Ontario

Number Sense and Numeration, Grades 4 to 6

Volume 2
Addition and Subtraction

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6

Ontario Education excellence for all

2006

Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.

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CONTENTS

INTRODUCTION

Number Sense and Numeration, Grades 4 to 6 is a practical guide, in six volumes, that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Number Sense and Numeration strand of *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. This guide provides teachers with practical applications of the principles and theories that are elaborated on in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

The guide comprises the following volumes:

- Volume 1: The Big Ideas
- Volume 2: Addition and Subtraction
- Volume 3: Multiplication
- Volume 4: Division
- Volume 5: Fractions
- Volume 6: Decimal Numbers

The present volume – Volume 2: Addition and Subtraction – provides:

- a discussion of mathematical models and instructional strategies that support student understanding of addition and subtraction;
- sample learning activities dealing with addition and subtraction for Grades 4, 5, and 6.

A glossary that provides definitions of mathematical and pedagogical terms used throughout the six volumes of the guide is included in Volume 1: The Big Ideas. Each volume contains a comprehensive list of references for the guide.

The content of all six volumes of the guide is supported by "eLearning modules" that are available at www.eworkshop.on.ca. The instructional activities in the eLearning modules that relate to particular topics covered in this guide are identified at the end of each of the learning activities (see pages 51, 68, and 80).

Relating Mathematics Topics to the Big Ideas

> *The development of mathematical knowledge is a gradual process. A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the "big ideas" of mathematics – that is, the interrelated concepts that form a framework for learning mathematics in a coherent way.*
>
> (*The Ontario Curriculum, Grades 1–8: Mathematics, 2005*, p. 4)

In planning mathematics instruction, teachers generally develop learning opportunities related to curriculum topics, such as fractions and division. It is also important that teachers design learning opportunities to help students understand the big ideas that underlie important mathematical concepts. The big ideas in Number Sense and Numeration for Grades 4 to 6 are:

- quantity
- operational sense
- relationships
- representation
- proportional reasoning

Each of the big ideas is discussed in detail in Volume 1 of this guide.

When instruction focuses on big ideas, students make connections within and between topics, and learn that mathematics is an integrated whole, rather than a compilation of unrelated topics. For example, in a lesson about division, students can learn about the relationship between multiplication and division, thereby deepening their understanding of the big idea of operational sense.

The learning activities in this guide do not address all topics in the Number Sense and Numeration strand, nor do they deal with all concepts and skills outlined in the curriculum expectations for Grades 4 to 6. They do, however, provide models of learning activities that focus on important curriculum topics and that foster understanding of the big ideas in Number Sense and Numeration. Teachers can use these models in developing other learning activities.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills. The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The learning activities described in this guide demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful for students. The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

Problem Solving: Each of the learning activities is structured around a problem or inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;
- work collaboratively and talk about mathematics;
- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;
- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

Reasoning and Proving: The learning activities described in this guide provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The learning activities include questions teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

Reflecting: Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

Selecting Tools and Computational Strategies: Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this guide provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and represent and communicate mathematical ideas at their own level of understanding.

Connecting: The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections among concrete, pictorial, and symbolic mathematical representations. Advice on helping students connect procedural knowledge and conceptual understanding is also provided. The problem-solving experiences in many of the learning activities allow students to connect mathematics to real-life situations and meaningful contexts.

Representing: The learning activities provide opportunities for students to represent mathematical ideas using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students' own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

Communicating: Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

Addressing the Needs of Junior Learners

Every day, teachers make many decisions about instruction in their classrooms. To make informed decisions about teaching mathematics, teachers need to have an understanding of the big ideas in mathematics, the mathematical concepts and skills outlined in the curriculum document, effective instructional approaches, and the characteristics and needs of learners.

The table on pp. 9–10 outlines general characteristics of junior learners, and describes some of the implications of these characteristics for teaching mathematics to students in Grades 4, 5, and 6.

Characteristics of Junior Learners and Implications for Instruction		
Area of Development	**Characteristics of Junior Learners**	**Implications for Teaching Mathematics**
Intellectual development	Generally, students in the junior grades: • prefer active learning experiences that allow them to interact with their peers; • are curious about the world around them; • are at a concrete operational stage of development, and are often not ready to think abstractly; • enjoy and understand the subtleties of humour.	The mathematics program should provide: • learning experiences that allow students to actively explore and construct mathematical ideas; • learning situations that involve the use of concrete materials; • opportunities for students to see that mathematics is practical and important in their daily lives; • enjoyable activities that stimulate curiosity and interest; • tasks that challenge students to reason and think deeply about mathematical ideas.
Physical development	Generally, students in the junior grades: • experience a growth spurt before puberty (usually at age 9–10 for girls, at age 10–11 for boys); • are concerned about body image; • are active and energetic; • display wide variations in physical development and maturity.	The mathematics program should provide: • opportunities for physical movement and hands-on learning; • a classroom that is safe and physically appealing.
Psychological development	Generally, students in the junior grades: • are less reliant on praise but still respond well to positive feedback; • accept greater responsibility for their actions and work; • are influenced by their peer groups.	The mathematics program should provide: • ongoing feedback on students' learning and progress; • an environment in which students can take risks without fear of ridicule; • opportunities for students to accept responsibility for their work; • a classroom climate that supports diversity and encourages all members to work cooperatively.
Social development	Generally, students in the junior grades: • are less egocentric, yet require individual attention; • can be volatile and changeable in regard to friendship, yet want to be part of a social group; • can be talkative; • are more tentative and unsure of themselves; • mature socially at different rates.	The mathematics program should provide: • opportunities to work with others in a variety of groupings (pairs, small groups, large group); • opportunities to discuss mathematical ideas; • clear expectations of what is acceptable social behaviour; • learning activities that involve all students regardless of ability. *(continued)*

Characteristics of Junior Learners and Implications for Instruction		
Area of Development	**Characteristics of Junior Learners**	**Implications for Teaching Mathematics**
Moral and ethical development	Generally, students in the junior grades: • develop a strong sense of justice and fairness; • experiment with challenging the norm and ask "why" questions; • begin to consider others' points of view.	The mathematics program should provide: • learning experiences that provide equitable opportunities for participation by all students; • an environment in which all ideas are valued; • opportunities for students to share their own ideas and evaluate the ideas of others.

(Adapted, with permission, from *Making Math Happen in the Junior Grades*. Elementary Teachers' Federation of Ontario, 2004.)

LEARNING ABOUT ADDITION AND SUBTRACTION IN THE JUNIOR GRADES

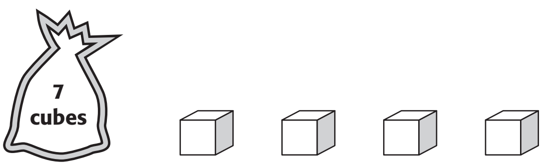
Introduction

Instruction in the junior grades should help students to extend their understanding of addition and subtraction concepts, and allow them to develop flexible computational strategies for adding and subtracting multidigit whole numbers and decimal numbers.

PRIOR LEARNING

In the primary grades, students develop an understanding of part-whole concepts – they learn that two or more parts can be combined to create a whole (addition), and that a part can be separated from a whole (subtraction).

Young students use a variety of strategies to solve addition and subtraction problems. Initially, students use objects or their fingers to model an addition or subtraction problem and to determine the unknown amount. As students gain experience in solving addition and subtraction problems, and as they gain proficiency in counting, they make a transition from using direct modelling to using counting strategies. *Counting on* is one such strategy: When two sets of objects are added together, the student does not need to count all the objects in both sets, but instead begins with the number of objects in the first set and counts on from there.



"7 . . . 8 . . . 9 . . . 10 . . . 11.

There are 11 cubes altogether."

As students learn basic facts of addition and subtraction, they use this knowledge to solve problems, but sometimes they need to revert to direct modelling and counting to support their thinking. Students learn certain basic facts, such as doubles (e.g., $3+3$ and $6+6$), before others, and they can use such known facts to derive answers for unknown facts (e.g., $3+4$ is related to $3+3$; $6+7$ is related to $6+6$).

By the end of Grade 3, students add and subtract three-digit numbers using concrete materials and algorithms, and perform mental computations involving the addition and subtraction of two-digit numbers.

In the primary grades, students also develop an understanding of properties related to addition and subtraction:

- **Identity property:** Adding 0 to or subtracting 0 from any number does not affect the value of the number (e.g., $6+0=6$; $11-0=11$).
- **Commutative property:** Numbers can be added in any order, without affecting the sum (e.g., $2+4=4+2$).
- **Associative property:** The numbers being added can be regrouped in any way without changing the sum (e.g., $7+6+4=6+4+7$).

It is important for teachers of the junior grades to recognize the addition and subtraction concepts and skills that their students developed in the primary grades – these understandings provide a foundation for further learning in Grades 4, 5, and 6.

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES

In the junior grades, instruction should focus on developing students' understanding of meaningful computational strategies for addition and subtraction, rather than on having students memorize the steps in algorithms.

The development of computational strategies for addition and subtraction should be rooted in meaningful experiences (e.g., problem-solving contexts, investigations). Students should have opportunities to develop and apply a variety of strategies, and to consider the appropriateness of strategies in various situations.

Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to addition and subtraction, listed in the following table.

Curriculum Expectations Related to Addition and Subtraction, Grades 4, 5, and 6		
By the end of Grade 4, students will:	**By the end of Grade 5, students will:**	**By the end of Grade 6, students will:**
Overall Expectation • solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies. ***Specific Expectations*** • add and subtract two-digit numbers, using a variety of mental strategies; • solve problems involving the addition and subtraction of four-digit numbers, using student-generated algorithms and standard algorithms; • add and subtract decimal numbers to tenths, using concrete materials and student-generated algorithms; • add and subtract money amounts by making simulated purchases and providing change for amounts up to $100, using a variety of tools; • use estimation when solving problems involving the addition, subtraction, and multiplication of whole numbers, to help judge the reasonableness of a solution.	***Overall Expectation*** • solve problems involving the multiplication and division of multidigit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies. ***Specific Expectations*** • solve problems involving the addition, subtraction, and multiplication of whole numbers, using a variety of mental strategies; • add and subtract decimal numbers to hundredths, including money amounts, using concrete materials, estimation, and algorithms; • use estimation when solving problems involving the addition, subtraction, multiplication, and division of whole numbers, to help judge the reasonableness of a solution.	***Overall Expectation*** • solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies. ***Specific Expectations*** • use a variety of mental strategies to solve addition, subtraction, multiplication, and division problems involving whole numbers; • add and subtract decimal numbers to thousandths, using concrete materials, estimation, algorithms, and calculators; • use estimation when solving problems involving the addition and subtraction of whole numbers and decimals, to help judge the reasonableness of a solution.

(*The Ontario Curriculum, Grades 1–8: Mathematics, 2005*)

The following sections explain content knowledge related to addition and subtraction concepts in the junior grades, and provide instructional strategies that help students develop an understanding of these operations. Teachers can facilitate this understanding by helping students to:

- solve a variety of problem types;
- relate addition and subtraction;
- model addition and subtraction;
- extend knowledge of basic facts;
- develop a variety of computational strategies;
- develop estimation strategies;
- add and subtract decimal numbers.

Solving a Variety of Problem Types

Solving different types of addition and subtraction problems allows students to think about the operations in different ways. There are four main types of addition and subtraction problems: joining, separating, comparing, and part-part-whole.

A *joining* problem involves increasing an amount by adding another amount to it. The situation involves three amounts: a start amount, a change amount (the amount added), and a result amount. A *joining* problem occurs when one of these amounts is unknown.

Examples:

- Gavin saved $14.50 from his allowance. His grandmother gave him $6.75 for helping her with some chores. How much money does he have altogether? (*Result unknown*)
- There were 127 students from the primary grades in the gym for an assembly. After the students from the junior grades arrived, there were 300 students altogether. How many students from the junior grades were there? (*Change unknown*)
- The veterinarian told Camilla that the mass of her puppy increased by 3.5 kg in the last month. If the puppy weighs 35.6 kg now, what was its mass a month ago? (*Start unknown*)

A *separating* problem involves decreasing an amount by removing another amount. The situation involves three amounts: a start amount, a change amount (the amount removed), and a result amount. A *separating* problem occurs when one of these amounts is unknown.

Examples:

- Damian earned $21.25 from his allowance and helping his grandmother. If he spent $12.45 on comic books, how much does he have left? (*Result unknown*)
- There were 300 students in the gym for the assembly. Several classes went back to their classrooms, leaving 173 students in the gym. How many students returned to their classrooms? (*Change unknown*)
- Tika drew a line on her page. The line was longer than she needed it to be, so she erased 2.3 cm of the line. If the line she ended up with was 8.7 cm long, what was the length of the original line she drew? (*Start unknown*)

A *comparing* problem involves the comparison of two quantities. The situation involves a smaller amount, a larger amount, and the difference between the two amounts. A *comparing* problem occurs when the smaller amount, the larger amount, or the difference is unknown.

Examples:

- Antoine collected $142.15 in pledges for the read-a-thon, and Emma collected $109.56. How much more did Antoine collect in pledges? (*Difference unknown*)
- Boxes of Goodpick Toothpicks come in two different sizes. The smaller box contains 175 toothpicks, and the larger box contains 225 more. How many toothpicks are in the larger box? (*Larger quantity unknown*)

- Evan and Liddy both walk to school. Liddy walks 1.6 km farther than Evan. If Liddy's walk to school is 3.4 km, how far is Evan's walk? (*Smaller quantity unknown*)

A *part-part-whole* problem involves two parts that make the whole. Unlike *joining* and *separating* problems, there is no mention of adding or removing amounts in the way that a *part-part-whole* problem is worded. A *part-part-whole* problem occurs when either a part or the whole is unknown.

Examples:

- Shanlee has a collection of hockey and baseball cards. She has 376 hockey cards and 184 baseball cards. How many cards are in Shanlee's collection? (*Whole unknown*)
- Erik bought 3.85 kg of fruit at the market. He bought only oranges and apples. If 1.68 kg of the fruit was oranges, what was the mass of the apples? (*Part unknown*)

Varying the types of problem helps students to recognize different kinds of addition and subtraction situations, and allows them to develop a variety of strategies for solving addition and subtraction problems.

Relating Addition and Subtraction

The relationship between *part* and *whole* is an important idea in addition and subtraction – any quantity can be regarded as a whole if it is composed of two or more parts. The operations of addition and subtraction involve determining either a part or the whole.

Students should have opportunities to solve problems that involve the same numbers to see the connection between addition and subtraction. Consider the following two problems.

> *"Julia's class sold 168 raffle tickets in the first week and 332 the next. How many tickets did the class sell altogether?"*

> *"Nathan's class made it their goal to sell 500 tickets. If the students sold 332 the first week, how many will they have to sell to meet their goal?"*

The second problem can be solved by subtracting 332 from 500. Students might also solve this problem using addition – they might think, "What number added to 332 will make 500?" Discussing how both addition and subtraction can be used to solve the same problem helps students to understand part-whole relationships and the connections between the operations.

It is important that students continue to develop their understanding of the relationship between addition and subtraction in the junior grades, since this relationship lays the foundation for algebraic thinking in later grades. When faced with an equation such as $x + 7 = 15$, students who interpret the problem as "What number added to 7 makes 15?" will also see that the answer can be found by subtracting 7 from 15.

Modelling Addition and Subtraction

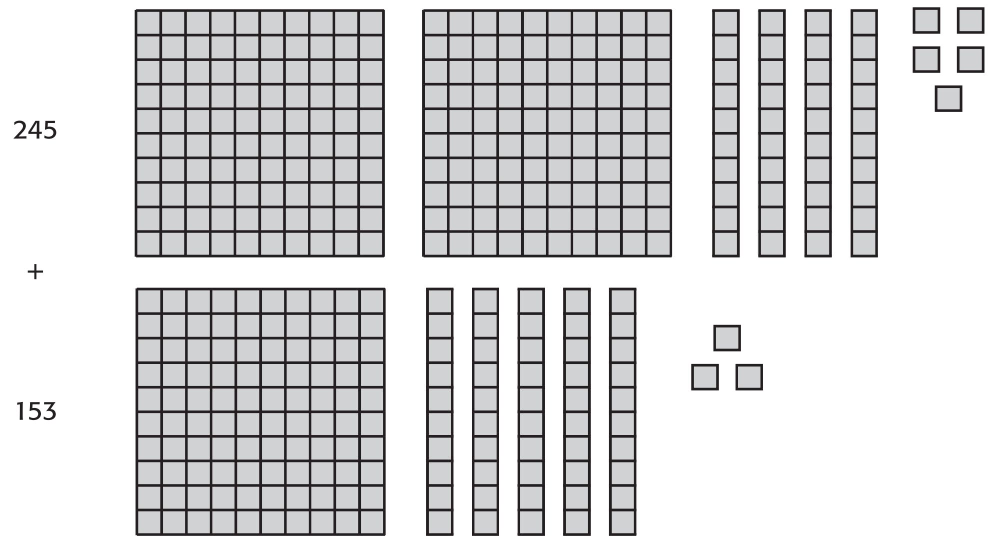
In the primary grades, students learn to add and subtract by using a variety of concrete and pictorial models (e.g., counters, base ten materials, number lines, tallies, hundreds charts).

