. How many apples will the grocer put in each bag?

Quotative division involves situations in which a quantity is separated into groups of a specified size. The quotient refers to the number of groups that can be formed.

EXAMPLE OF A QUOTATIVE DIVISION PROBLEM

A grocer has 30 apples. She wants to put 5 apples in each bag. How many bags will the grocer need?

It is not important that students be able to identify division problems as either partitive or quotative. It is important, however, to provide opportunities for students to solve both kinds of problems and deepen their understanding of division concepts.

It is also essential to allow students to solve problems in ways that make sense to them, so that they can construct a meaningful understanding of division and its relationship to other operations. Students' solution strategies vary in sophistication – some students use simple counting strategies, while others apply their understanding of number strategies, such as using repeated subtraction, using multiplying as the inverse of dividing, or using basic division facts.

GETTING STARTED

Explain to students that it is often necessary to divide a class into equal-sized groups for games, sports, and learning activities. Display the following problems on the board or on chart paper, and discuss them with students:

- We want to play checkers. How many groups will there be if we divide our class into groups of 2?
- We want to play soccer. How many students will there be on each team if we divide our class into 2 equal groups?

Ask students to work with a partner to solve each problem by using a strategy that makes sense to them. Suggest that students use manipulatives (e.g., counters, base ten blocks, square tiles) and/or diagrams to help them determine a solution.

Provide each pair of students with markers and a sheet of chart paper or large sheet of newsprint. Ask them to fold the paper in half and record a solution for each problem on each part of the paper. Ask each pair to be prepared to share their strategies and solutions with their classmates.

Watch and listen to students as they work on the problems, observe the various strategies being used, and provide guidance when necessary. If some students finish before others, encourage them to find other ways to solve the problems.

Note: If there is an odd number of students in the class, students will need to discuss how the extra person can be included in the groups (e.g., forming one group of three for the checker games or having a soccer team with an extra player).

STRATEGIES STUDENTS MIGHT USE

COUNTING

Students might use manipulatives (e.g., counters, base ten blocks, square tiles) to represent the students in the class. To solve the first problem, they might count out the number of manipulatives that corresponds to the number of students in the class, then arrange the manipulatives into groups of two, and then count the number of groups. For the second problem, students could divvy the manipulatives into two equal groups and count the number of manipulatives in each group.

(continued)

USING REPEATED SUBTRACTION

Students might use repeated subtraction to solve the first problem. If there are 26 students in the class, they might subtract 2 from 26, and then continue to subtract 2 from the remaining difference. Finally, students would determine that there were 13 groups of two by counting the number of times they subtracted 2.

The second problem does not lend itself to a process of repeated subtraction. Instead, students might use a halving process (e.g., recognizing that one half of 26 is 13).

$$\begin{array}{r} 26 \\ -\ 2 \\ \hline 24 \\ -\ 2 \\ \hline 22 \\ -\ 2 \\ \hline 20 \\ -\ 2 \\ \hline \end{array}$$

and so on

DECOMPOSING A NUMBER INTO PARTS

Students can determine the number of groups in the first problem by decomposing the number of students into tens and ones. For example, if there are 26 students, they might break 26 into 2 tens and 6 ones. There are 5 pairs in each group of 10 students and 3 pairs in the group of 6 students.

10 students → 5 pairs

10 students → 5 pairs

6 students → 3 pairs

13 pairs altogether

Gather students together after they have had sufficient time to solve the problems. Select a few pairs to present and discuss their solutions, choosing students who used different strategies.

During students' presentations, avoid making comments that suggest that some strategies are better than others. Instead, encourage students to consider the effectiveness and efficiency of each strategy by asking the following questions after each presentation:

- "Was it easy to find a solution using your strategy?"
- "What worked well?"
- "What did not work well?"
- "How would you change your strategy if you solved the problem again?"

WORKING ON IT

Explain to students that their help is needed in solving a problem about organizing intramural teams. Present the context for the problem:

> "Seventy-eight students signed up for intramural sports. All the students will play both soccer and four-square. Ms. Boswell [*Note:* You might want to use the name of a teacher in your school who is involved in organizing intramural teams] would like to create the different teams and then make a chart with the names for each team. The chart will allow students to see which teams they belong to. But first of all, Ms. Boswell needs to solve some problems."

Present the following problems on the board or on chart paper:

- There will be 4 soccer teams. How many players will there be on each team?
- There are 4 players on each four-square team. How many teams will there be?

Ask students to work with a partner. (You might decide to have students work with the same partner as in Getting Started, or you might form different pairs.) Explain that students may solve the problems in any order.

Encourage students to consider whether any of the various strategies that were demonstrated earlier could help them solve the problems. Encourage them, as well, to modify any of the strategies or to develop new ones. Ensure that manipulatives (e.g., counters, base ten blocks, square tiles) are available, and invite students to use them.

Provide each student with a copy of **Div4.BLM1: Intramural Dilemmas**. Explain that although they are working in pairs, each student is responsible for recording solutions to the problems. Remind students to think about ways to use words, symbols, and/or diagrams to explain their ideas.

REFLECTING AND CONNECTING

After students have solved the problems, use an inside-outside circle strategy (described below) so that they can share their solutions with each other. Have students use their completed copy of **Div4.BLM1: Intramural Dilemmas**, and organize them for the activity:

- Divide the class into two equal groups.
- Ask one group to form a circle with students facing outwards, away from the centre of the circle.
- Ask the other group to form another circle around the first circle. Each student in the inside circle faces a partner in the outside circle.

To begin the activity, ask students to discuss with their partner how they determined the number of players on each soccer team. Encourage students to refer to their work on **Div4.BLM1: Intramural Dilemmas** as they explain their strategies. Remind students to be courteous by allowing time for their partners to present their ideas, by listening attentively, and by making positive comments (e.g., "I think you had a clever idea!"). Allow three to four minutes for students to share their strategies for this problem.

Next, ask the outside circle to move counterclockwise by three people, and have students share their strategies and solutions for the soccer teams problem with their new partners.

Ask the inside circle to move counterclockwise by four people. Have students, with their new partners, discuss the strategies they used to determine the number of four-square teams. Again, allow three to four minutes for students to share their strategies.