In the junior grades, teachers should provide learning experiences in which students continue to use models to develop understanding of mental and paper-and-pencil strategies for adding and subtracting multidigit whole numbers and decimal numbers.

In the junior grades, base ten materials and open number lines provide significant models for addition and subtraction.

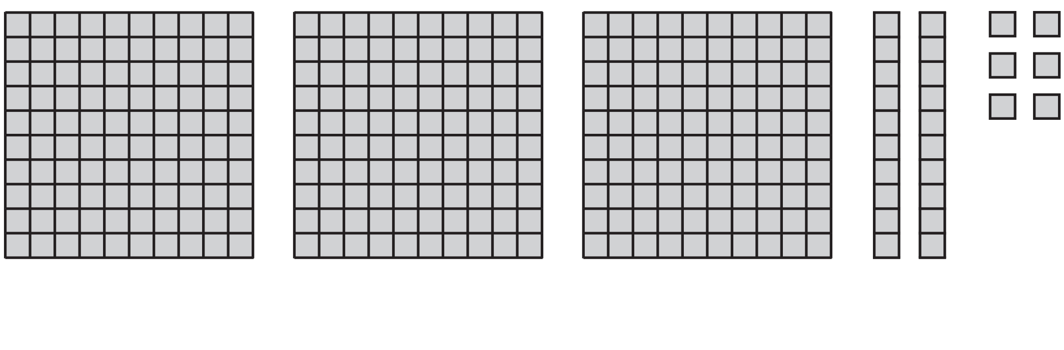
BASE TEN MATERIALS

Base ten materials provide an effective model for addition because they allow students to recognize the importance of adding ones to ones, tens to tens, hundreds to hundreds, and so on. For example, to add 245 + 153, students combine like units (hundreds, tens, ones) separately and find that there are 3 hundreds, 9 tens, and 8 ones altogether. The sum is 398.



Students can also use base ten blocks to demonstrate the processes involved in regrouping. Students learn that having 10 or more ones requires that each group of 10 ones be grouped to form a ten (and that 10 tens be regrouped to form a hundred, and so on). After combining like base ten materials (e.g., ones with ones, tens with tens, hundreds with hundreds), students need to determine whether the quantity is 10 or greater and, if so, regroup the materials appropriately.

Concepts about regrouping are important when students use base ten materials to subtract. To solve 326 – 184, for example, students could represent 326 by using the materials like this:



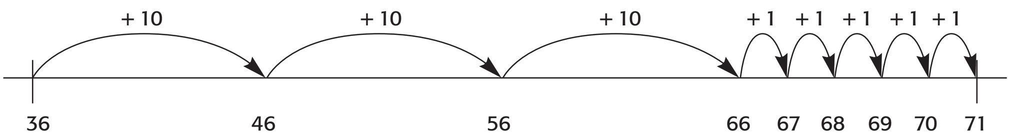
To begin the subtraction, students might remove 4 ones, leaving 2 ones. Next, students might want to remove 8 tens but find that there are only 2 tens available. After exchanging 1 hundred for 10 tens (resulting in 12 tens altogether), students are able to remove 8 tens, leaving 4 tens. Finally students remove 1 hundred, leaving 1 hundred. Students examine the remaining pieces to determine the answer: 1 hundred, 4 tens, 2 ones is 142.

Because base ten materials provide a concrete representation of regrouping, they are often used to develop an understanding of algorithms. (See Appendix 10–1 in Volume 5 of *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6* for a possible approach for developing understanding of the standard algorithm by using base ten materials.) However, teachers should be aware that some students may use base ten materials to model an operation without fully understanding the underlying concepts. By asking students to explain the processes involved in using the base ten materials, teachers can determine whether students understand concepts about place value and regrouping, or whether students are merely following procedures mechanically, without fully understanding.

OPEN NUMBER LINES

Open number lines (number lines on which only significant numbers are recorded) provide an effective model for representing addition and subtraction strategies. Showing computational steps as a series of "jumps" (drawn by arrows on the number line) allows students to visualize the number relationships and actions inherent in the strategies.

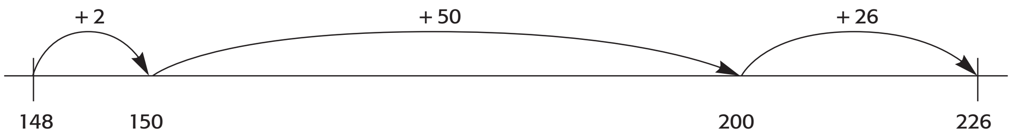
In the primary grades, students use open number lines to represent simple addition and subtraction operations. For example, students might show 36 + 35 as a series of jumps of 10's and 1's.



In the junior grades, open number lines continue to provide teachers and students with an effective tool for modelling various addition and subtraction strategies. For example, a student might explain a strategy for calculating 226 – 148 like this:

> *"I knew that I needed to find the difference between 226 and 148. So I started at 148 and added on 2 to get to 150. Next, I added on 50 to get to 200. Then I added on 26 to get to 226. I figured out the difference between 226 and 148 by adding 2 + 50 + 26. The difference is 78."*

The teacher, wanting to highlight the student's method, draws an open number line on the board and represents the numbers the student added on to 148 as a series of jumps.



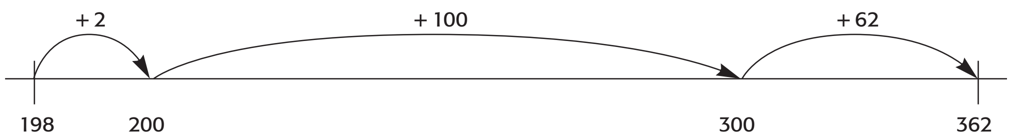
$$\begin{array}{r} 2 \\ 50 \\ +\ 26 \\ \hline 78 \end{array}$$

By using a number line to illustrate the student's thinking, the teacher gives all students in the class access to a visual representation of a particular strategy. Representing addition and subtraction strategies on a number line also helps students to develop a sense of quantity, by thinking about the relative position of numbers on a number line.

Students can also use open number lines as a tool in problem solving. For example, the teacher might have students solve the following problem.

> *"I am reading a very interesting novel. Last weekend, I read 198 pages. I noticed that there are 362 pages in the book. How many more pages do I have to read?"*

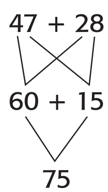
The teacher encourages students to solve the problem in a way that makes sense to them. Some students interpret the problem as the *distance* between 198 and 362, and they choose to use an open number line to solve the problem. One student works with friendly numbers – making a jump of 2 to get from 198 to 200, a jump of 100 to get from 200 to 300, and a jump of 62 to get from 300 to 362. The student then adds the jumps to determine that the distance between 198 and 362 is 164.



"2 + 100 + 62 = 164. You have 164 more pages to read."

SELECTING APPROPRIATE MODELS

Although base ten materials and open number lines are powerful models to help students add and subtract whole numbers and decimal numbers, it is important for teachers to recognize that these are not the only models available. At times, a simple diagram is effective in demonstrating a particular strategy. For example, to calculate 47 + 28, the following diagram shows how numbers can be decomposed into parts, then the parts added to calculate partial sums, and then the partial sums added to calculate the final sum.



Teachers need to consider which models are most effective in demonstrating particular strategies. Whenever possible, more than one model should be used so that students can observe different representations of a strategy. Teachers should also encourage students to demonstrate their strategies in ways that make sense to them. Often, students create diagrams of graphic representations that help them to clarify their own strategies and allow them to explain their methods to others.

Extending Knowledge of Basic Facts

In the primary grades, students develop fluency in adding and subtracting one-digit numbers, and apply this knowledge to adding and subtracting multiples of 10 (e.g., $2 + 6 = 8$, so $20 + 60 = 80$).

Teachers can provide opportunities for students to explore the impact of adding and subtracting numbers that are multiples of 10, 100, and 1000 – such as 40, 200, and 5000. For example, teachers might have students explain their answers to questions such as the following:

- "What number do you get when you add 200 to 568?"
- "If you subtract 30 from 1252, how much do you have left?"
- "What number do you get when you add 3000 to 689?"
- "What is the difference between 347 and 947?"

It is important for students to develop fluency in calculating with multiples of 10, 100, and 1000 in order to develop proficiency with a variety of addition and subtraction strategies.

Developing a Variety of Computational Strategies

In the primary grades, students learn to add and subtract by using a variety of mental strategies and paper-and-pencil strategies. They use models, such as base ten materials, to help them understand the procedures involved in addition and subtraction algorithms.

In the junior grades, students apply their understanding of computational strategies to determine sums and differences in problems that involve multidigit whole numbers and decimal numbers. Given addition and subtraction problems, some students may tend to use a standard algorithm and carry out the procedures mechanically – without thinking about number meaning in the algorithm. As such, they have little understanding of whether the results in their computations are reasonable.

It is important that students develop a variety of strategies for adding and subtracting. If students develop skill in using only standard algorithms, they are limited to paper-and-pencil strategies that are often inappropriate in many situations (e.g., when it is more efficient to calculate numbers mentally).

Teachers can help students develop flexible computational strategies in the following ways:

- Students can be presented with a problem that involves addition or subtraction. The teacher encourages students to use a strategy that makes sense to them. In so doing, the teacher allows students to devise strategies that reflect their understanding of the problem, the

numbers contained in the problem, and the operations required to solve the problem. Student-generated strategies vary in complexity and efficiency. By discussing with the class the various strategies used to solve a problem, students can judge the effectiveness of different methods and learn to adopt these methods as their own. (The learning activities in this document provide examples of this instructional approach.)

- Teachers can help students develop skill with specific computational strategies through mini-lessons (Fosnot & Dolk, 2001a). With this approach, students are asked to solve a sequence of related computations – also called a "string" – which allows students to understand how a particular strategy works. (In this volume, see Appendix 2–1: Developing Computational Strategies Through Mini-Lessons for more information on mini-lessons with math strings.)

The effectiveness of these instructional methods depends on students making sense of the numbers and working with them in flexible ways (e.g., by decomposing numbers into parts that are easier to calculate). Learning about various strategies is enhanced when students have opportunities to visualize how the strategies work. By representing various methods visually (e.g., drawing an open number line that illustrates a strategy), teachers can help students understand the processes used to add and subtract numbers in flexible ways.

ADDITION STRATEGIES

This section explains a variety of addition strategies. Although the examples provided often involve two- or three-digit whole numbers, it is important that the number size in problems aligns with the grade-level curriculum expectations and is appropriate for the students' ability level.

The examples also include visual representations (e.g., diagrams, number lines) of the strategies. Teachers can use similar representations to model strategies for students.

It is difficult to categorize the following strategies as either mental or paper-and-pencil. Often, a strategy involves both doing mental calculations and recording numbers on paper. Some strategies may, over time, develop into strictly mental processes. However, it is usually necessary – and helpful – for students to jot down numbers as they work through a new strategy.

Splitting strategy: Adding with base ten materials helps students to understand that ones are added to ones, tens to tens, hundreds to hundreds, and so on. This understanding can be applied when using a splitting strategy, in which numbers are decomposed according to place value and then each place-value part is added separately. Finally, the partial sums are added to calculate the total sum.

$38 + 26$	$168 + 384$	$4.8 + 3.5$
$50 + 14$	$400 + 140 + 12$	$7 + 1.3$
64	$540 + 12 = 552$	8.3

The splitting strategy is often used as a mental addition strategy. For example, to add $25 + 37$ mentally, students might use strategies such as the following:

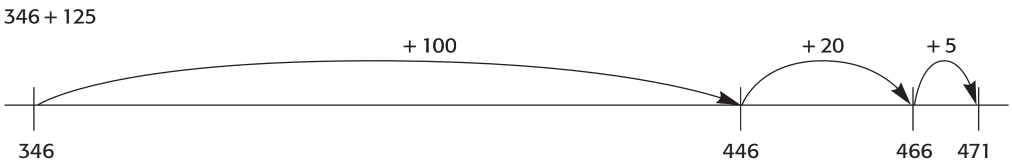
- add the tens first ($20 + 30 = 50$), then add the ones ($5 + 7 = 12$), and then add the partial sums ($50 + 12 = 62$); or
- add the ones first ($5 + 7 = 12$), then add the tens ($20 + 30 = 50$), and then add the partial sums ($12 + 50 = 62$).

The splitting strategy is less effective for adding whole numbers with four or more digits (and with decimal numbers to hundredths and thousandths), because adding all the partial sums takes time, and students can get frustrated with the amount of adding required.

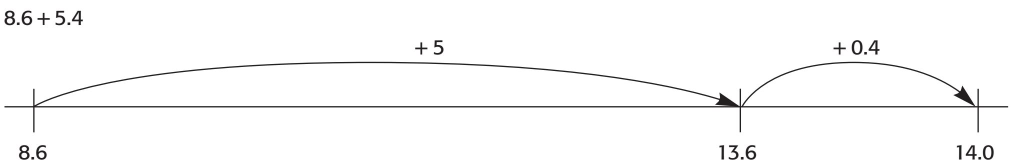
Adding-on strategy: With this strategy, one addend is kept intact, while the other addend is decomposed into friendlier numbers (often according to place value – into ones, tens, hundreds, and so on). The parts of the second addend are added onto the first addend. For example, to add $36 + 47$, students might:

- add the first addend to the tens of the second addend ($36 + 40 = 76$), and then add on the ones of the second addend ($76 + 7 = 83$);
- add the first addend to the ones of the second addend ($36 + 7 = 43$), and then add on the tens of the second addend ($43 + 40 = 83$).

The adding-on strategy can be modelled using an open number line. The following example shows $346 + 125$. Here, 125 is decomposed into 100, 20, and 5.



The adding-on strategy can also be applied to adding decimal numbers. To add $8.6 + 5.4$, for example, students might add $8.6 + 5$ first, and then add $13.6 + 0.4$. The following number line illustrates the strategy.



Moving strategy: A moving strategy involves "moving" quantities from one addend to the other to create numbers that are easier to work with. This strategy is particularly effective when one addend is close to a friendly number (e.g., a multiple of 10). In the following example, 296 is close to 300. By "moving" 4 from 568 to 296, the addition question can be changed to 300 + 564.

$$\overset{\curvearrowleft\,4}{296 + 568}$$

$$300 + 564 = 864$$

The preceding example highlights the importance of examining the numbers in a problem in order to select an appropriate strategy. A splitting strategy or an adding-on strategy could have been used to calculate 296 + 568; however, in this case, these strategies would be cumbersome and less efficient than a moving strategy.

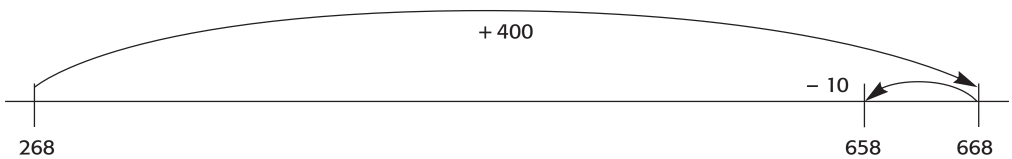
Compensation strategy: A compensation strategy involves adding more than is needed, and then taking away the extra at the end. This strategy is particularly effective when one addend is close to a friendly number (e.g., a multiple of 10). In the following example, 268 + 390 is solved by adding 268 + 400, and then subtracting the extra 10 (the difference between 390 and 400).

$$268 + 390$$

$$268 + 400 = 668$$

$$668 - 10 = 658$$

A number line can be used to model this strategy.