Conduct another rotation to provide an opportunity for students to share their strategies and solutions for the four-square teams problem with another partner.

Following the inside-outside circle activity, discuss the problems, one at a time, with the entire group by asking the following questions.

QUESTIONS FOR THE SOCCER TEAMS PROBLEM

- "How many players will there be on each soccer team?"
- "Was there a remainder (a leftover quantity) when you divided 78 by 4? What does this leftover quantity represent? How did your solution include these 2 extra students?"
- "Is it reasonable to have 19 (or 20, if the leftover students are placed on teams) players on each team?" (Students might suggest that a team of 19 players is too large and that some players would have little opportunity to play.)
- "If a team of 19 players is too large, what would you recommend to the teacher?"

QUESTIONS FOR THE FOUR-SQUARE TEAMS PROBLEM

- "How many four-square teams will there be?"
- "Was there a remainder when you solved the problem? What does this leftover quantity represent? How did your solution include these 2 extra students?"
- "Do you think that it is appropriate to have 19 four-square teams? Why or why not?"

Ask a few students to explain their strategies to the entire group. Select different strategies, and emphasize the idea that a variety of strategies are possible. Encourage students to think about the efficiency of different strategies by asking the following questions:

- "Which strategies work well? Why?"
- "Which strategy makes the most sense to you? Why?"
- "Which strategies are similar? How are they alike?"
- "How could you change a strategy to make it more efficient?"

Next, focus on the relationships between the problems and between their solutions. Ask: "Which problems are similar?" Students might observe that the problems are alike because 4 is the divisor in both situations. Discuss the differences between the problems (e.g., in the soccer teams problem, the number of teams was known, but the size of each team was unknown; in the four-square teams problem, the size of each team was known, but the number of teams was unknown).

ADAPTATIONS/EXTENSIONS

Some students may need to work with smaller numbers. Rather than dealing with problems that involve 78 students, they could determine how 24 students could be arranged on teams. Encourage these students to use manipulatives (e.g., counters, cubes, square tiles) to represent and model the problems.

Extend the activity for students requiring a greater challenge by asking them to determine different ways in which 78 students could be divided exactly into equal teams (i.e., with no remainders).

ASSESSMENT

Observe students while they are solving the problems. Assess how well they are able to determine and apply a strategy that allows them to solve the problems effectively. Ask questions such as the following:

- "How are you solving these problems?"
- "What is working well with your strategy?"

- "What is not working well?"
- "Did you change your strategy? If yes, how?"
- "How are the problems the same?"
- "How are the problems different?"
- "How are you recording your solution?"
- "How do you know that your solution will be understood by others?"
- "Did solving one problem help you solve another? If yes, how?"

Provide the following problems for students to solve individually:

- Suppose we need to divide our class into 3 groups for science activities. How many students would there be in each group?
- Suppose we need to divide our class into groups of 3 for a rock-paper-scissors challenge. How many groups would there be?

Observe completed solutions to assess how well students:

- apply an appropriate strategy to solve the problems;
- use an efficient strategy;
- explain their strategies;
- recognize and apply the relationship between the problems.

HOME CONNECTION

Send home **Div4.BLM2: The Remainders Game**. This Home Connection activity provides an opportunity for students to observe that reminders often occur when a set of items is divided into equal groups.

LEARNING CONNECTION 1
Divide and Draw

MATERIALS

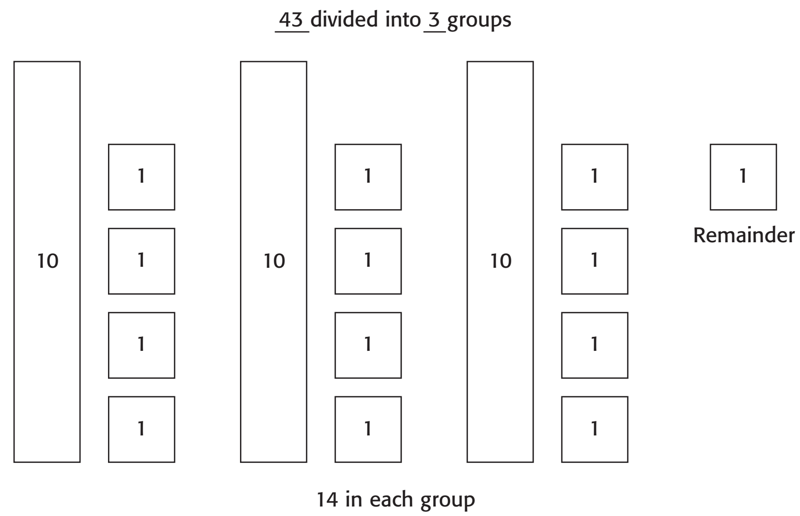
- base ten blocks, including ones cubes and tens rods (10 small cubes and 10 rods for each pair of students)
- paper bag labelled "ones", containing number cards for 1 to 9 (1 per pair of students)
- paper bag labelled "tens", containing number cards for 1 to 9 (1 per pair of students)
- six-sided number cubes (1 per pair of students)
- **Div4.BLM3: Divide and Draw** (1 per student)

Arrange students into pairs. Explain the activity:

- The first student draws a number card from both the "tens" and the "ones" bags, and selects the corresponding number of tens rods and ones cubes.
- The second student rolls the number cube to determine the number of groups into which the base ten blocks are to be divided.
- Students work together to divide the blocks into equal groups, exchanging tens rods for ones cubes when necessary.

- After students have divided the blocks into groups, they record the results on **Div4.BLM3: Divide and Draw** using diagrams and symbols. (Students can use blank sheets of paper if they complete all sections of **Div4.BLM3: Divide and Draw**.)

Show students how they might record 43 divided by 3.



As students participate in the activity, ask them to explain their strategies for dividing the base ten blocks. Observe whether students divide tens before ones, or vice versa. (Both methods are acceptable.)

You can modify the activity by having students work to solve quotative problems. As before, students draw "tens" and "ones" cards, and represent the number using base ten blocks. In this modified activity, the number cube indicates the number in each group, and students need to divide the blocks to determine the number of groups.