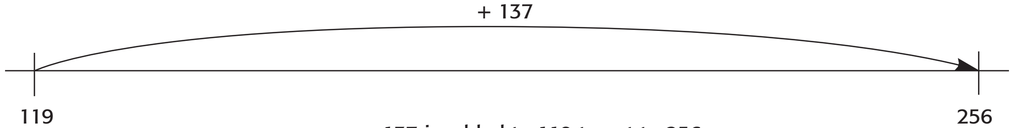


SUBTRACTION STRATEGIES

The development of subtraction strategies is based on two interpretations of subtraction:

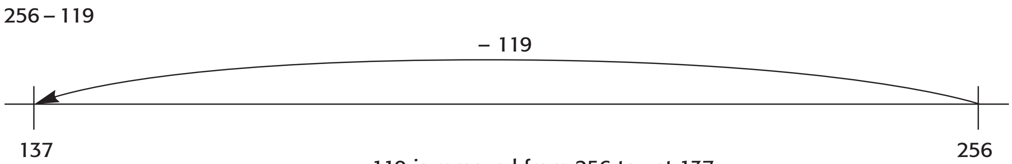
- Subtraction can be thought of as the distance or difference between two given numbers. On the following number line showing 256 – 119, the *difference* (137) is the space between 119 and 256. Thinking about subtraction as the *distance* between two numbers is evident in the adding-on strategy described below.

256 – 119



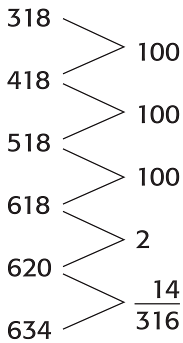
137 is added to 119 to get to 256.

- Subtraction can be thought of as the removal of a quantity from another quantity. On the following number line, the *difference* is found by removing (taking away) 119 from 256. Thinking about subtraction as *taking away* between two numbers is evident in the partial-subtraction strategy and the compensation strategy described below.



119 is removed from 256 to get 137.

Adding-on strategy: This strategy involves starting with the smaller quantity and adding on numbers until the larger quantity is reached. The sum of the numbers that are added on represent the difference between the larger and the smaller quantities. The following example illustrates how an adding-on strategy might be used to calculate 634 – 318:

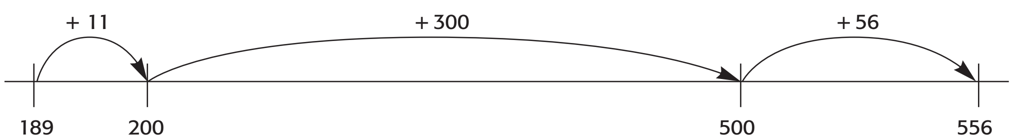


Students also might begin by adding on 300, rather than 3 hundreds, to get from 318 to 618.

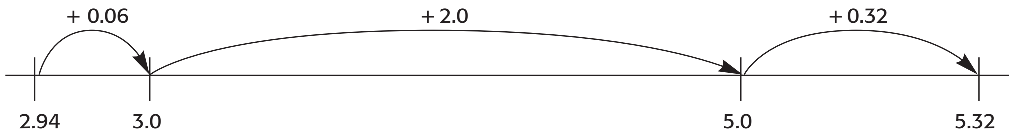
Another version of the adding-on strategy involves adding on to get to a friendly number first, and then adding hundreds, tens, and ones. For example, students might calculate 556 – 189 by:

- adding 11 to 189 to get to 200; then
- adding 300 to 200 to get to 500; then
- adding 56 to 500 to get to 556; then
- adding the subtotals, 11 + 300 + 56 = 367. The difference between 556 and 189 is 367.

A number line can be used to model the thinking behind this strategy.



An adding-on strategy can also be used to solve subtraction problems involving decimal numbers. For example, the following number line shows 5.32 – 2.94.



In this example, 0.06, 2.0, and 0.32 are added together to calculate the difference between 5.32 and 2.94. (0.06 + 2.0 + 0.32 = 2.38)

With an adding-on strategy, students need to keep track of the quantities that are added on. Students might use pencil and paper to record the numbers that are added on, or they might keep track of the numbers mentally.

Partial-subtraction strategy: With a partial-subtraction strategy, the number being subtracted is decomposed into parts, and each part is subtracted separately. In the following example, 325 is decomposed according to place value (into hundreds, tens, and ones).

856 – 325
300   20   5

856 – 300 = 556
556 – 20 = 536
536 – 5 = 531

The number being subtracted can also be decomposed into parts that result in a friendly number, as shown below.

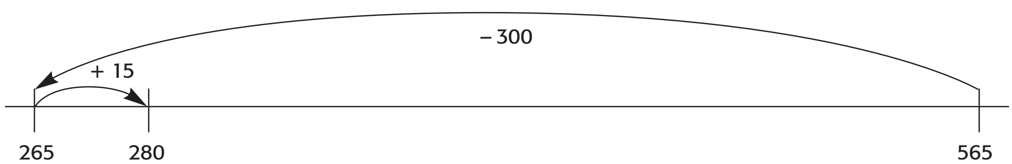
843 – 254
243   11

843 – 243 = 600 (11 left to subtract)
600 – 11 = 589

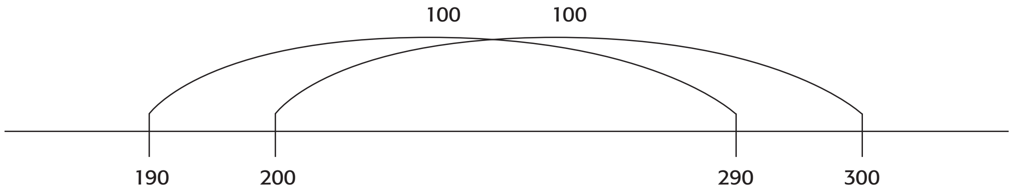
Compensation strategy: A compensation strategy for subtraction involves subtracting more than is required, and then adding back the extra amount. This strategy is particularly effective when the number being subtracted is close to a friendly number (e.g., a multiple of 10). In the following example, 565 – 285 is calculated by subtracting 300 from 565, and then adding back 15 (the difference between 285 and 300).

565 – 285
565 – 300 = 265
265 + 15 = 280

Modelled on the number line, compensation strategies look like big jumps backwards, and then small jumps forward:



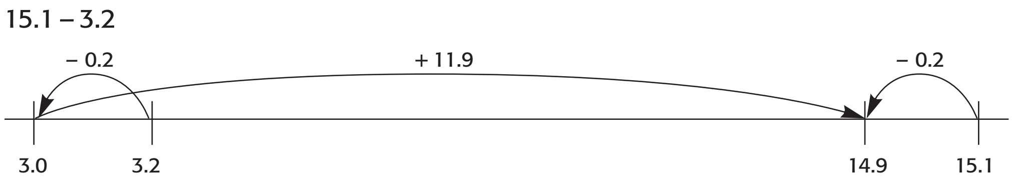
Constant-difference strategy: An effective strategy for solving subtraction problems mentally is based on the idea of a constant difference. Constant difference refers to the idea that the difference between two numbers does not change after adding or subtracting the *same quantity* to both numbers. In the following example, the difference between 290 and 190 is 100. Adding 10 to both numbers does not change the difference – the difference between 300 and 200 is still 100.



This strategy can be applied to subtraction problems. For example, a student might solve a problem involving 645 – 185 in the following way:

> *"If I add 15 to 185, it becomes 200, which is an easy number to subtract. But I have to add 15 to both numbers, so the question becomes 660 – 200, which is 460."*

A constant-difference strategy usually involves changing the number being subtracted into a friendlier number. As such, the strategy is useful in subtraction with decimal numbers, especially in problems involving tenths. To solve 15.1 – 3.2, for example, 0.2 could be subtracted from both 15.1 and 3.2 to change the problem to 14.9 – 3.0. The subtraction of a whole-number value (3.0), rather than the decimal number in the original problem, simplifies the calculation. The example is illustrated on the following number line.



SELECTING AN APPROPRIATE STRATEGY

As with all computational strategies, students should first examine the numbers in the problem before choosing a strategy. Removing hundreds, tens, and ones does not always work neatly with regrouping. For example, to calculate 731 – 465, a partial-subtraction strategy of subtracting 400, 60, and 5 is not necessarily an efficient strategy because of the regrouping required to subtract 6 tens from 3 tens. However, an adding-on strategy might be used: Add 35 to 465 to get to 500, add 200 to get to 700, add 31 to get to 731, and add 35 + 200 + 31 to calculate a total difference of 266. A constant-difference strategy could also be applied: Add 35 to both numbers to change the subtraction to 766 – 500.

Developing Estimation Strategies

It is important for students to develop skill in estimating sums and differences. Estimation is a practical skill in many real-life situations. It also provides a way for students to judge the reasonableness of a calculation performed with a calculator or on paper.

Selecting an appropriate strategy depends on the context of a given problem and on the numbers involved in the problem. Consider the following situation.

> *"Aaron needs to buy movie tickets for $8.25, popcorn for $3.50, and a drink for $1.75. About how much money should Aaron bring to the movies?"*

In this situation, students should recognize that an appropriate estimation strategy would involve rounding up each money amount to the closest whole-number value, so that Aaron has enough money.

The table below lists several estimation strategies for addition and subtraction. It is important to note that the word "rounding" is used loosely – it does not refer to any set of rules or procedures for rounding numbers (e.g., look to the number on the right, if it is greater than 5 then round up…).

Strategy	Example
Rounding each number to the nearest multiple of 10, 100, 1000, and so on	891 + 667 is about 900 + 700 = 1600 891 – 667 is about 890 – 670 = 220
Rounding numbers to friendly numbers	891 + 667 is about 900 + 650 = 1550 891 – 667 is about 900 – 650 = 250
Rounding one number but not the other	891 – 667 is about 900 + 667 = 1567
Rounding one number up and the other down (This strategy is more appropriate for addition than for subtraction.)	891 + 667 is about 900 + 660 = 1560
Rounding both numbers up or both numbers down (This strategy is more appropriate for subtraction than for addition.)	891 – 667 is about 900 – 700 = 200 891 – 667 is about 800 – 600 = 200
Finding a range	538 + 294 is between 700 (500 + 200) and 900 (600 + 300) 418 – 126 is between 200 (400 – 200) and 400 (500 – 100)
Using compatible numbers	626 + 328 is about 626 + 324 = 950 747 – 339 is about 747 – 347 = 400

Adding and Subtracting Decimal Numbers

Many of the addition and subtraction strategies described above also apply to computations with decimal numbers. (See the preceding examples involving decimal numbers under "Splitting strategy", "Adding-on strategy" for addition, "Adding-on strategy" for subtraction, and "Constant-difference strategy".)

Using the standard algorithm is a practical strategy for adding and subtracting decimal numbers in many situations. (The standard algorithm is impractical if the calculations can easily be performed mentally, or if the problem involves numbers that are best calculated using a calculator.) When teaching addition and subtraction with decimal numbers, teachers should develop strategies through problem-solving situations and strive to create meaningful contexts for the operations. For example, problems involving money expressed as decimal numbers provide contexts that can be relevant to students. As well, measurement problems (e.g., involving length or mass) often involve working with decimal numbers.

Perhaps the most difficult challenge students face with decimal-number computation is adding or subtracting numbers that do not share a common "end point" (e.g., adding tenths to thousandths, subtracting hundredths from tenths). Part of the difficulty arises from the lack of contextual referents – rarely are people called on in real-life situations to add or subtract numbers like 18.6, 125.654, and 55.26 in the same situational context.

It is more important that teachers emphasize place-value concepts when they help their students understand decimal-number computations by using algorithms. Rather than simply following the rule of "lining up the decimals" in an algorithm, students should recognize that like-units need to be added or subtracted – ones are added to or subtracted from ones, tenths to and from tenths, hundredths to and from hundredths, and so on. With an understanding of place value in an algorithm, students recognize that annexing zeros to the decimal part of a number does not change the value of the number. The following example shows how an addition expression can be rewritten by including zeros in the hundredths and thousandths places in one of the addends.

$$\begin{array}{r} 18.6 \\ +125.654 \\ \hline \end{array} \qquad \begin{array}{r} 18.600 \\ +125.654 \\ \hline \end{array}$$

Estimation plays an important role when adding and subtracting decimal numbers using algorithms. For example, students can recognize that 34.96 – 29.04 is close to 35 – 30, and estimate that the difference will be about 5. After completing the algorithm, students can refer back to their estimate to determine whether the result of their calculation is reasonable.

A Summary of General Instructional Strategies

Students in the junior grades benefit from the following instructional strategies:

- solving a variety of addition and subtraction problems, including joining, separating, comparing, and part-part-whole problems;
- using concrete and pictorial models, such as base ten materials and open number lines, to develop an understanding of addition and subtraction concepts and strategies;
- providing opportunities to connect subtraction to addition through problem solving;

- solving addition and subtraction problems that serve different instructional purposes (e.g., to introduce new concepts, to learn a particular strategy, to consolidate ideas);
- providing opportunities to develop and practise mental computation and estimation strategies.

The Grades 4–6 Addition and Subtraction module at www.eworkshop.on.ca provides additional information on developing addition and subtraction concepts with students. The module also contains a variety of learning activities and teaching resources.

eworkshop.on.ca

APPENDIX 2-1: DEVELOPING COMPUTATIONAL STRATEGIES THROUGH MINI-LESSONS

Introduction

> *"A number of researchers have argued that mental arithmetic . . . can lead to deeper insights into the number system"*
>
> (Kilpatrick, Swafford, & Findell, 2001, p. 214).

Developing efficient mental computational strategies is an important part of mathematics in the junior grades. Students who learn to perform mental computations develop confidence in working with numbers and are able to explore more complex mathematical concepts without being hindered by computations.

Mini-Lessons With Mental Math Strings

One method for developing mental computational skills is through the use of mini-lessons – short, 10- to 15-minute lessons that focus on specific computational strategies (Fosnot & Dolk, 2001a). Unlike student-centred investigations, mini-lessons are more teacher-guided and explicit. Each mini-lesson is designed to develop or "routinize" a particular mental math strategy.

A computational mini-lesson often involves a "string" – a structured sequence of four to seven related computations that are designed to elicit a particular mental computational strategy. The following is an example of a string that focuses on a compensation strategy for addition. This strategy involves adding more than is needed (often a multiple of 10) and then subtracting the extra amount.

46 + 10

46 + 9

64 + 20

64 + 19

36 + 19

The computations in this string are related to one another. Students know the answer to $46 + 10$, and they also know that $46 + 9$ is one less than $46 + 10$. The third computation, $64 + 20$, is like the first, only this time students are adding 20 instead of 10. They can calculate $64 + 19$ by knowing that the answer is one less than $64 + 20$. The last computation, $36 + 19$, has no "helper" (e.g., $46 + 10$ and $64 + 20$, shown in bold type, are "helper" computations for $46 + 9$ and $64 + 19$). However, the previous four computations follow a pattern that helps students to apply a compensation strategy. Students might consider $36 + 19$ and think: "$36 + 20 = 56$. But $36 + 19$ is one less, so $36 + 19 = 55$."

A mini-lesson usually proceeds in the following way:

- The teacher writes the first computation horizontally on the board and asks students to calculate the answer.
- Students are given time to calculate mentally. Students may jot down numbers on paper to help them keep track of figures, but they should not perform paper-and-pencil calculations that can be done mentally.
- The teacher asks a few students to explain how they determined the answer.
- The teacher models students' thinking on the board by using diagrams, such as open number lines, to illustrate various strategies.
- The teacher presents the remaining computations, one at a time. Strategies for each computation are discussed and modelled.
- After all computations have been solved, the focus strategy is identified and discussed.

Following the mini-lesson, the teacher should reflect on the effectiveness of the string in helping students to develop an understanding of the focus strategy. Reflecting on the mini-lesson will help to provide direction for future lessons. For example, the teacher may realize that students are not ready for a particular strategy and that they need more experience with a related concept first. Or the teacher might determine that students use a strategy effectively and are ready to learn a new one.

Mini-lessons can be used throughout the year, even when the main mathematics lesson deals with concepts from other strands of mathematics. Mini-lessons can take place before the regular math lesson or at any other time during the day.

In a mini-lesson, teachers might also pose an individual computation instead of strings. This approach encourages students to examine the numbers in the expression in order to determine an appropriate strategy (rather than looking at the helper computations to determine a strategy). The various strategies used by students are discussed and modelled.

Note: To develop confidence in teaching computational strategies with strings, teachers might work with a small group of students before they use mini-lessons with the whole class.

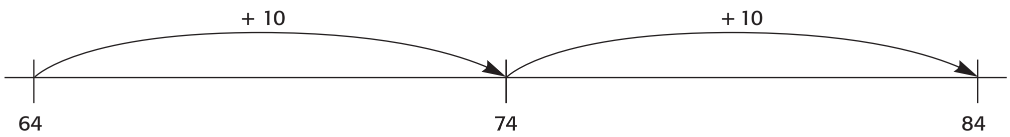
Modelling Student Thinking and Strategies

It is important for teachers to encourage students to communicate their thinking when they perform computations during mini-lessons. When students explain their thinking, they clarify their strategies for themselves and their classmates, and they make connections between different strategies. During mini-lessons, the teacher records student thinking on the board in order to demonstrate various strategies for the class.

The open number line provides an effective model for representing students' thinking and strategies. For example, a student might explain how he determined the answer to 64 + 20 in the string given above like this:

> *"Well, I started at 64, and then I added on 10's . . . 64 and 10 is 74, and 74 plus 10 is 84. So 64 + 20 is 84."*

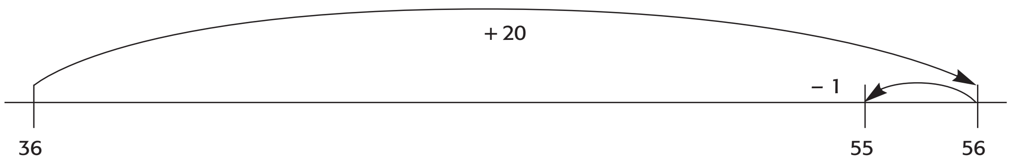
The teacher could illustrate the student's strategy by drawing a number line on the board.



Later, in the mini-lesson, another student might explain how she solved 36 + 19:

> *"I added 36 + 20 and got 56, but I knew that was too much because I was adding 19 and not 20, so I had to go back 1 to 55."*

The teacher's drawing of a number line helps the class understand the student's thinking.



The modelling of students' thinking helps the class to visualize strategies that might not be clearly understood if only oral explanations of those strategies are given. Recorded models also allow students to develop a mental image of different strategies. These images can help students to reason towards a solution when presented with other computations.

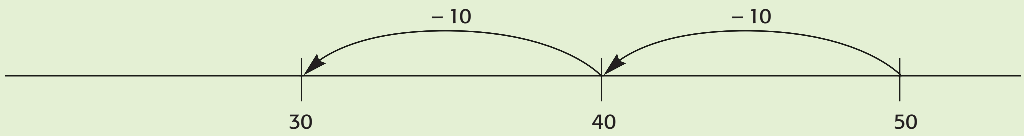
A Mini-Lesson in Action

The following scenario provides a description of a mini-lesson with a Grade 4 class. The teacher wants to highlight a compensation strategy for subtraction. This strategy involves subtracting more than is needed (often a multiple of 10), and then adding back the extra amount. In this lesson, the teacher uses the string shown at right. She developed the string prior to the lesson, putting considerable thought into developing a sequence of questions that highlight the focus strategy.

50 – 10
50 – 20
50 – 19
75 – 20
75 – 19
87 – 18
145 – 28

The teacher begins the mini-lesson by writing the first computation, 50 – 10, on the board. She asks, "Who knows the answer to this question? Show me a thumbs-up when you know it." Most students know the answer right away. Devon responds: "40. I subtracted 1 from the 5 to get 4, so the answer is 40." The teacher asks, "Really? When I subtract 1 from 5, I get 4 – not 40. How did you get 40?" Devon clarifies that the 1 he subtracted was actually a 10, since it was in the tens column. The teacher draws an open number line to show the jump backwards from 50 to 40.

Next, the teacher writes 50 – 20 on the board and again most students show their thumbs quickly. Keri offers her solution: "30. I just jumped backwards another 10." The teacher models Keri's thinking on the board by using an open number line.



When the teacher writes 50 – 19 on the board, the students are pensive, and only a few quickly offer a thumbs-up. She gives the class time to think about the question.

"Who knows this one?"

Laura answers "31", and the teacher asks her to explain how she figured out the answer.

"Well, on the second question we started at 50 and jumped back 20. That got us to 30. But for this one, I didn't have to jump back 20, I only needed to jump back 19, so I added 1 when I was done."

"When you were done?" asks the teacher.

"Yeah, when I was done jumping 20."

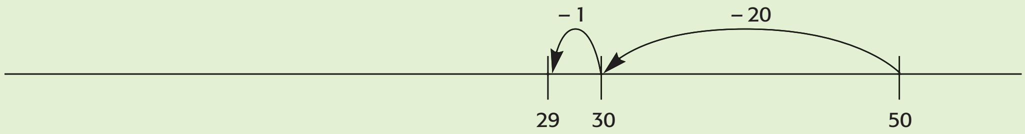
The teacher gives the class some time to think about this, and then asks if anyone can explain Laura's strategy.

Moira says, "I know what Laura was trying to do, but I don't get it. I know the answer is 31. I made 19 into 20, and then took 20 away from 50 to get 30. Then I added one more to get 31, but I don't get it."

The teacher asks, "What don't you get?"

"Well, if I added 1 to 19 to make 20, shouldn't I take it away at the end? That would give me 29, not 31."

The teacher models Moira's idea.

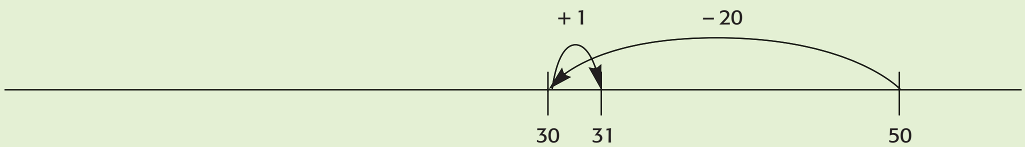


Moira is making a connection to a compensation strategy for addition that she is comfortable using. (To add $56 + 29$, 1 is added to 29 to make a friendly number of 30. At the end, she must compensate – she needs to take away the extra amount that was added to make the friendly number.)

Moira wonders aloud why her strategy would give the wrong answer. The teacher asks the class to consider Moira's question. Very few hands go up, and she wonders whether most students follow the discussion. After a while, Dennis thinks he has the answer to Moira's question.

"It's like this, Moira. You took 1 from somewhere to make 19 into 20. Now you have to put it back. If you take it away at the end, you're taking away 21, not 19. You're only supposed to take away 19, but 20 is easier, so you borrowed 1 from somewhere to take away 20. At the end you put it back."

The teacher draws another open number line.

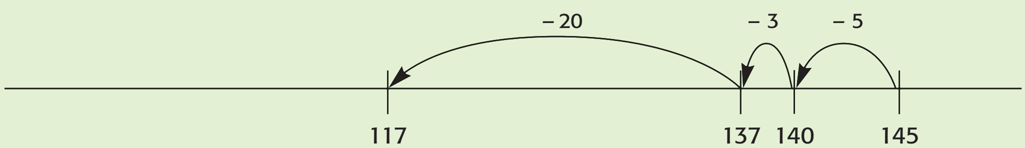


Dennis's explanation makes sense to many of the students. Moira sums up his explanation nicely. "It's like Dennis said – if I take away 20, I'm taking too much, so at the end I have to put some back."

The teacher continues with the string, and the students calculate $75 - 20$, $75 - 19$, and $87 - 18$. The class discusses strategies, and the teacher models the ideas on the board.

When she writes the final computation, $145 - 28$, the extra digit intimidates some students. "Whoa, now hundreds? We can't do this mentally."

Izzy determines the answer by decomposing 28 into $20 + 3 + 5$: "145 minus 5 is 140, take away 3 is 137. Then I took 20 away to get 117."



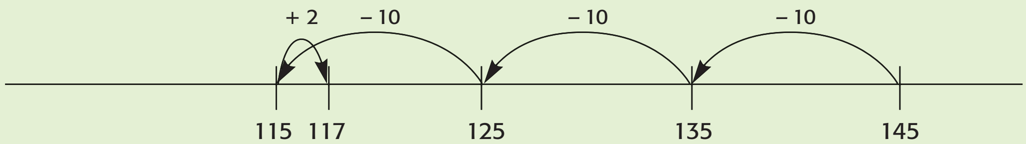
Izzy's strategy is effective and efficient, although it is not as efficient as a compensation strategy that involves subtracting 30 and adding back 2.

(*continued*)

Dennis refers back to his strategy. "Well, you could use 'put-it-back' too. 145 minus 30 is 115. Then because I only needed to take away 28, I add the 2 back at the end, so the answer is 117."

The teacher asks, "Did you jump 30 all at once, or make jumps of 10?"

"Jumps of 10," answers Dennis.



At the conclusion of the mini-lesson, the teacher recognizes that only a few students can confidently use the compensation strategy for subtraction. She observed that many students "jump back" by 10's, as did Dennis, rather than subtract a multiple of 10 (e.g., students think "145 – 10 – 10 – 10", rather than "145 – 30"). She decides to focus on a strategy that involves subtracting multiples of 10 in the next mini-lesson.

This mini-lesson provided the teacher with valuable feedback and direction for strategies to pursue in the future. She plans to revisit this strategy when students are more confidently able to subtract multiples of 10.

The effectiveness of the mini-lesson depends on the teacher's efforts to engage students in the activity. Specifically, the teacher:

- expects all students to try the computations in the strings;
- has students use a thumbs-up signal to show when they have completed the computation – this technique encourages all students to determine an answer;
- encourages students to explain their strategies;
- asks students to respond to others' strategies;
- asks students to clarify their explanations for others;
- accepts and respects students' thinking, even though their strategies may reflect misconceptions;
- poses questions that help students clarify their thinking;
- models strategies on the board so that students can "see" one another's thinking.

Developing Strings for Addition and Subtraction

In order to design strings and plan mini-lessons effectively, teachers must have an understanding of various mental computational strategies. The following are some different strategies for mental addition and subtraction.

Addition Strategies	Subtraction Strategies
Adding On With this strategy, the number being added is decomposed into parts, and each part is added separately. 136 + 143 136 + 100 is 236. 236 + 40 is 276. 276 + 3 is 279. So, 136 + 143 is 279.	***Partial Subtraction*** With this strategy, the number being subtracted is decomposed into parts, and each part is subtracted separately. 387 – 146 387 – 100 is 287. 287 – 40 is 247. 247 – 6 is 241. So, 387 – 146 is 241.
Compensation This strategy involves adding more than is required, and then subtracting the extra amount. 236 + 297 236 + 300 is 536. Subtract 3 (the difference between 297 and 300): 536 – 3 is 533. So, 236 + 297 is 533.	***Compensation*** This strategy involves subtracting more than is required, and then adding back the extra amount. 547 – 296 547 – 300 is 247. Add back 4 (the difference between 296 and 300): 247 + 4 is 251. So, 547 – 296 is 251.
Moving This strategy involves "moving" a quantity from one addend to another to create an expression with friendly numbers. 153 + 598 Move 2 from 153 to 598. 151 + 600 is 751. So, 153 + 598 is 751.	***Constant Difference*** The difference between two numbers does not change after adding or subtracting the same *quantity* to both numbers. 146 – 38 Add 2 to both numbers to create an expression with friendly numbers: 148 – 40 is 108. So, 146 – 38 is 108.

Strings are usually made up of pairs or groups of computations that are related. "Helper" computations (questions that students are able to answer easily) are followed by a computation that can be solved by applying the focus strategy. The following example shows the structure of a string that focuses on the adding-on strategy. In this case, the strategy involves adding the tens from the second addend first, and then adding on the ones.

42 + 10

56 + 10

56 + 12

35 + 20

35 + 26

73 + 24

136 + 42

The helpers activate prior knowledge that will help to develop the strategy.

These computations are related to the helpers and can be solved using the target strategy.

This final computation is given without any helpers to see whether students can apply the strategy developed through the previous computations.

The following are examples of strings that are based on the computation strategies explained above.

Examples of Addition Strings		
Adding On	***Using Compensation***	***Moving***
47 + 20	**34 + 40**	**45 + 30**
47 + 3	34 + 39	46 + 29
47 + 23	34 + 38	47 + 28
147 + 20	**36 + 200**	**24 + 300**
147 + 25	36 + 199	25 + 299
147 + 35	36 + 198	27 + 297
341 + 36	134 + 396	216 + 496

Examples of Subtraction Strings		
Using Partial Subtraction	***Using Compensation***	***Using Constant Difference***
85 – 20	**56 – 30**	**50 – 25**
85 – 3	56 – 29	51 – 26
85 – 23	56 – 28	49 – 24
275 – 100	**344 – 200**	**72 – 30**
275 – 40	344 – 199	73 – 31
275 – 3	344 – 197	71 – 29
275 – 143	546 – 196	64 – 29

Mini-lessons with strings are not intended to be a means of teaching a prescribed list of computational procedures. Rather than simply following a series of computations in a resource book, teachers should develop their own strings based on the needs of their students. In developing strings, teachers need to focus on particular computational strategies that will extend students' skill in mental computation. Whenever possible, a string should relate to, or be an extension of, a mental strategy that students have already practised.

Careful thought should go into the development of a string. Thinking about the computations presented in a string, as well as possible student responses, allows teachers to anticipate how the mini-lesson might unfold. Teachers should consider *alternative* strategies students might use (strategies that are different from the intended focus strategy). Teachers need to ask themselves: Why might students come up with alternative strategies? How are these alternative strategies related to the focus strategy? How can models, such as open number lines, help students to see the relationship between different strategies?

Often, student responses determine the direction teachers should take in developing subsequent strings. If students experience difficulties in using a focus strategy, teachers should consider whether students need practise with a related, more fundamental, strategy first. As well, teachers need to consider whether the string used in the mini-lesson was well crafted and constructed, or whether other computations would have been more effective in developing the strategy.

Strings for Multiplication and Division

Strings used for multiplication and division are similar to those used for addition and subtraction:

- Each string focuses on a particular strategy.
- A string comprises helper computations as well as computations that can be solved by applying the focus strategy.
- Teachers should model students' strategies to illustrate students' thinking.

The following is an example of a multiplication string that focuses on the use of the distributive property in mental computation.

8 × 5

8 × 40

8 × 45

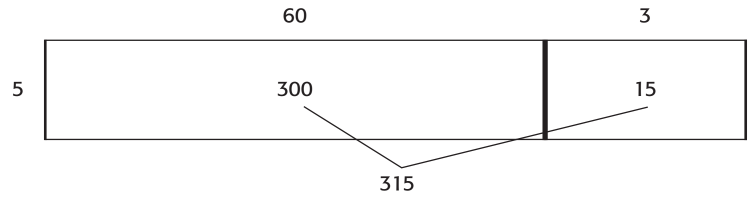
6 × 4

6 × 30

6 × 34

5 × 63

This string helps students to understand that a multiplication expression, such as 63×5, can be calculated by multiplying ones by tens ($5 \times 60 = 300$), then multiplying ones by ones ($5 \times 3 = 15$), and then adding the partial products ($300 + 15 = 315$).



The open array provides a model for demonstrating this strategy.

The open array helps students to visualize how 63 can be decomposed into 60 and 3, then each part can be multiplied by 5, and then the partial products can be added to determine the total product.

Conclusion

Learning mathematics is effective when it is done collaboratively among students. The same can be said for teachers as they begin to develop strings and develop computational strategies using mini-lessons. Working with other teachers allows for professional dialogue about strategies and student thinking.

Teachers can find more information on developing mini-lessons with math strings in several of the resources listed on the following page; specifically, the three *Young Mathematicians at Work* volumes by Fosnot and Dolk (2001a, 2001b, 2001c), and the books on mini-lessons by Fosnot, Dolk, Cameron, and Hersch (2004) and Fosnot, Dolk, Cameron, Hersch, and Teig (2005).

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Learning Activities for Addition and Subtraction

Introduction

The following learning activities for Grades 4, 5, and 6 provide teachers with instructional ideas that help students achieve some of the curriculum expectations related to addition and subtraction. The learning activities also support students in developing their understanding of the big ideas outlined in Volume 1: The Big Ideas.

The learning activities do not address all concepts and skills outlined in the curriculum document, nor do they address all the big ideas – one activity cannot fully address all concepts, skills, and big ideas. The learning activities demonstrate how teachers can introduce or extend mathematical concepts; however, students need multiple experiences with these concepts to develop a strong understanding.

Each learning activity is organized as follows:

OVERVIEW: A brief summary of the learning activity is provided.