LEARNING CONNECTION 2
Decisions, Decisions

MATERIALS

- six-sided number cubes (3 per pair of students)
- variety of manipulatives (e.g., counters, square tiles, base ten blocks)

Have students play this game with a partner. Explain that the goal of the game is to be the player who creates the division sentence with the greater quotient.

For each round of the game, both players record the following empty division expression on a sheet of paper.

_____ _____ ÷ _____

To begin, the first player rolls three number cubes and records each number in one of the blanks in the empty division expression. Once a number is recorded, it cannot be moved. The second player follows the same procedure to create a division expression on his or her paper.

Next, players determine the solutions to their division expressions using any strategy (e.g., using manipulatives, using basic facts, drawing a diagram, using paper-and-pencil calculations), and record the answer on their paper.

Example:
Player A rolls 6, 3, and 5 and completes the division expression in the following way:

$$\underline{\ 5\ }\ \underline{\ 6\ } \div \underline{\ 3\ } = 18 \text{ R2}$$

Player B rolls 4, 4, and 6 and completes the division expression in the following way:

$$\underline{\ 6\ }\ \underline{\ 4\ } \div \underline{\ 4\ } = 16$$

Players check each other's work. Players earn 2 points if they get the correct answer. As well, the player with the greater quotient earns 5 points.

The player with the greater quotient begins the next round.

The first player to earn 50 points wins the game.

After students have played the game, discuss the strategies they used to determine the greatest possible quotient.

Variations of the game can involve:
- using a 10-sided number cube with numbers 0 to 9;
- trying to create the lesser quotient;
- trying to create the greater remainder;
- trying to create the lesser remainder.

LEARNING CONNECTION 3
Fair Shares

MATERIALS
- paper bag containing between 20 and 40 square tiles (1 bag per group of 4 students)

Divide students into groups of four. Provide each group with a bag of square tiles. Ask students to pour the tiles onto their desks and to count the tiles. Have them print this number on the bag.

Next, ask students to discuss the following question with their group members: "Can you share the tiles fairly between 2 of the group members and have no remainders?" After students have discussed the question, invite them to check their prediction by divvying the tiles between 2 students.

Next, ask: "Can you share the tiles fairly among 3 of the group members and have no remainders?" Have students discuss the question with group members before checking their predictions.

Finally, ask students to discuss whether there will be a remainder if the tiles are shared among 4 group members, and have them verify their conjectures.

Have groups exchange bags and repeat the activity.

As a whole class, discuss the following questions:
- "How did you predict whether or not there would be a remainder?"
- "When was it easy to predict that there would be no remainder? Why?"
- "Which number of tiles could be shared the greatest or the least number of ways? Why do you think this?"

As an extension, ask students to predict whether remainders would occur if the tiles in their bag were shared among 5 students, 6 students, and 7 students.

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on division concepts. On the homepage, click "Toolkit". In the "Numeracy" section, find "Multiplication and Division (4 to 6)", and then click the number to the right of it.

eworkshop.on.ca

Intramural Dilemmas

There are 78 students who signed up for intramural sports.

There are 4 soccer teams. How many players are there on each team?
There are 4 players on each four-square team. How many teams are there?

The Remainders Game

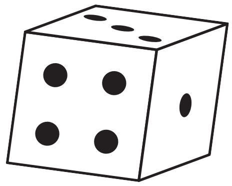
Dear Parent/Guardian:

We have been learning to solve division problems. Sometimes there are remainders in a division problem. If you take 23 buttons and divide them into 4 equal groups, there will be 5 buttons in each group and 3 leftover buttons.

Play the Remainders Game with your child, to help him or her practise making groups with equal amounts and determining whether there are any remainders.

Thank you for doing this activity with your child.

THE REMAINDERS GAME



Play this game with a partner. You will need a number cube (die) and 20 small objects (e.g., buttons, paper clips, pennies, pieces of paper).

To begin, a player rolls the number cube. The player needs to divide the 20 small objects into the number of groups shown on the number cube. For example, if a player rolls a 3, the player divides the 20 small objects into 3 groups.

If there is a remainder (there would be 2 leftover objects in the previous example), the player records the number of remaining objects on a piece of paper.

Now the turn passes to the other player who rolls the number cube, makes equal groups, and records the remainder on his or her paper.

After each player has had 10 turns, the players add up the remainders on their piece of paper. The player with the greater total wins the game.

Divide and Draw

________ divided into ______ groups

________ divided into ______ groups

________ divided into ______ groups

Grade 5 Learning Activity
Family Math Night

OVERVIEW

In this learning activity, students solve a problem by determining the number of tables that are needed for 165 people if 6 people sit at each table. The focus in this learning activity is on having students use strategies that make sense to them, rather than on applying learned procedures.

BIG IDEAS

This learning activity focuses on the following big ideas:

Operational sense: Allowing students to develop and apply their own strategies helps them develop an understanding of division situations and of flexible approaches for solving division problems.

Relationships: The learning activity provides an opportunity for students to explore the relationship involving a quantity, the number of groups the quantity can be divided into, and the size of each group.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- divide three-digit whole numbers by one-digit whole numbers, using concrete materials, estimation, student-generated algorithms, and standard algorithms;
- use estimation when solving problems involving the addition, subtraction, multiplication, and division of whole numbers, to help judge the reasonableness of a solution.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- solve problems involving the multiplication and division of multidigit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies.

ABOUT THE LEARNING ACTIVITY

TIME: approximately two 60-minute periods

MATERIALS

- sheets of paper (1 per pair of students)
- pencils
- sheets of chart paper or large sheets of newsprint (1 per pair of students)
- markers (a few per pair of students)
- sheets of paper or math journals (1 per student)
- **Div5.BLM1: Sharing Pennies** (1 per student)

MATH LANGUAGE

- divide
- division
- divisor
- quotient
- remainder
- divisible
- algorithm
- multiples

INSTRUCTIONAL GROUPING: pairs, individuals

INSTRUCTIONAL SEQUENCING

This learning activity provides an introductory exploration of strategies for dividing a three-digit number by a one-digit number. Before starting this learning activity, students should have had experiences in solving problems involving the division of two-digit numbers by one-digit numbers.