BIG IDEAS: The big ideas that are addressed in the learning activity are identified. The ways in which the learning activity addresses these big ideas are explained.

CURRICULUM EXPECTATIONS: The curriculum expectations are indicated for each learning activity.

ABOUT THE LEARNING ACTIVITY: This section provides guidance to teachers about the approximate time required for the main part of the learning activity, as well as the materials, math language, instructional groupings, and instructional sequencing for the learning activity.

ABOUT THE MATH: Background information is provided about the mathematical concepts and skills addressed in the learning activity.

GETTING STARTED: This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or task.

WORKING ON IT: In this part, students work on the mathematical activity, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.

REFLECTING AND CONNECTING: This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.

ADAPTATIONS/EXTENSIONS: These are suggestions for ways to meet the needs of all learners in the classroom.

ASSESSMENT: This section provides guidance for teachers on assessing students' understanding of mathematical concepts.

HOME CONNECTION: This section is addressed to parents or guardians, and includes an activity for students to do at home that is connected to the mathematical focus of the main learning activity.

LEARNING CONNECTIONS: These are suggestions for follow-up activities that either extend the mathematical focus of the learning activity or build on other concepts related to the topic of instruction.

BLACKLINE MASTERS: These pages are referred to and used throughout the learning activities.

Grade 4 Learning Activity
Rising Waters

OVERVIEW

In this learning activity, students find different ways to fill a Kindergarten water table basin with exactly 25 L of water using different-sized containers. The activity provides an opportunity for students to explore different strategies for adding and subtracting decimal numbers to tenths.

BIG IDEAS

This learning activity focuses on the following big ideas:

Quantity: Students explore the "howmuchness" of decimal numbers to tenths by comparing and ordering the capacity of different containers.

Operational sense: Students apply their understanding of addition and subtraction to perform calculations with decimal numbers.

Relationships: This learning activity allows students to see the relationship between tenths and wholes (e.g., that 10 tenths makes 1 whole).

Representation: Students represent decimal amounts visually by using concrete materials, and symbolically by using decimal notation.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.
Students will:
- represent, compare, and order decimal numbers to tenths, using a variety of tools (e.g., concrete materials such as paper strips divided into tenths and base ten materials, number lines, drawings) and using standard decimal notation;
- add and subtract decimal numbers to tenths, using concrete materials (e.g., paper strips divided into tenths, base ten materials) and student-generated algorithms (e.g., "When I added 6.5 and 5.6, I took five tenths in fraction circles and added six tenths in fraction circles to give me one whole and one tenth. Then I added 6 + 5 + 1.1, which equals 12.1.").

These specific expectations contribute to the development of the following **overall expectation**.
Students will:
- solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies.

ABOUT THE LEARNING ACTIVITY

TIME: approximately 60–90 minutes

MATERIALS

- overhead transparency of **AddSub4.BLM1: What's The Capacity?**
- overhead projector
- overhead marker
- manipulatives for modelling decimal numbers to tenths (e.g., fraction circles, paper strips cut from **AddSub4.BLM2: Paper Strips Divided Into Tenths**, base ten blocks)
- sheets of paper (1 per pair of students)
- half sheet of chart paper or a large sheet of newsprint (1 per pair of students)
- markers (a few per pair of students)
- **AddSub4.BLM3a–c: Addition and Subtraction Puzzles** (1 per student)

MATH LANGUAGE

- decimal number
- tenths
- addition
- sum
- difference
- subtraction
- capacity
- litres
- friendly number

Note: To connect decimal numbers to their meaning, it is helpful to read 2.6 as "two and six tenths", rather than as "two point six" or "two decimal six".

INSTRUCTIONAL GROUPING: pairs

INSTRUCTIONAL SEQUENCING

Before starting this learning activity, students should have had opportunities to represent, compare, and order decimal numbers to tenths using a variety of tools (e.g., fraction circles, paper strips divided into tenths, base ten blocks, number lines, drawings). This learning activity allows students to solve a problem by applying their understanding of decimal numbers and of addition and subtraction.

ABOUT THE MATH

In Grade 4 students learn to add and subtract decimal number to tenths. In this learning activity, students have an opportunity to add decimal numbers using strategies that make sense to them (e.g., using manipulatives, using drawings, using number lines, using reasoning), without having to follow the steps in an algorithm. The learning activity also provides experience in adding decimal numbers to determine friendly whole-number values (e.g., $1.1 + 2.9 = 4$, $1.4 + 3.6 = 5$).

GETTING STARTED

Display an overhead transparency of **AddSub4.BLM1: What's the Capacity?** Draw students' attention to the numbers on the bottom of the overhead transparency, and explain that these numbers represent the capacities of the different containers. Review the meaning of "capacity" (the greatest amount or volume that a container can hold).

Ask:

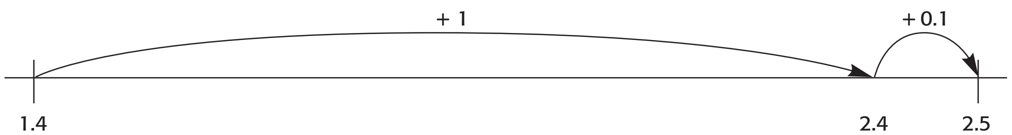
- "Which container do you think has the greatest capacity?"
- "Which capacity listed at the bottom of the transparency corresponds to this container?"
- "How do you know that 3.6 L is the greatest capacity?"

Have students work with a partner to match the other four capacities with containers. Have students explain their rationales for matching each capacity with a container. Use an overhead marker to record the capacity below each container (dish soap, 1.1 L; laundry soap, 2.9 L; water, 0.5 L; juice, 1.4 L).

Explain that containers are sometimes reused to carry water to pour into other containers. Ask: "How much water would fill both the dish soap container and the juice container?"

Provide access to manipulatives (e.g., fraction circles, paper strips cut from **AddSub4.BLM2: Paper Strips Divided Into Tenths**, base ten blocks), and invite students to use the manipulatives to help them determine a solution. Allow approximately one minute for students to work individually, and then have them explain their strategies to a partner.

Invite a few students to share their strategies with the class. Encourage them to demonstrate their thinking using manipulatives or diagrams. For example, students might use base ten blocks or fraction strips to represent the capacity of each container (1.1 L and 1.4 L), and then combine the materials to determine the total capacity. Other students might add the two capacities mentally. A possible mental computation might involve adding 1.4 L and 1 L to get 2.4 L, and then adding 0.1 to get 2.5 L. Model addition strategies by drawing a number line on the board to help students visualize addition processes.



WORKING ON IT

Explain the problem situation.

> "The Kindergarten teacher often needs to fill the water table basin with water. The basin is too heavy to carry when it is filled with water, so the teacher wants to use empty plastic containers to fill the basin. He doesn't want to fill the basin to the top, because the water will spill over the sides when the children are playing in it. He has discovered that 25 L of water is the ideal capacity."

Refer to the overhead transparency of **AddSub4.BLM1: What's the Capacity?** and explain the problem:

> "The Kindergarten teacher found 5 empty containers that he can use to fill the water basin. The containers have capacities of 0.5 L, 1.1 L, 1.4 L, 2.9 L, and 3.6 L. He can use 1, 2, 3, 4, or all 5 containers. How can he use these containers to fill the water table basin with 25 L of water? Is there more than one way to fill the water basin?"

Organize students into pairs. Explain that students will work with their partner to solve the problem. Encourage them to use manipulatives (e.g., fraction circles, fraction strips, base ten blocks) or diagrams (e.g., open number lines) to help them determine a solution. Provide each pair of students with a sheet of paper on which they can record their work.

STRATEGIES STUDENTS MIGHT USE

USING TRIAL AND ERROR
Students might try different combinations of containers to reach a total of 25 L.

USING FRIENDLY NUMBERS
Students might find that certain combinations of containers provide whole-number capacities that are easy to work with. For example, the 3.6 L and 1.4 L containers can be added to get 5 L. Students would reason that they would need to combine five 3.6 L and five 1.4 L containers to get 25 L.

USING REPEATED ADDITION
Students might try to maximize the use of the largest container and repeatedly add 3.6 L six times until they get to 21.6 L. (Adding 3.6 seven times results in 25.2, a value that is greater than 25.) To get from 21.6 L to 25 L, students would add on containers that provide a capacity of 3.4 L (e.g., 2.9 L + 0.5 L).

Observe students as they work. Ask them questions about their strategies and solutions:
- "What strategy are you using to solve the problem?"
- "What is working well with your strategy? What is not working well?"
- "How can you prove that your solution is correct?"
- "Can you find another solution?"

After pairs of students have found one or more solutions, provide them with markers and a half sheet of chart paper or a large sheet of newsprint. Ask students to show how they solved the problem in a way that can be clearly understood by the Kindergarten teacher. Encourage them to use diagrams (e.g., open number lines) to show their thinking.

REFLECTING AND CONNECTING

Have pairs of students present their solutions to the class. Try to include a variety of strategies, solutions, and formats. Have each pair justify their solution(s) by asking them to prove that each combination of containers would provide 25 L of water. Encourage students to clarify their understanding of presented solutions and strategies by asking questions such as:
- "What strategy did the presenters use to determine the total number of litres?"
- "Why do you think the presenters used that strategy?"
- "What questions about the strategy or solution do you have for the presenters?"

Post students' work in the classroom following each presentation. After several pairs have explained their strategies, ask:

- "Which solutions would you recommend to the Kindergarten teacher? Why?"
- "Which solutions would you not recommend? Why?"

(Students might assess the suitability of different solutions by considering the number of times containers need to be filled.)

Draw students' attention to the different formats used to record solutions. Ask questions such as:

- "In what different ways did pairs record their strategies and solutions?"
- "Which forms are easy to understand? Why is the work clear and easy to understand?"

ADAPTATIONS/EXTENSIONS

Simplify the problem for students who experience difficulties by reducing the capacity of the water table basin to 10 L and/or reducing the number of containers. Prompt students to search for friendly numbers by asking:

- "What combination of containers would create a friendly number?"
- "How can you use this friendly number to get close to the capacity of the basin?"

Extend the activity for students requiring a greater challenge by posing the following problems:

- "What is the fewest number of containers that you could use to fill the basin? How many times would you need to fill each container?"
- "What is the greatest number of containers that you could use? How many times would you need to fill each container?"

ASSESSMENT

Have students, individually, solve the following problem. Ask students to record their solutions, reminding them to show their ideas in a way that can be clearly understood by others.

> "How can you fill a bucket with 20 L of water using any of these five containers with the following capacities: 0.8 L, 1.3 L, 1.5 L, 2.2 L, 2.7 L? You may use one or more of the containers in your solution."

Encourage students to use manipulatives (e.g., fraction circles, fraction strips, base ten blocks) or diagrams (e.g., open number lines) to help them determine a solution. Observe students' work to assess how well they:

- add the capacities of containers using manipulatives and/or diagrams;
- solve the problem by finding a combination of containers with a capacity of 20 L;
- demonstrate their thinking using materials, drawings, and/or written explanations;
- explain their strategy and solution.

HOME CONNECTION

Send home **AddSub4.BLM3a–c: Addition and Subtraction Puzzles**. In this Home Connection activity, students and parents solve puzzles involving the addition and subtraction of decimal numbers. Include a copy of **AddSub4.BLM2: Paper Strips Divided Into Tenths**, and encourage students to use the strips when they work on the puzzles with their parents. (You may want to send home the puzzle solutions on **AddSub4.BLM3c** at the same time.) Make time in class for students to share their puzzle solutions and strategies.

LEARNING CONNECTION 1
Bedtime Treats!

MATERIALS

- base ten blocks (rods and small cubes) or paper strips cut from **AddSub4.BLM2: Paper Strips Divided Into Tenths**
- chocolate bar with ten sections*
- sheets of paper (1 per pair of students)

Describe the following scenario:

> "Every night before going to bed, Lee's grandparents have some chocolate. Because they are trying not to eat too much candy, they eat only part of a chocolate bar each time. The last time Lee visited his grandparents, he noticed that they had four chocolate bars in their "treat cupboard".

Show students a chocolate bar divided into tenths. Ask:
- "How many equal sections does the chocolate bar have?"
- "How can you use base ten blocks or paper strips to represent all the chocolate bars Lee's grandparents had in their cupboard?"

Have a student demonstrate how 4 rods or 4 paper strips could be used to represent the chocolate bars.

Explain that on the first night of Lee's visit, his grandparents opened one chocolate bar and ate 6 sections. Ask the following questions:
- "What fraction of the bar did Lee's grandparents eat?" (6 tenths)
- "What amount of the chocolate bars is left? (3 and 4 tenths) How do you know?"

Have students use base ten blocks or paper strips to explain the remaining amount of the chocolate bars. Together, discuss how to represent the situation using a number sentence. Record "$4.0 - 0.6 = 3.4$" on the board.

Record the following on the board:
- Night 2: 6 sections
- Night 3: 8 sections
- Night 4: 10 sections

* Ensure that the chocolate bar used is free of allergens.

Explain that Lee's grandparents ate 6 sections on the second night of his visit, 8 sections on the third night, and 10 sections on the last night.

Have students work with a partner. Ask them to find the amount of chocolate that was left each night after Lee's grandparents ate their treat. Encourage students to use base ten blocks, paper strips, and open number lines. Provide each pair of students with a sheet of paper on which they can record their work. Ask students to record a number sentence that represents the subtraction situation for each night.

Ask pairs of students to explain how they determined the remaining amount of chocolate each night. Discuss the meaning of the number sentences that students used to represent the subtraction situations.

LEARNING CONNECTION 2
Closest to One

MATERIALS

- spinners made from **AddSub4.BLM4: Closest-to-One Spinner**, a paper clip, and a pencil (1 per pair of students)
- base ten blocks (rods and small cubes)
- paper strips from **AddSub4.BLM2: Paper Strip Divided Into Tenths**
- sheets of paper (1 per student)

Explain the game to the class:

- Students play with a partner. Both players begin with a score of 10.
- One player spins the spinner, reads the number indicated by the spinner, and subtracts the number from 10. The player may use base ten blocks, paper strips divided into tenths, and/or open number lines to help him or her subtract. The player records the new score (the difference between 10 and the spinner number) on a piece of paper.
- The second player takes a turn.
- Players continue to take turns. On each turn, the player subtracts the number indicated on the spinner from the score achieved on his or her previous turn.
- Each player decides when he or she will stop spinning.
- The player with a final score that is closest to 1 wins the game.

Provide an opportunity for students to play a few rounds of the game. As students play the game, observe how they represent decimal amounts using concrete materials and decimal notation. Ask students to demonstrate how they use materials to help them subtract.

LEARNING CONNECTION 3
Decimal Number Triathlon

MATERIALS

- metre sticks
- paper and pencil
- **AddSub4.BLM5: Decimal Number Triathlon** (1 per group of 4 students)
- overhead transparency of **AddSub4.BLM5: Decimal Number Triathlon**
- overhead projector
- overhead marker

Create, or have your students suggest, three activities that meet the following requirements:

- The results of the activity can be measured to the nearest tenth of a metre.
- The activity can be done safely in the classroom (or in the gymnasium or on the playground).
- The activity requires minimal equipment.
- The activity is fun for all students.

Here are some examples of activities in which the results (distance) can be measured to the nearest tenth of a metre:

- flicking a penny with your thumb or finger;
- kicking a crumpled tissue or scrap of paper;
- blowing a cotton ball or ping-pong ball along a flat surface.

Organize students into groups of four. Have groups rotate through the three activities. (If possible, set up two centres for each event. Have half the class rotate through one set of three events, and the other half of the class through the other set.) At each centre, students perform the activity, measure one another's results (to the closest tenth of a metre) using metre sticks, and record the results on a copy of **AddSub4.BLM5: Decimal Number Triathlon**.

After students have completed all three activities, reconvene the class. Invite individual students to share their results for the different activities, and use an overhead marker to fill in the chart on an overhead transparency of **AddSub4.BLM5: Decimal Number Triathlon**.

Pose questions that can be answered by adding or subtracting the data found in the chart (e.g., "What was the combined distance of Student A and Student B in the flick-a-penny event?", "How much farther did Student C blow a cotton ball than Student D?").

In groups of four, have students write three or four addition and subtraction questions that can be answered using the data in the chart. Ask students to work together as a group to answer the questions. Encourage students to use concrete materials and/or open number lines to help them find solutions.

LEARNING CONNECTION 4
Number-Line Bingo

MATERIALS

- spinners made from **AddSub4.BLM6: Number-Line Bingo Spinner**, a paper clip, and a pencil (1 per pair of students)
- sheets of paper (1 per student)
- **AddSub4.BLM7: Number-Line Bingo** (1 per pair of students)

Explain the game to the class:

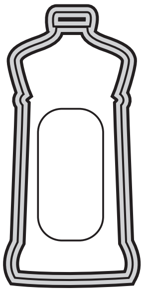
- Students play the game with a partner.
- Players each spin the spinner twice and record the two numbers indicated by the spinner on a sheet of paper.
- Players use the numbers to create addition and subtraction sentences. For example, if a player's numbers are 0.6 and 0.7, he or she could create the following addition and subtraction sentences: $0.6 + 0.7 = 1.3$; $0.7 + 0.6 = 1.3$; or $0.7 - 0.6 = 0.1$.
- Players exchange papers to check each other's calculations.
- If players' answers are correct, they circle the corresponding numbers (the sums or differences) on their individual number line on **AddSub4.BLM7: Number-Line Bingo**.
- Players continue to take turns spinning the spinner twice, creating addition and subtraction sentences, and circling answers on their number lines. (If a number is already circled, players do not need to circle the number again.)
- The first player to circle all the numbers on his or her number line, or the player who has the most numbers circled at the end of a specified time, wins the game.

eWORKSHOP CONNECTION

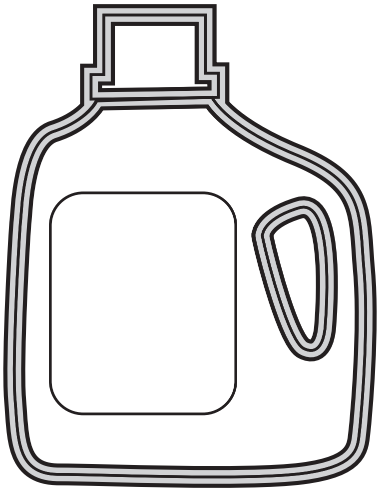
Visit www.eworkshop.on.ca for other instructional activities that focus on addition and subtraction concepts. On the homepage, click "Toolkit". In the "Numeracy" section, find "Addition and Subtraction (4 to 6)", and then click the number to the right of it.

eworkshop.on.ca

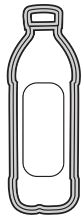
What's the Capacity?



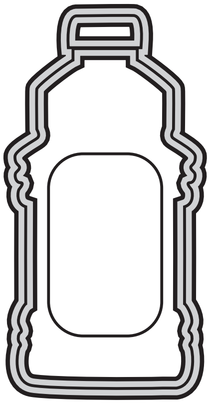
Dishwashing Detergent



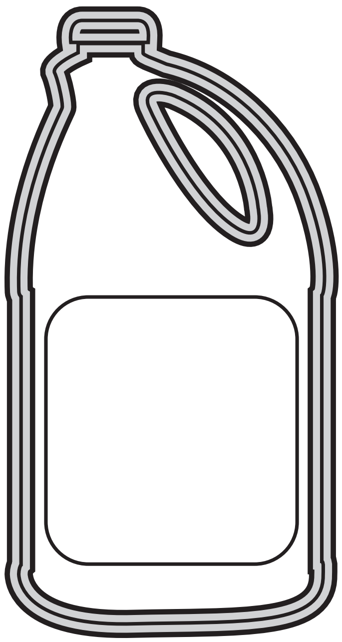
Laundry Detergent



Water



Juice



Fabric Softener

1.4 L   0.5 L   3.6 L   2.9 L   1.1 L

Paper Strips Divided Into Tenths

Addition and Subtraction Puzzles

Dear Parent/Guardian:

We have been learning about adding and subtracting decimal numbers. Attached are some addition and subtraction puzzles for you and your child to solve together.

Have your child cut out the six circles containing the decimal numbers for the first puzzle. Ask your child to place the number circles on the empty circles in the triangle so that a true addition or subtraction sentence is formed on each side of the puzzle. Have your child glue the number circles onto the puzzle or write the numbers in the empty circles. Ask him or her to repeat for each of the remaining puzzles.

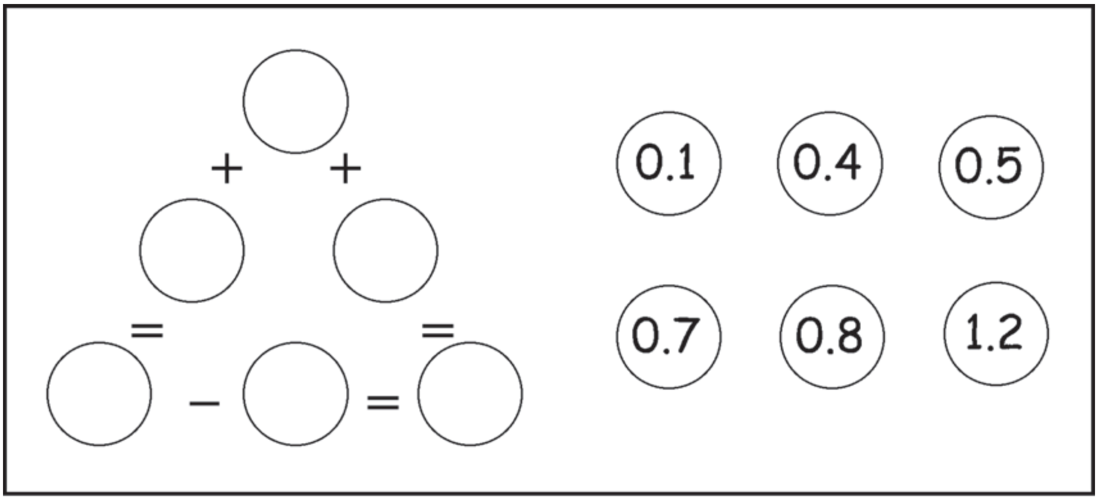
As you and your child work on these puzzles, encourage him or her to add and subtract decimal numbers using the paper strips divided into tenths. (You might look at the solution after finishing the puzzle, or use one of the correct numbers as a clue to help you get started.)

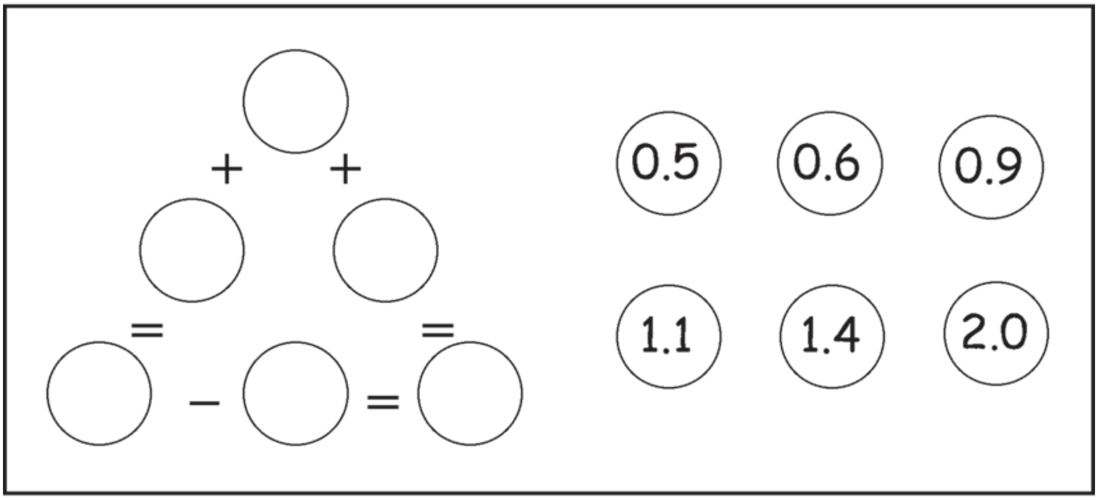
In class, students may want to share their solutions and strategies for these puzzles.

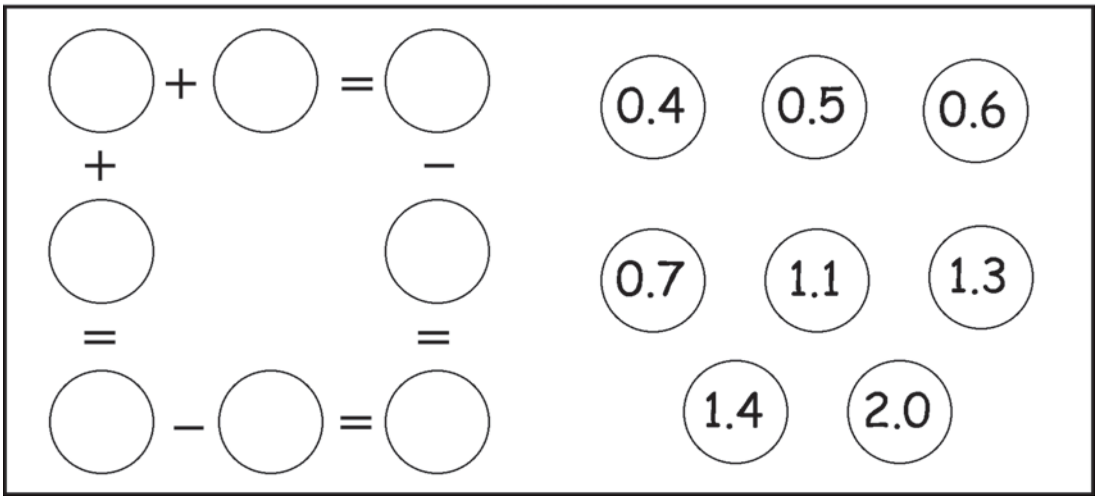
You and your child might try to create your own decimal number puzzle. If you do, have your child bring the puzzle to school for our class to solve.

Thank you for doing this activity with your child.

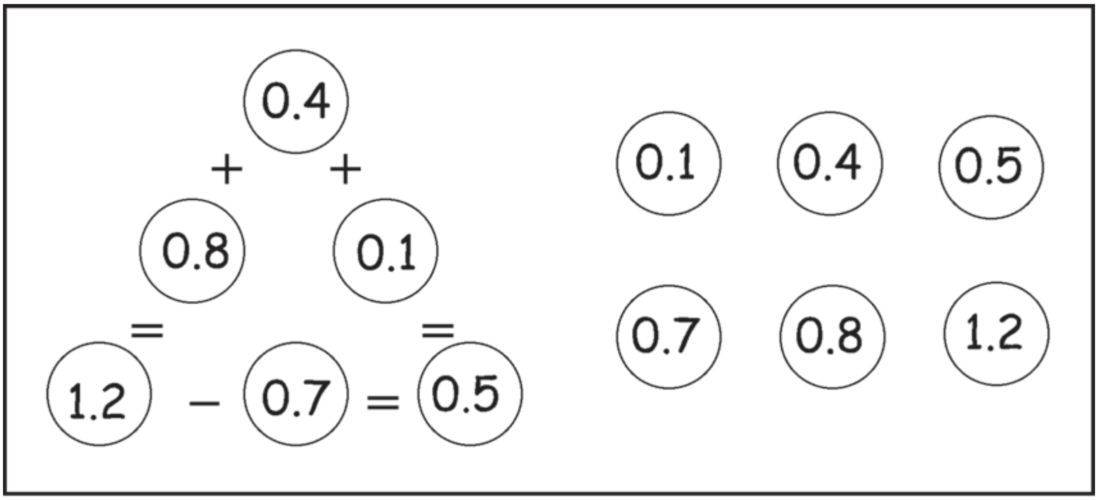
Addition and Subtraction Puzzles

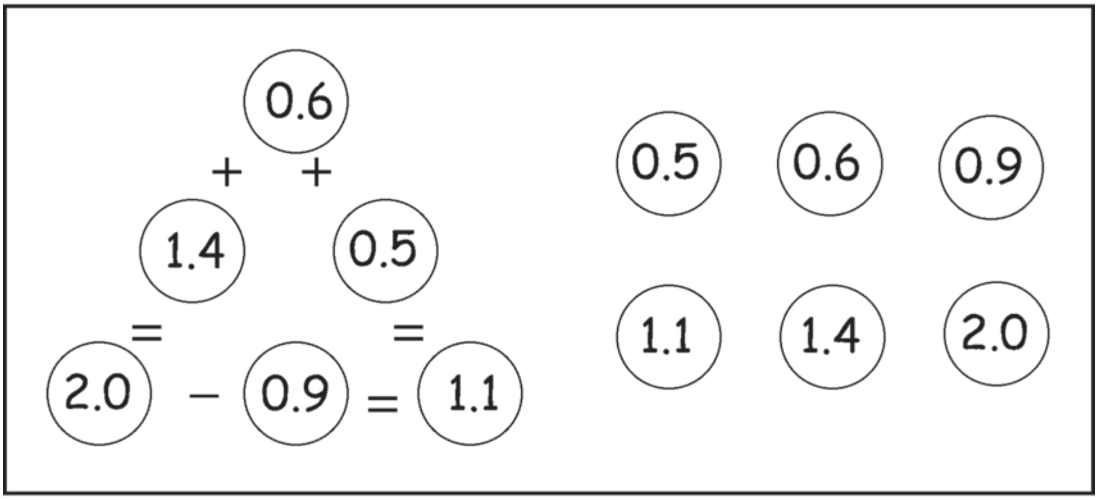


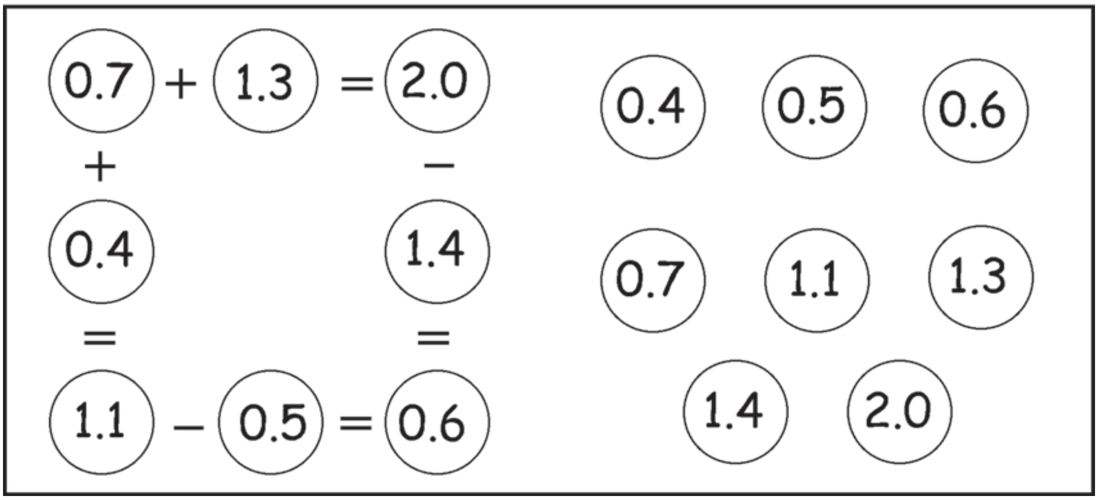




Addition and Subtraction Puzzles (Solutions)