ABOUT THE MATH

In Grade 5, students are expected to divide three-digit whole numbers by a one-digit divisor. Although students are working with larger numbers than in previous grades, instructional activities should continue to focus on the meaning of division, not merely on teaching paper-and-pencil algorithms.

In this learning activity, students solve a problem by using strategies that make sense to them. When students design their own strategies, they need to interpret the problem situation and apply their understanding of number operations, rather than simply following the steps in an algorithm. Given this opportunity to solve division problems in ways that make sense to them, students use a variety of strategies of varying complexity and efficiency. At the conclusion of the learning activity, students discuss the efficiency of different strategies for division.

GETTING STARTED

Explain to students that their help is needed in organizing a Family Math Fair at the school. Tell them that the principal has conducted a survey and that 165 people from the community have indicated that they will attend the fair. Explain to students that the first part of the math night involves a presentation in the gym and that people will sit at tables in groups of 6. Ask: "How many tables need to be set up?"

Record important information about the problem on the board:

- 165 people
- 6 people at each table
- How many tables?

Ask students to estimate the number of tables that will be needed. Discuss the different strategies that students used.

Organize the class into pairs. Ask students to work collaboratively to solve the problem in a way that makes sense to both partners. Provide each pair with a sheet of paper on which students can record their work.

WORKING ON IT

As students work on the problem, observe the various strategies they use to solve it. Pose questions to help students think about their strategies and solutions:

- "What strategy are you using to determine the number of tables that will be needed?"
- "Why did you choose this method?"
- "What is working well? What is not working well?"
- "Did you change your strategy? Why did you change it?"
- "How are you recording your ideas?"

STRATEGIES STUDENTS MIGHT USE

COUNTING

Students might draw diagrams (e.g., make tally marks) to represent 165 people, and then group the people (e.g., by circling tally marks) into sets of 6. Students then count the number of sets to determine the number of tables that are needed. Students will find that there are 27 groups of 6 people with 3 people left over and conclude that another table will be needed.

USING REPEATED ADDITION

Students might draw tables (e.g., rectangles) and indicate 6 people at each table (e.g., by sketching 6 chairs at each table, by recording "6" at each table). As they draw the tables, students keep a running count of the number of people by repeatedly adding 6 until they reach 162. Students might realize that 168, the next multiple of 6, is greater than 165, but that an extra table will be needed to accommodate the last 3 people. Students then count the number of groups of 6.

Students might also use repeated addition without the use of a diagram. For example, they might repeatedly add 6, and then count the number of 6's that were added together.

USING PROPORTIONAL REASONING

Students might use proportional reasoning, for example, a doubling strategy – 1 table for 6 people, 2 tables for 12 people, 4 tables for 24 people, and so on. Students might organize this information in a table.

Tables	1	2	4	8	16	32
People	6	12	24	48	96	192

If students use a doubling strategy, they will observe that 16 tables are too few and that 32 tables are too many. They might combine different table-people ratios to determine the total number of tables needed; for example, 16 + 8 + 4 tables (28 tables) will seat 96 + 48 + 24 people (168 people).

USING REPEATED SUBTRACTION

Students might begin with 165 and repeatedly subtract groups of 6 until they reach 3. To determine the number of tables, students count the number of times that 6 was subtracted and include an extra table for the remaining 3 people.

USING "CHUNKING"

Students might subtract "chunks" (multiples of 6) from 165.

$$\begin{array}{rl} 165 & \\ \underline{-\ 60} & \text{(10 tables)} \\ 105 & \\ \underline{-\ 60} & \text{(10 tables)} \\ 45 & \\ \underline{-\ 42} & \text{(7 tables)} \\ 3 & \end{array}$$

USING PARTIAL QUOTIENTS

Students might use a strategy in which they calculate partial quotients by using their knowledge of multiplication. For example, they might know that 20 tables would seat 120 people ($20 \times 6 = 120$), and then determine that another 8 tables ($8 \times 6 = 48$) would be needed for the remaining 45 people. The partial quotients, 20 and 8, are added to determine the number of tables.

USING AN ALGORITHM

Students might have been taught an algorithm and apply these procedures to solve the problem. If students are unable explain the meaning of the procedures and numbers in the algorithm, suggest that they select a method that they can explain.

When students have solved the problem, provide each pair with markers and a sheet of chart paper or a large sheet of newsprint. Ask students to record their strategies and solutions on the paper and to clearly show how they solved the problem.

Make a note of pairs who might share their strategies and solutions during Reflecting and Connecting. Aim to include pairs who used various methods that range in their degree of efficiency (e.g., using counting, using repeated addition, using proportional reasoning, using partial quotients).

REFLECTING AND CONNECTING

Gather the class. Ask a few pairs to share their problem-solving strategies and solutions. Try to order the presentations so that students observe inefficient strategies (e.g., counting, using repeated addition) first, followed by increasingly efficient methods. Post students' work following each presentation.

Avoid making comments that suggest that some strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

As students explain their work, ask questions that encourage them to explain the reasoning behind their strategies:

- "How did you determine the number of tables that are needed?"
- "Why did you use this strategy?"
- "What worked well with this strategy? What did not work well?"
- "How do you know that your solution makes sense?"

Following the presentations, encourage students to consider the effectiveness and efficiency of the various strategies that have been presented. Ask the following questions:

- "In your opinion, which strategy worked well?"
- "Why is the strategy effective in solving this kind of problem?"
- "How would you explain this strategy to someone who has never used it?"

Provide an opportunity for students to extend their understanding of division strategies by posing the following problem:

> "After the presentation in the gym, the 165 math fair participants will be divided into teams of 4 people to play math games. How many teams will there be?"

Have students work independently to solve the problem. Encourage them to think back to the different strategies presented by their classmates, and to use an efficient strategy that makes sense to them. Have students show their strategies and solutions on a sheet of paper or in their math journals.

ADAPTATIONS/EXTENSIONS

Encourage students to solve the problem by using a strategy that makes sense to them. Recognize that some students may need to use simple strategies (e.g., counting, using repeated addition, using repeated subtraction). It may be necessary to model the use of manipulatives and simple counting strategies for students who experience difficulty in solving the problem. These students might also benefit from working with a partner who is able to explain different strategies.

For students requiring a greater challenge, have them solve the problem in different ways, and ask them to explain how the various strategies are alike and different.