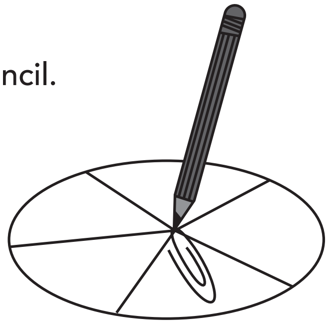


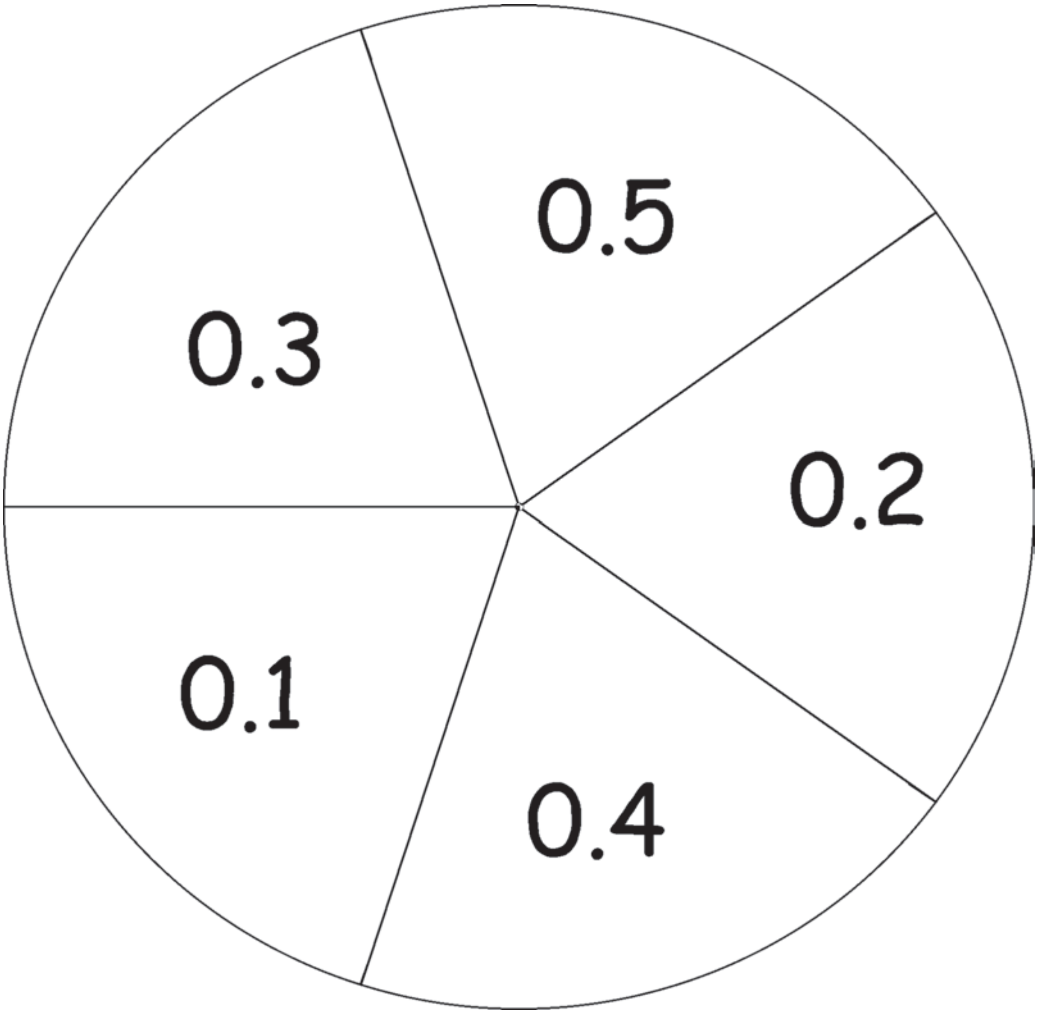




Closest-to-One Spinner

Make a spinner using this page, a paper clip, and a pencil.



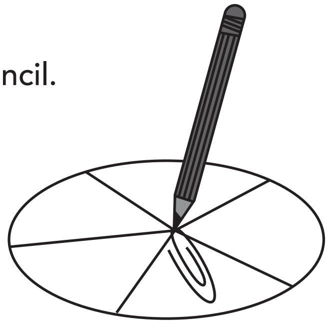


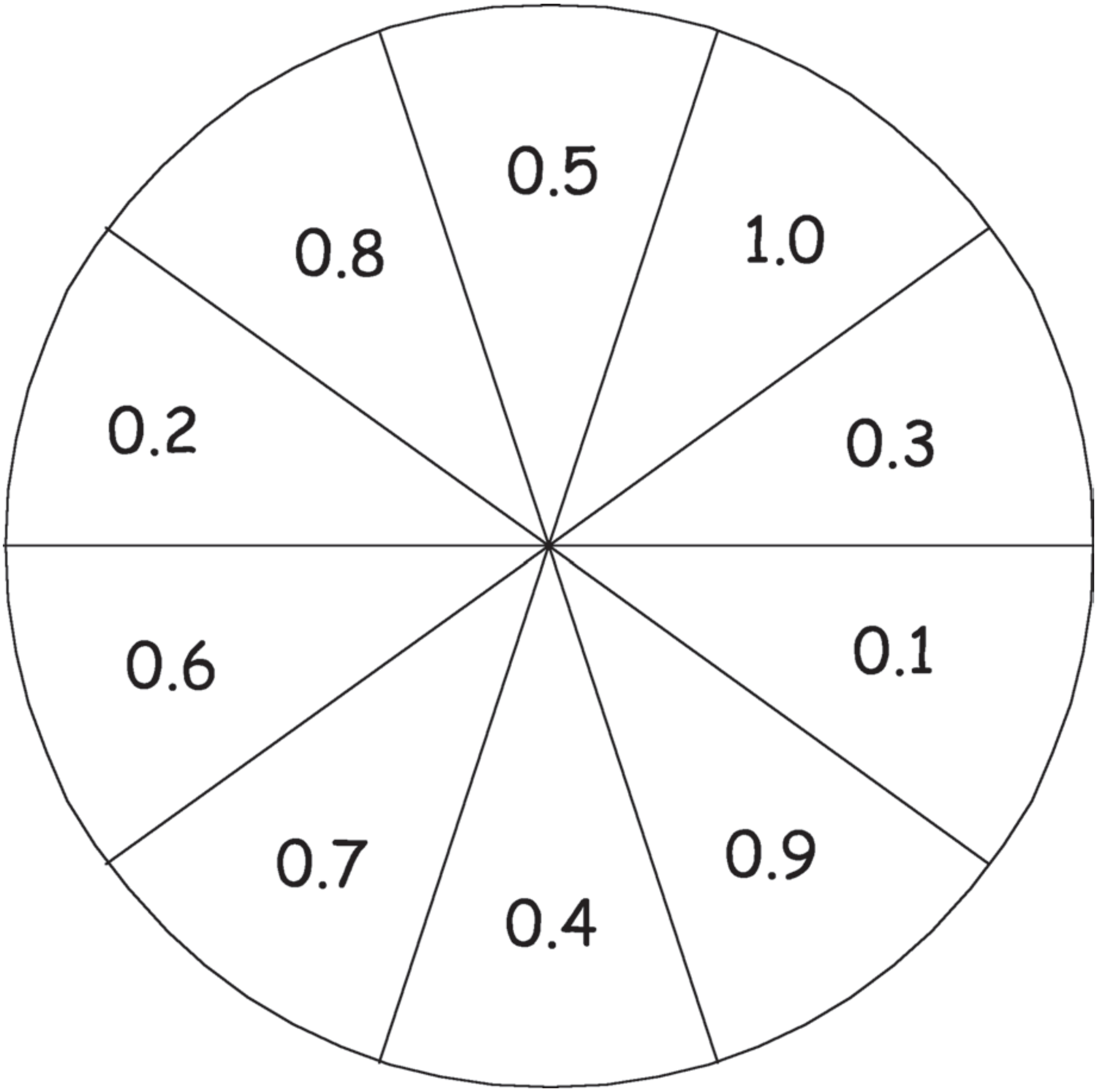
Decimal Number Triathlon

Students	Activity		

Number-Line Bingo Spinner

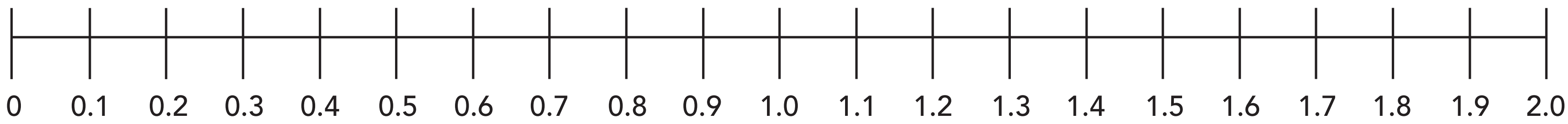
Make a spinner using this page, a paper clip, and a pencil.



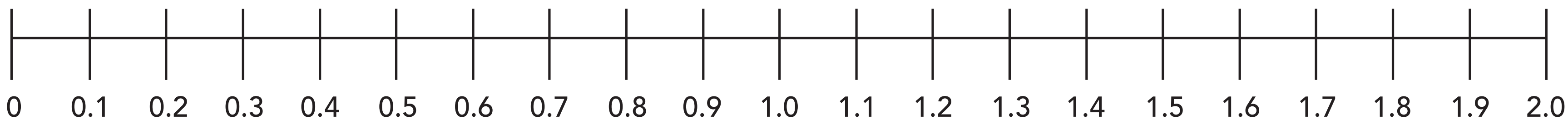


Number-Line Bingo

Player: ______________________



Player: ______________________



Grade 5 Learning Activity
Reaching a Goal

OVERVIEW

In this learning activity, students determine the amount of money that needs to be raised by students in junior grades in a school fundraising project. To solve this problem, students use a variety of addition and subtraction strategies, including paper-and-pencil and mental computational methods. After solving the problem, students discuss and reflect on the efficiency of various strategies.

BIG IDEAS

This learning activity focuses on the following big ideas:

Operational sense: Students explore a variety of strategies for adding and subtracting decimal numbers to hundredths (money amounts).

Relationships: Students compare decimal numbers expressed as money amounts. They also consider how number relationships help to determine appropriate and efficient computational strategies (e.g., when a mental strategy is more efficient than using a standard algorithm).

Representation: Students represent decimal numbers visually by using concrete materials, and symbolically by using decimal notation.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectation**.
Students will:

- add and subtract decimal numbers to hundredths, including money amounts, using concrete materials, estimation, and algorithms (e.g., use 10 × 10 grids to add 2.45 and 3.25).

This specific expectation contributes to the development of the following **overall expectation**.
Students will:

- solve problems involving the multiplication and division of multidigit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies.

ABOUT THE LEARNING ACTIVITY

TIME: approximately 60 minutes

MATERIALS

- sheets of paper (1 per pair of students)
- a variety of concrete materials for representing decimal numbers to hundredths (e.g., money sets, 10 × 10 grids, base ten materials)
- half sheets of chart paper or large sheets of newsprint (1 per pair of students)
- markers (a few per pair of students)

MATH LANGUAGE

- decimal number
- tenths
- hundredths
- sum
- difference
- mental computation
- algorithm

Note: Decimal numbers, such as 0.6 or 3.25, are often read as "point six" (or "decimal six") and "three point two five". To connect decimal numbers to their meaning, it is helpful to read 0.6 as "six tenths" and 3.25 as "three and twenty-five hundredths".

INSTRUCTIONAL GROUPING: pairs

INSTRUCTIONAL SEQUENCING

Before starting this learning activity, students should have had experiences with representing decimal numbers to hundredths using concrete materials, reading and writing money amounts to $1000, and adding and subtracting whole numbers using algorithms and mental strategies. This learning activity allows students to apply their understanding of decimal numbers (money amounts) and computational methods in solving a problem.

ABOUT THE MATH

By Grade 5, students have developed mental strategies and algorithms for adding and subtracting whole numbers. These whole-number strategies provide a foundation for learning computational methods with decimal numbers.

In this learning activity, students solve a problem that involves addition and subtraction of decimal numbers to hundredths (money amounts) by using strategies that make sense to them. The numbers in the problem have been chosen to allow for the use of mental strategies as well as algorithms. Students examine various approaches to solving the problem, reflect on the efficiency of various strategies, and discuss cases in which different strategies are appropriate.

GETTING STARTED

Explain the following problem:

> "The Pine Hill School community hopes to raise $500 for a charity organization. The parent group held a craft sale and raised $265.50. The students in the primary grades raised $104.25 by selling tickets to a play they performed. Next week, the students

in the junior grades will be holding a Skip-a-Thon to the raise the rest of the money. How much money do the students in the junior grades need to raise?"

Discuss the problem with students, and record important information about the problem on the board:

- Fundraising goal: $500
- Parent group: $265.50
- Primary classes: $104.25
- Junior classes: ?

Pose the following questions. Have students explain their thinking.

- "About how much money has been raised so far?"
- "Do the students in the junior grades have to raise more money than the parent group did?"
- "Do the students in the junior grades have to raise more money than the students in the primary grades did?"
- "About how much money will the students in the junior grades need to raise to reach the goal?"

Explain that students will work with a partner to solve the problem.

WORKING ON IT

Provide each pair of students with a sheet of paper on which they can record their work. Encourage students to think about how they might use different strategies, including algorithms (paper-and-pencil calculations) and mental computations, to help them solve the problem. Explain that students must be able to explain their strategies later, when the class discusses different ways to solve the problem.

Note: Performing mental computations does not mean that students may not use paper and pencil. Often strategies involve both mental calculations and recording numbers on page. Students may jot down numbers on paper to help them keep track of figures, but they should not perform paper-and-pencil calculations that can be done mentally.

Have available concrete materials for representing decimal numbers to hundredths (e.g., money sets, 10 × 10 grids, base ten materials) and explain to students that representing the problem by using the materials can help them solve the problem.

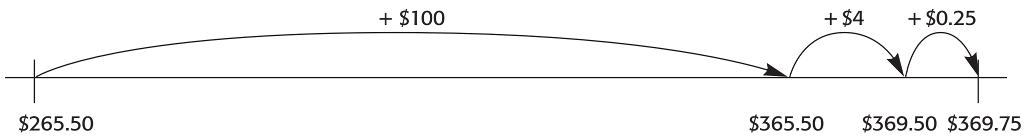
STRATEGIES STUDENTS MIGHT USE

FINDING THE DIFFERENCE BETWEEN $500 AND THE AMOUNT OF MONEY RAISED SO FAR

Students might add $265.50 and $104.25 to determine the amount of money raised so far ($369.75), and then calculate the difference between $369.75 and $500. Some students may use traditional addition and subtraction algorithms to perform these computations; however, other students may

choose to use mental computations. To add $265.50 and $104.25, for example, they might use an adding-on strategy (see p. 21) by breaking $104.25 into parts, and adding on each part:

- adding $100 to $265.50 to get to $365.50;
- then adding $4 to $365.50 to get to $369.50;
- then adding $0.25 to $369.50 get to $369.75.

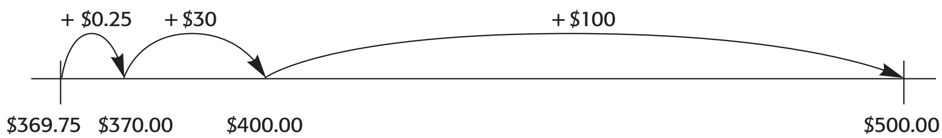


This strategy can be modelled by using an open number line. (In an open number line, the jumps are not to scale.)

To calculate the difference between $369.75 and $500, students might use an adding-on strategy (see p. 21):

- add $0.25 to $369.75 to get to $370;
- then add $30 to $370 to get to $400;
- then add $100 to $400 to get to $500;
- then calculate the difference by adding $0.25 + $30 + $100. The difference is $130.25.

The following open number line illustrates this strategy



$$\begin{array}{r} \$\ \ \ 0.25 \\ \$\ 30.00 \\ +\ \$100.00 \\ \hline \$130.25 \end{array}$$

SUBTRACTING THE AMOUNTS OF MONEY RAISED SO FAR FROM $500

Students might subtract $265.50 from $500 first, and then subtract $104.25 to determine the amount that remains to be raised. (Alternatively, they might subtract $104.25 first, and then subtract $265.50.) Students might decide to use a subtraction algorithm to perform these computations.

$$\begin{array}{r} {}^{499\ 1} \\ \$\cancel{500}.00 \\ -\ \$265.50 \\ \hline \$234.50 \end{array} \qquad \begin{array}{r} {}^{4\ 1} \\ \$23\cancel{4}.\cancel{5}0 \\ -\ \$104.25 \\ \hline \$130.25 \end{array}$$

Note: Students may experience difficulty in using the standard algorithm, and may demonstrate little understanding of the regrouping required to perform the computations. If students are unable to explain the meaning of the algorithm, encourage them to find a method that they can understand and explain to others.

Students might also perform the subtraction computations in other ways. For example, they might combine parts of $265.50 and $104.25 according to place value, and subtract these parts from $500:

- combine the hundreds (i.e., $200 + $100 = $300), and subtract that amount from $500 (i.e., $500 – $300 = $200); then
- combine the tens (i.e., $60 + 0 = $60), and subtract that amount from $200 (i.e., $200 – $60 = $140); then
- combine the ones (i.e., $5 + $4 = $9), and subtract that amount from $140 (i.e., $140 – $9 = $131); then
- combine the hundredths (i.e., $0.50 + $0.25 = $0.75), and subtract that amount from $131 (i.e., $131 – $0.75 = $ 130.25).