The following problem could also be used as an extension to the learning activity:

> "78 children and 87 adults are planning to attend the math fair. Each child will receive 2 glasses of juice, and each adult will receive 1 glass of juice. A jug of juice holds 7 glasses. How many jugs of juice will be needed?"

ASSESSMENT

Observe students as they solve the problem to assess how well they:

- understand the problem;
- apply an appropriate problem-solving strategy;
- judge the efficiency and accuracy of their strategy;
- find and explain a solution;
- determine whether the solution is reasonable;
- explain their strategies and solutions clearly and concisely, using mathematical language.

Collect students' solutions to the problem in which they determined the number of math teams of 4 people. Observe the work to determine how well they apply an efficient strategy to solve the division problem.

HOME CONNECTION

Send home **Div5.BLM1: Sharing Pennies**. This Home Connection activity provides an opportunity for parents and students to discuss division strategies.

LEARNING CONNECTION 1
Apples at the Math Fair

MATERIALS

- sheets of paper (1 per pair of students)

Arrange students in pairs. Ask students to solve the following problem with their partner and to record their strategy and solution on a sheet of paper.

> "The principal purchased 12 dozen apples for the math fair. The apples are placed on trays that hold 8 apples each. How many trays of apples are there?"

After students have solved the problem, have each pair partner with another pair to create groups of four. Have students compare the strategies they used to solve the problem.

LEARNING CONNECTION 2
Exploring Divisibility

MATERIALS

- **Div5.BLM2: Hundreds Chart** (1 per student)
- pencil crayons

Recognizing divisibility (i.e., knowing that numbers divide evenly without a remainder) is important in developing mental division skills.

Provide each student with a copy of **Div5.BLM2: Hundreds Chart**, and ask them to use a red pencil crayon to circle the multiples of 2. Ask students to describe numerical patterns that they notice (e.g., all multiples of 2 are even numbers; multiples of 2 end in 0, 2, 4, 6, or 8).

Next, have students use a green pencil crayon to circle the multiples of 3. Again, discuss numerical patterns (e.g., the sum of the digits in a multiple of 3 is divisible by 3).

Continue by having students use a yellow pencil crayon to circle multiples of 5. Discuss numerical patterns (e.g., multiples of 5 end in 0 and 5).

Challenge students to suggest numbers greater than 100 that are divisible by 2, 3, or 5, and ask them to explain why the numbers can be divided exactly by the multiple. Have students test their conjectures by having them perform the division calculation.

LEARNING CONNECTION 3
Divisibility Challenge

MATERIALS

- sheets of paper (1 per pair of students)

This activity is best completed after students have had an opportunity to investigate the divisibility of numbers (e.g., after completing Learning Connection 2).

Arrange students in pairs. Challenge them to work together to determine:

- a two-digit number that is divisible by 2;
- a three-digit number that is divisible by 2;
- a four-digit number that is divisible by 2;
- a five-digit number that is divisible by 2.

Next, have students find:

- a two-digit number that is divisible by 3;
- a three-digit number that is divisible by 3;
- a four-digit number that is divisible by 3;
- a five-digit number that is divisible by 3.

Continue the activity by having students determine two-, three-, four-, and five-digit numbers that are divisible by 4, by 5, and by 6.

LEARNING CONNECTION 4
Striving for Small Remainders

MATERIALS

- six-sided number cubes (1 number cube per pair of students)
- paper (a few sheets per pair of students)

Organize students into pairs. Explain the activity:

- In this game, the player with fewer points will be the winner.
- Together, partners pick any three numbers between 50 and 100, and record them on a piece of paper.
- The first player rolls the number cube to determine a divisor. If the player rolls a 1, he or she rolls the number cube again.
- The player selects one of the numbers recorded on the piece of paper and divides it by the divisor shown on the number cube. Students may use any calculation method they want (e.g., using partial quotients, using an algorithm).
- The remainder determines the number of points that the player earns.
  Example: A player rolls a 6 on the number cube and chooses 87 from the recorded numbers. After dividing 87 by 6 and calculating the answer of 14 R3, the player receives 3 points. If there is no remainder, the player receives no points.
- The second player takes a turn to determine the number of points he or she receives.
- The player with fewer points after five rounds wins the game.

After students have played the game, discuss the strategies they used to obtain the smallest possible remainders. Students might explain that they tried to select numbers from the piece of paper that are divisible by the divisor shown on the number cube (e.g., selecting an even number after rolling a 2; selecting a number ending in 0 or 5 after rolling a 5).

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on division concepts. On the homepage, click “Toolkit”. In the “Numeracy” section, find “Multiplication and Division (4 to 6)”, and then click the number to the right of it.

eworkshop.on.ca

Sharing Pennies

Dear Parent/Guardian:

We have been learning about different ways to solve division problems.

Ask your child to solve the following problem.

Four children collected 627 pennies. They want to share the pennies equally. • How many pennies will each child get? • How many pennies will be left over? • How many more pennies will they need to collect so that they all have the same number and no pennies are left over?

Have your child explain how he or she solved the problem.

Discuss other ways to solve the problem.

Thank you for doing this activity with your child.

Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Grade 6 Learning Activity
Gearing Up for a Biking Trip

OVERVIEW

In this learning activity, students are given the distance of a cycling trip (1550 km) and the average distance two cyclists can travel per day (95 km per day). They are asked to determine the number of days the trip will take. The emphasis in this learning activity is on interpreting the problem situation, applying meaningful procedures rather than simply using an algorithm, and making sense of the solution.

BIG IDEAS

This learning activity focuses on the following big ideas:

Operational sense: Students solve a division problem by using strategies that make sense to them. They discuss and analyse the various strategies used, in order to judge their efficiency and accuracy.

Relationships: An understanding of number relationships helps students solve the problem in this learning activity. For example, students need to think about how 1550 km can be broken down into 95 km parts, to determine the number of travel days.

Proportional reasoning: Students' work that involves rate (kilometres per day) contributes to their understanding of proportional reasoning.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- solve problems involving the multiplication and division of whole numbers (four-digit by two-digit), using a variety of tools (e.g., concrete materials, drawings, calculators) and strategies (e.g., estimation, algorithms);
- represent relationships using unit rates.

These specific expectations contribute to the development of the following **overall expectations**.

Students will:

- solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies;
- demonstrate an understanding of relationships involving percent, ratio, and unit rate.

ABOUT THE LEARNING ACTIVITY

TIME:
two 60-minute periods

MATERIALS

- sheets of paper (2 per group of 2 or 3 students)
- overhead transparency of **Div6.BLM1: Gearing Up for a Biking Trip**
- overhead projector
- sheets of chart paper or large sheets of newsprint (1 per group of 2 or 3 students)
- markers (a few per group of 2 or 3 students)
- **Div6.BLM2: Detour to Edmonton** (1 per student)
- **Div6.BLM3: Finding Travel Times** (1 per student)

MATH LANGUAGE

- divide
- division
- divisor
- dividend
- quotient
- remainder
- algorithm

INSTRUCTIONAL GROUPING:
groups of 2 or 3

INSTRUCTIONAL SEQUENCING

This learning activity serves as an introduction to strategies for solving problems that involve the division of four-digit whole numbers by two-digit whole numbers. It should be used before students learn the division algorithms for four-digit whole numbers by two-digit whole numbers, although students might use algorithms that they have learned in previous grades.

ABOUT THE MATH

As students progress through the junior grades, they are expected to perform division computations with increasingly larger numbers. Traditionally, the approach to teaching computations was the same at each grade – teachers reviewed the procedures for the standard division algorithm and then had students practise the algorithm using number sizes that were consistent with grade-level expectations. Although some students mastered the steps in performing the standard algorithm, fewer were successful in doing so as the number size and complexity of the computations increased. Even fewer students really understood the meaning behind the steps in the algorithm; they simply followed a memorized procedure.

The traditional approach to teaching division computations reinforces a belief in students that the standard algorithm is the only correct way to solve division problems. Students, especially those who struggle with the algorithm, focus on performing each step of the algorithm correctly, rather than on understanding the problem and the meaning of the solution. A lack of understanding is often apparent when students attempt to explain the meaning of a remainder in a division problem. In one study, 70 percent of the 45 000 Grade 8 students correctly performed the long division for the following problem.

> "An army bus holds 36 soldiers. If 1128 soldiers are being bussed to their training site, how many buses are needed?"

However, some wrote that "31 remainder 12" buses were needed, or just 31 – ignoring the remainder. Only 23 percent of the total group gave the correct answer of 32 buses (Schoenfeld, 1987).

In the following learning activity, students are given the distance of a cycling trip (1550 km) and the average distance the cyclists can travel per day (95 km per day). They are asked to determine the number of days the trip will take. Working collaboratively, students have the opportunity to share their understanding of the problem, discuss possible approaches, and help one another arrive at a solution by using strategies that make sense to them.

The answer to the division computation is 16 with a remainder of 30. Because students solve the problem in ways that are meaningful to them, they are more likely to understand the significance of the "16" and the "30" than if they merely calculated an answer using the standard algorithm. Understanding that the remainder represents a quantity that is a part of the solution allows students to interpret, account for, and represent the remainder in an appropriate way. For example, students need to realize that the "30" represents the remaining kilometres that are not travelled if the cyclists bike 95 km per day for 16 days. (The cyclists would be 30 km away from their destination at the end of the 16th day.) To deal with this leftover amount, students might have the cyclists travel more than 95 km per day, or they might add another day of travel.

GETTING STARTED

Ask students: "How long do you think it would take you to bike 100 km?" Have a few students estimate the time it might take, and ask them to explain how they made their estimates.

Continue the discussion by asking: "What information would help you answer the question more accurately?" Students might suggest, for example, that it would be helpful to know an actual 100 km distance or an average biking speed (e.g., kilometres per hour). Explain to students that you will provide some information to help them refine their estimates. Display the following statements on the board or on chart paper.

- The distance from the school to the [local site] is 5 km.
- It takes about an hour for a typical recreational cyclist to bike 15 km to 20 km.

Divide the class into groups of two or three students. Instruct students to work in their groups to determine the length of time it would take to bike 100 km. Invite them to use information (from the displayed statements or based on their own knowledge) to determine a solution. Provide each group of students with a sheet of paper on which they can record their work. Ask them to record their solution and to be prepared to share it with the class.

As students work on the problem, examine the various strategies they are using. For example, students might:

- refer to a familiar 100 km distance (e.g., the distance between two nearby towns) and estimate the time it would take to bike the distance;
- estimate the time it takes to bike 5 km and multiply this time by 20;
- consider the time it takes a recreational cyclist to bike 20 km and multiply this time by 5.

When students have finished recording their solutions, ask different groups to present their strategies and solutions to the class. Attempt to include groups who used a variety of strategies. Discuss the variety of approaches by asking questions such as the following:

- "Which strategies are similar? How are they alike?"
- "Why do the solutions differ? Is it possible to have an exact time for the solution?"
- "Which solutions seem reasonable? Why do you think they are reasonable?"
- "Which strategy, do you think, provides the most accurate solution? Why?"
- "What variables or factors might affect the time it takes to bike 100 km?"

For the last question, students might respond that factors such as the terrain, the kind of bike and condition of the bike, the physical condition of the rider, and the weather will influence the amount of time it would take to bike 100 km.

WORKING ON IT

Tell students that they are going to solve a problem encountered by Ben and Jen, two cyclists who are planning a biking trip from Winnipeg to Lake Louise. Have students locate these places on a map.

Display an overhead transparency of **Div6.BLM1: Gearing Up for a Biking Trip**, and discuss the problem:

> "The distance from Winnipeg to Lake Louise, travelling west on the Trans-Canada Highway through Calgary, is 1550 km. From past experiences, Ben and Jen know that they can bike an average of 95 km/day. If they cycle at this speed, how many days will it take them to complete the trip?"

Arrange students in groups of two or three. Explain that each group will work to solve the problem in a way that makes sense to all its members. Provide each group with a sheet paper on which they can record their work.

Allow sufficient time for students to solve the problem and to record their strategies and solutions. Observe the strategies that students use, and provide guidance when necessary. If some students use a division algorithm, remind them that they need to be prepared to explain how and why the algorithm works. If they are unable to do so, suggest that they find a method that they are able to explain.

Move from group to group, and ask questions that encourage students to reflect on and articulate their reasoning:

- "What strategy are you using to solve this problem? Why did you choose this strategy?"
- "Is there a remainder? What does the remainder mean? How can you use the remainder in your solution?"
- "Is your answer reasonable? How do you know?"
- "How can you show your work so that others will understand what you are thinking?"

STRATEGIES STUDENTS MIGHT USE

USING REPEATED SUBTRACTION

Students might begin with 1550 and repeatedly subtract 95 until they reach a remainder of 30. They then count the number of times 95 was subtracted.

$$
\begin{array}{r}
1550 \\
\underline{-\ 95} \\
1455 \\
\underline{-\ 95} \\
1360 \\
\underline{-\ 95} \\
1265 \\
\underline{-\ 95}
\end{array}
$$

and so on

USING DOUBLING

Students might double 95 to calculate the distance travelled in 2 days, and then continue to double the distance and the number of days until they reach a distance close to 1550.

95 + 95 = 190 (2 days)
190 + 190 = 380 (4 days)
380 + 380 = 760 (8 days)
760 + 760 = 1520 (16 days)
1550 – 1520 = 30 (30 kilometres more to travel)

USING "CHUNKING"

Students might subtract "chunks" (multiples of 95) from 1550.

$$
\begin{array}{rl}
1550 & \\
\underline{-\ 950} & \text{(10 days)} \\
600 & \\
\underline{-\ 190} & \text{(2 days)} \\
410 & \\
\underline{-\ 190} & \text{(2 days)} \\
220 & \\
\underline{-\ 190} & \text{(2 days)} \\
30 &
\end{array}
$$

USING AN ALGORITHM

Students might use an algorithm to divide 1550 by 95.

$$\begin{array}{r|r} 95\,\overline{)1550} & \\ \underline{950} & 10 \\ 600 & \\ \underline{190} & 2 \\ 410 & \\ \underline{190} & 2 \\ 220 & \\ \underline{190} & \underline{2} \\ 30 & 16 \end{array}$$

Note: If students attempt to use an algorithm that they have learned in previous grades, encourage them to think about the meaning of each procedural step.

When students have solved the problem, provide each group with markers and a sheet of chart paper or large sheet of newsprint. Ask students to record their strategies and solutions on the paper and to clearly demonstrate how they solved the problem.

Make a note of the various strategies used by students, and consider which groups might present their strategies during Reflecting and Connecting. Aim to include a variety of strategies (e.g., using repeated subtraction, using doubling, using “chunking”, using an algorithm).

REFLECTING AND CONNECTING

After students have finished solving the problem and recording their solutions, bring the class together to share their work. Ask a few groups of students to explain their strategies and solutions to the class. Pose guiding questions to help students explain their procedures:

- “What strategy did you use to solve the problem? Why did you use this strategy?”
- “How did you know that you were on the right track?”
- “Did you alter your strategy as you worked on the problem?”
- “What is your solution to the problem?”
- “Is the solution to the problem reasonable? How do you know?”
- “What did you do with the remainder?”

It is important that students have an opportunity to examine and discuss various strategies and evaluate their efficiency in terms of ease of use and effectiveness, in order to provide an accurate and meaningful solution. The purpose of this evaluation is not to have the class make definitive conclusions about which strategies are best, but to allow students, individually, to make decisions about which strategies make sense to them.

Encourage students to consider the effectiveness and efficiency of each strategy by asking the following questions after each presentation:

- "Was it easy to find a solution using your strategy?"
- "What are the advantages of this method? What are the disadvantages?"
- "How would you change your strategy if you solved the problem again?"

Conduct a think-pair-share activity. Provide 30 seconds for students to think about the different strategies they observed and to choose the strategy that they think worked best to solve the problem. Next, have them share their thoughts with a partner.

Ask a few students to share their thoughts about effective strategies with the class. Pose the following questions:
- "In your opinion, which strategy worked well?"
- "Why is the strategy effective in solving this kind of problem?"
- "How would you explain this strategy to someone who has never used it?"

ADAPTATIONS/EXTENSIONS

Simplify the problem for students who experience difficulties because of the size of numbers in the problem (e.g., "How many days will it take Ben and Jen to complete a trip of 260 km if they travel 65 km each day?"). It may be necessary to demonstrate a simple strategy, such as repeated subtraction, or to pair students with classmates who can explain a simple problem-solving method.

For students who require a challenge, ask them to solve the following problems:
- If Ben and Jen were to cycle 45 km in 3 1/2 hours, about how many kilometres would they cycle in 8 hours?
- If a 4-day cycle trip costs approximately $635, about how much would Ben and Jen spend on their trip from Winnipeg to Lake Louise?

ASSESSMENT

Observe students as they solve the problem to assess how well they:
- understand the problem;
- use an appropriate problem-solving strategy;
- judge the efficiency and accuracy of their strategy;
- solve the problem;
- explain the meaning of the remainder within the context of the problem and their solutions;
- explain their strategies and solutions clearly and concisely, using mathematical language;
- determine whether the solution is reasonable.

Provide an additional assessment opportunity by having students solve an additional problem. Provide students with copies of **Div6.BLM2: Detour to Edmonton**, and discuss the problem.

> "If Ben and Jen take the Yellowhead Highway from Winnipeg to Edmonton and then travel south to Lake Louise, the total distance is 1910 km. If Ben and Jen travel at a more leisurely pace of 85 km a day, how many days will it take them to complete the trip?"

Encourage students to think about the various strategies that the class used to solve the previous problem, and to apply one that would work well to solve this problem. Remind students to show their strategy and solution clearly so that others will know what they are thinking.

Observe students' completed work and assess how well they apply an appropriate strategy, solve the problem, and explain their strategy and solution.

HOME CONNECTION

Send home **Div6BLM3: Finding Travel Times**. In this Home Connection activity, students solve a problem in which they determine the time it takes to travel by car between two cities and discuss their strategies with their parents.

LEARNING CONNECTION 1
Exploring a Flexible Division Algorithm

Learning the standard North American division algorithm can be difficult for students if they do not know basic multiplication facts, or if they are unsure of the steps involved in the algorithm. Exploring non-traditional algorithms provides students with an alternative to the standard North American algorithm and can help them understand the processes of division.

In the flexible algorithm explained below, students use known multiplication facts to determine parts that can be subtracted from the dividend. Students repeatedly subtract parts from the dividend until no multiples of the divisor are left. Students keep track of the pieces as they are subtracted, to the right of the algorithm.

Record the following on the board, and explain that the structure will allow students to calculate $1450 \div 43$.

$$\begin{array}{rr|l} 43 & \overline{)1450} & \\ & & \\ & & \end{array}$$

Ask: "Is there at least one group of 43 in 1450? Are there at least 2 groups? At least 10 groups?"

When students agree that there are at least 10 groups of 43 (since $10 \times 43 = 430$, and 430 is less than 1450), complete the next step in the algorithm.

$$\begin{array}{rr|l} 43 & \overline{)1450} & \\ & \underline{430} & 10 \\ & 1020 & \\ & & \end{array}$$

Explain that 1020 remains, and ask: "How many groups of 43 could we take from 1020?" Students might explain that another 10 groups of 43 could be taken from 1020. Record the next step in the algorithm.

43)	1450	
	430	10
	1020	
	430	10
	590	

Continue to have students subtract multiples of 43 until no more multiples of 43 remain.

43)	1450	
	430	10
	1020	
	430	10
	590	
	430	10
	160	
	86	2
	74	
	43	1
	31	33

After the algorithm has been completed, ask:
- "How many groups of 43 are there in 1450?"
- "What is the remainder?"
- "Why is there a remainder?"

Provide other opportunities for students to use the flexible algorithm.

LEARNING CONNECTION 2
Making Sense of Remainders

MATERIALS
- sheets of paper (1 per group of 2 or 3 students)

Solutions to division problems often involve remainders. The way in which remainders are dealt with depends on the context in the problem situation. For example, remainders can:
- be discarded;
- be partitioned into fractional pieces and distributed equally;
- remain a quantity;
- force the answer to the next highest whole number.

In other situations, the quotient can be rounded to the nearest whole number for an approximate answer. (See p. 17 for examples of different ways of dealing with remainders.)

This learning connection provides an opportunity for students to think about the meaning of a remainder within the context of a problem.

Organize students into groups of two or three. Ask each group to compose a word problem that involves 162 ÷ 12, and to record it on a sheet of paper. Next, have groups exchange papers. Ask students to solve the problem in a way that makes sense to all group members. (See pp. 18–19 for possible strategies.)

Observe students as they solve the problem, and ask:

- "What strategy are you using to solve the problem?"
- "How do you know that this strategy is working?"
- "Is there some way to modify your strategy so that it will work better?"
- "Is there a remainder? How will you deal with the remainder so that it makes sense in your solution?"

Have groups present their strategies and solutions to the class. Discuss the meaning of the quotient and remainder within the context of each problem. Compare the different ways in which the remainder is dealt with in different situations.

LEARNING CONNECTION 3
Asking Questions

MATERIALS

- **Div6.BLM4: Asking Questions** (1 per pair of students)

Provide each pair of students with a copy of **Div6.BLM4: Asking Questions**. Explain that the page provides the answers to four questions, and that students are to determine what the questions might be, based on the information given at the top of the page. Have students work with their partner to discuss possible questions and to record them on the page.

Have pairs of students share their questions with the class.

Some possibilities are:

- What is the question if the answer is $16.50? (How much did Joe earn per hour?)
- What is the question if the answer is 48? (How many hours did Joe work?)
- What is the question if the answer is $66? (How much did Joe earn each day?)
- What is the question if the answer is $132? (How much did Joe earn in 2 days?)

LEARNING CONNECTION 4
Base Ten Towers

MATERIALS

- base ten blocks, including hundreds flats, tens rods, and ones cubes (a collection for each group of 2 or 3 students)
- sheets of paper (1 per group of 2 or 3 students)
- pencils
- metre sticks (1 per group of 2 or 3 students)

Divide students into groups of two or three. Invite each group to build a tower using base ten blocks. Allow five minutes for students to build their towers. Challenge groups to calculate the cost of their towers if each hundreds flat is worth $100, each tens rod is worth $10, and each ones cube is worth $1.

Provide each group with a metre stick, and ask students to calculate the cost of each centimetre of the structure's height.

Invite groups to explain the strategies they used throughout the activity.

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on division concepts. On the homepage, click "Toolkit". In the "Numeracy" section, find "Multiplication and Division (4 to 6)", and then click the number to the right of it.

eworkshop.on.ca

Gearing Up for a Biking Trip

The distance from Winnipeg to Lake Louise, travelling west on the Trans-Canada Highway through Calgary, is 1550 km. From past experiences, Ben and Jen know that they can bike an average of 95 km/day. If they cycled at this speed, how many days will it take them to complete the trip?

Detour to Edmonton

If Ben and Jen take the Yellowhead Highway from Winnipeg to Edmonton and then travel south to Lake Louise, the total distance is 1910 km. If Ben and Jen travel at a more leisurely pace of 85 km a day, how many days will it take them to complete the trip?

Finding Travel Times

Dear Parent/Guardian:

We have been learning about different ways to solve division problems.

In math class, we solved a problem that involved finding the number of days it would take to bike 1550 km (kilometres) at a speed of 95 km per day. Students were encouraged to use methods that made sense to them, rather than follow a procedure that they might not understand. We then examined several ways to solve this problem and discussed the advantages and disadvantages of each method.

Have your child solve the following problem in a way that makes sense to him or her.

> The distance from Barrie to Thunder Bay is 1275 km. How long would it take to travel this distance by car if you travel at an average speed of 85 km per hour?

Ask your child to explain how he or she solved the problem. You might also demonstrate how you would solve the problem.

As an extension activity, have your child find the distance between two provincial capitals, and have him or her determine the approximate time it would take to travel by car between the two cities.

Thank you for doing this activity with your child.

Asking Questions

Joe earned $792 for 12 days of work. Each day, he worked 4 hours.

- What is the question if the answer is $16.50?

- What is the question if the answer is 48?

- What is the question if the answer is $66?

- What is the question if the answer is $132?

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