Observe students as they are working. Ask them questions that allow them to explain and reflect on their strategies:

- "What strategy are you using to solve the problem?"
- "What did you do first? Why did you do that first? What did you do next?"
- "How are you using paper-and-pencil calculations? Mental computation?"
- "Which calculations did you do mentally? Why did you decide to do these calculations mentally?"
- "What is working well with your strategy? What is not working well?"
- "How could you show your strategy so that others can understand what you are thinking? How could you use a diagram, such as an open number line?"

It may be necessary to demonstrate how diagrams, such as open number lines, can be used to model strategies. (Examples of open number lines can be found on pages 17–18.)

After pairs of students have found a solution, provide them with markers and a half sheet of chart paper or a large sheet of newsprint. Ask students to record their strategies in a way that can be clearly understood by others. Remind students that they need to be prepared to explain their strategies to the class.

REFLECTING AND CONNECTING

Have pairs of students present their solutions to the class. Try to include a variety of computational strategies (e.g., various mental computation strategies, paper-and-pencil algorithms). Pose questions that encourage the presenters to explain the computational strategies they used to solve the problem:

- "How does your strategy work? What steps did you take to use this strategy? Why did you do that step?"
- "Why did you choose this strategy?"
- "What worked well with this strategy? What did not work well?"
- "Was this strategy easy to use? Why or why not?"
- "How do you know that your solution is correct?"

Assist the class in understanding the strategies that are presented. For example, have students explain a strategy in their own words to a partner. As well, model strategies by drawing diagrams (e.g., open number lines) on the board.

Post students' work in the classroom following each presentation. After several pairs have explained their strategies, ask questions that help the class to reflect on the efficiency of different strategies:

- "Which strategies worked well in solving the problem? Why?"
- "How would you explain this strategy to someone who has never used it?"
- "When is it appropriate to use this strategy? For example, with what kinds of numbers does this strategy work well?"
- "Which strategy would you use if you solved another problem like this again? Why?"
- "How would you change a strategy? Why would you change it?"

Keep students' work posted after the activity is finished. Students will need to use it during Learning Connection 1.

ADAPTATIONS/EXTENSIONS

Provide a simpler version of the problem for students who experience difficulties.

> "The school community hopes to raise $500. If the school has already raised $295.50, how much more money does the school need to raise to reach its goal?"

Extend the activity for students requiring a greater challenge by asking them to find different ways to solve the problem. Ask students to judge the efficiency of the different strategies.

You might also challenge students by having them determine the amount of money that needs to be raised if the goal changes (e.g., from $500 to $750).

ASSESSMENT

Observe students to assess how well they:
- understand the problem and formulate an approach to solving it;
- select and apply appropriate computational strategies (e.g., mental strategies, algorithms);
- demonstrate flexibility and skill in using various computational strategies;
- represent and explain computational strategies (e.g., by using an open number line);
- judge the efficiency of different computational strategies.

HOME CONNECTION

Send home copies of **AddSub5.BLM1: Adding and Subtracting Money Amounts**. In this Home Connection activity, students demonstrate different strategies for adding and subtracting money amounts.

LEARNING CONNECTION 1
How Can We Add?

MATERIALS

- spinners made from **AddSub5.BLM2: 0–9 Spinner**, a paper clip, and a pencil (1 per pair of students)
- sheets of paper (a few per pair of students)
- students' posted work samples from the main learning activity (half sheets of chart paper or large sheets of newsprint showing computational strategies)

Provide each pair of students with the materials they need to make a spinner (a copy of **AddSub5.BLM2: 0–9 Spinner**, a pencil, and a paper clip). Instruct pairs to draw the following "blank" number expression on a sheet of paper:

_____ . _____ _____ + _____ . _____ _____

Next, have partners take turns spinning the spinner and recording the digit it shows in one of the six spaces of the number expression. Once a digit has been recorded, it cannot be erased and recorded in another position.

After all spaces are filled, have students discuss how they can add the two decimal numbers mentally. Encourage them to refer to the posted work samples from the main learning activity to review different mental strategies and to select an appropriate method. Remind students that they may jot down numbers on paper to help them keep track of figures, but that they should not perform paper-and-pencil calculations that can be done mentally.

After pairs of students calculate the sum of their decimal numbers, have them record an explanation of the steps they took to add the numbers mentally. Encourage them to use diagrams, such as open number lines, to show their thinking.

Combine pairs of students to create groups of four. Have pairs explain to their new partners how they calculated the sum of the decimal numbers mentally.

Adaptation: Students could also spin the spinner six times and record the digits on a piece of paper. After obtaining six digits from the spinner, they record the digits in the blank number expression so as to determine the greatest possible sum (or the sum that is closest to 10).

LEARNING CONNECTION 2
What Is the Question?

MATERIALS

- **AddSub5.BLM3: What Is the Question?** (1 per pair of students)

Provide pairs of students with a copy of **AddSub5.BLM3: What Is the Question?** Have students place decimal points in the numbers being added or subtracted so that each equation is correct.

Discuss the methods that students used to determine solutions.

LEARNING CONNECTION 3
Problem Solving at the Fair

MATERIALS

- **AddSub5.BLM4a–b: Problem Solving at the Fair** (1 per pair of student)
- half sheets of chart paper or large sheets of newsprint (1 sheet per pair of students)
- markers (a few per pair of students)

Have pairs of students work together to solve the problems on **AddSub5.BLM4b: Problem Solving at the Fair** using the information given in the table on **AddSub5.BLM4a: Problem Solving at the Fair**. Provide each pair with markers and a half sheet of chart paper or a large sheet of newsprint on which they can record their strategies and solutions.

Ask students to present their strategies and solutions to the class. Ask questions such as:

- "Why did you use this strategy?"
- "What worked well with this strategy? What did not work well?"
- "How do you know that your solution is correct?"

Have students compare the different strategies.

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on addition and subtraction concepts. On the homepage, click "Toolkit". In the "Numeracy" section, find "Addition and Subtraction (4 to 6)", and then click the number to the right of it.

eworkshop.on.ca

Adding and Subtracting Money Amounts

Dear Parent/Guardian:

We have been learning to add and subtract money amounts.

There are different ways to add and subtract money amounts. Sometimes it is easy to calculate answers "in your head". At other times, it is easier to figure out answers using paper and pencil.

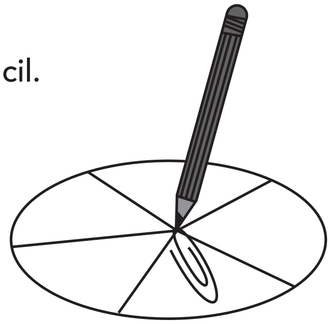
Provide an opportunity for your child to explain different ways to add and subtract money amounts:

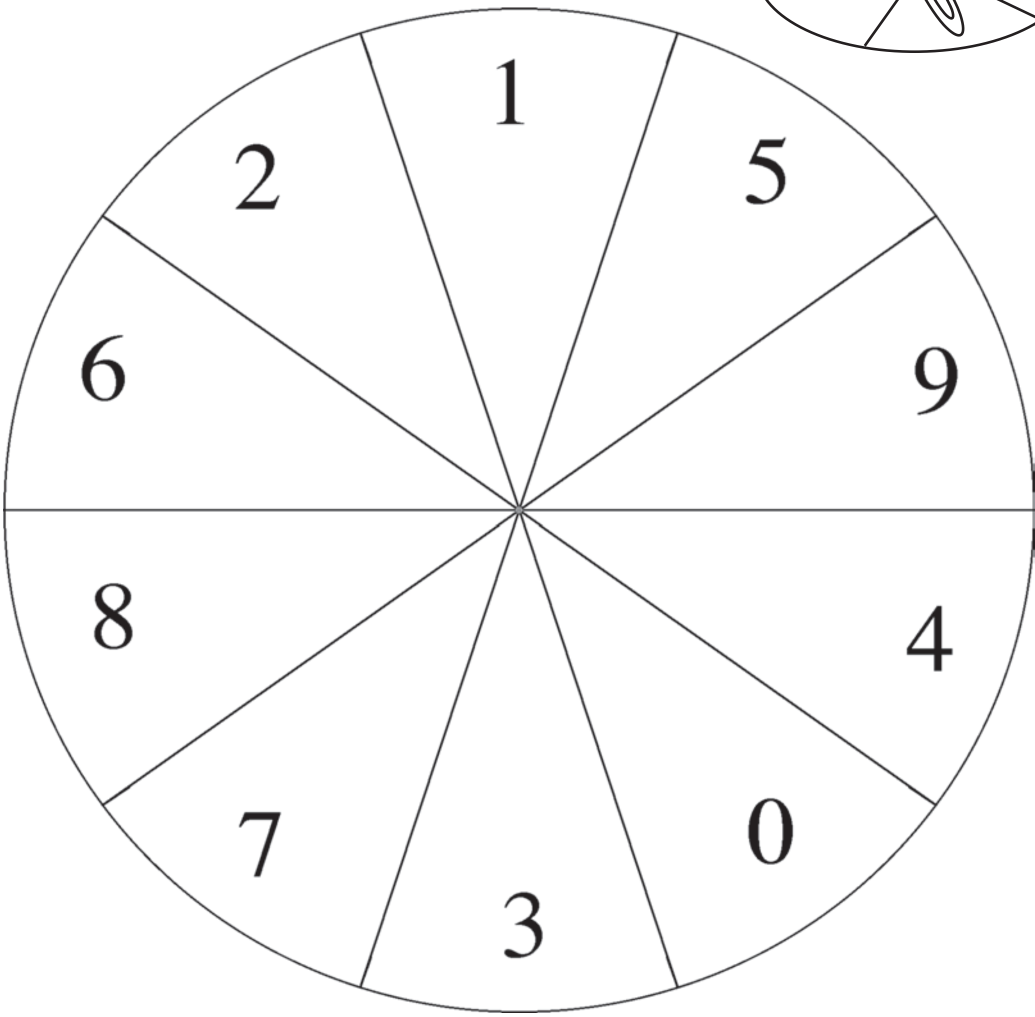
- Together, choose two items in a catalogue or grocery flyer that each cost between $5 and $10.
- Have your child determine the total cost of the two items. Ask your child to explain how he or she added the numbers.
- Ask your child to show a different way to determine the total cost.
- Have your child figure out how much money he or she would get back if he or she used $20 to purchase the two items. Again, ask your child to determine a different way to solve the problem.
- You might show your child how you would add and subtract the money amounts.

Thank you for providing an opportunity for your child to show different ways to add and subtract money amounts.

0–9 Spinner

Make a spinner using this page, a paper clip, and a pencil.





What Is the Question?

64 + 325 = 3.89

48 + 327 + 122 = 20.27

272 + 143 + 135 = 42.13

1821 + 74 + 109 + 24 = 29.1

2367 – 211 = 21.56

4974 – 366 = 493.74

321 – 158 + 6739 = 69.02

Problem Solving at the Fair

The fair is about to get started, and vendors are busy setting up their booths. The following table shows how much each of the vendors paid for a booth, and what they paid for their products. It also shows the amount of money they collected during the three-day fair.

Use the information in the table to solve the problems on the next page.

Booth	Cost of Booth	Cost of Products	Money Collected
Tom's Hand-Picked Apples	$25 for all 3 days	$15 (for apples)	$87.35
Marlena's Clay Creations	$8.75 per day	$55 (for clay and fuel for the kiln)	$173.85
Bjorn's "Bottle Blast" Game	$10.50 per day	$66.50 (for prizes)	$198
Rhoda's Refreshment Booth	$21 for all 3 days	$58.75 (for juice mix and supplies)	$163.50
McKendrick Family's Handmade Scarves and Mitts	$7.50 per day	$35 (for wool)	$156.30

Problem Solving at the Fair

Use information from the table on the previous page to solve the following problems. Explain how you calculated your answers.

1. Which vendor paid the least amount of money for a booth?

2. Which vendor made the most money (profit) at the end of the fair?

3. Which vendor made the least amount of money?

4. Would the vendor who made the most money at the fair still have made the most if the cost of the booths had been the same for everyone?

Grade 6 Learning Activity
A Weighty Matter

OVERVIEW

In this learning activity, students review the meaning of "thousandths" by representing decimal numbers using base ten materials. After discussing concerns about heavy backpacks and possible related injuries, students find combinations of backpack items whose total mass is close to, but does not exceed, the recommended maximum mass.

BIG IDEAS

This learning activity focuses on the following big ideas:

Quantity: Using base ten materials and mass sets, students explore the "howmuchness" of decimal numbers to thousandths.

Operational sense: Students explore a variety of strategies for adding decimal numbers.

Relationships: Representing decimal numbers using base ten materials allows students to understand the base ten relationships in our number system (e.g., 10 thousandths is 1 hundredth).

Representation: Students represent decimal numbers visually by using concrete materials, and symbolically by using decimal notation.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- represent, compare, and order whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools (e.g., number lines with appropriate increments, base ten materials for decimals);
- demonstrate an understanding of place value in whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools and strategies (e.g., use base ten materials to represent the relationship between 1, 0.1, 0.01, and 0.001);
- add and subtract decimal numbers to thousandths, using concrete materials, estimation, algorithms, and calculators.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies.

ABOUT THE LEARNING ACTIVITY

TIME: approximately 60–90 minutes

MATERIALS

- half sheets of chart paper or large sheets of newsprint (1 per group of 4 students)
- markers (a few per group of 4 students)
- base ten blocks (large cubes, flats, rods, small cubes)
- kilogram and gram masses (1 kg mass and 1 g mass per group of 4 students)
- bathroom scale (optional)
- **AddSub6.BLM1: Backpack Items** (1 per pair of students)
- sheets of paper (a few per pair of students)
- sheets of chart paper or large sheets of newsprint (1 per pair of students)
- **AddSub6.BLM2: Lessen the Load** (1 per pair of students)

MATH LANGUAGE

- decimal number
- tenths
- hundredths
- thousandths
- whole
- place value
- kilogram
- gram

Note: Decimal numbers, such as 0.61 or 3.254, are often read as "point six one" (or "decimal six one") and "three point two five four". To connect decimal numbers to their meaning, it is helpful to read 0.61 as "sixty-one hundredths" and 3.254 as "three and two hundred fifty-four thousandths".

INSTRUCTIONAL GROUPINGS: small groups and pairs

INSTRUCTIONAL SEQUENCING

This learning activity reviews the meaning of thousandths and provides an opportunity for students to explore strategies for adding decimal numbers. Before starting this learning activity, students should have had experiences with representing decimal numbers to hundredths using concrete materials, recording hundredths using decimal notation, and adding decimal numbers to hundredths.

ABOUT THE MATH

In Grade 6, students continue to build on their understanding of decimal numbers by exploring the meaning of thousandths. Opportunities to represent decimal numbers using concrete materials (e.g., base ten blocks, place-value mats) help students to develop an understanding of the base ten relationships in decimal numbers.

Grade 6 students also learn to add and subtract decimal numbers to thousandths. In this learning activity, students have an opportunity to add decimal numbers using a variety of methods (e.g., using manipulatives, using student-generated strategies, using algorithms). The learning activity also provides experience in estimating the sums of decimal numbers.

GETTING STARTED

The following two activities reinforce an understanding of "thousandths" and prepare students for the problem in the main learning activity.

ACTIVITY 1: BASE TEN INVESTIGATION

Organize students into groups of four. Provide each group with markers and a half sheet of chart paper or a large sheet of newsprint. Have them create a place-value mat by vertically folding the paper into fourths, outlining the columns with a marker, and labelling the columns "Ones", "Tenths", "Hundredths", "Thousandths".

Ones	Tenths	Hundredths	Thousandths

Provide each group with a collection of base ten blocks (large cubes, flats, rods, small cubes).

Record "1.3, 1.42, 2.09" on the board, and have students read the numbers orally ("one and three tenths", "one and forty-two hundredths", "two and nine hundredths").

Tell students that the flat represents one whole. Ask them to work together as a group to represent the numbers recorded on the board using base ten blocks and their place-value mats. Ask students to demonstrate and explain how they used the materials to represent each number.

Next, explain that the large cube now represents one whole, and again have students represent the numbers recorded on the board. Discuss students' representations with the base ten blocks, and how the change in the representation of one whole (i.e., the large cube instead of the flat) affected the concrete representation of the decimal numbers.

Remind students that the large cube still represents one whole. Show the class the following collections of base ten blocks:

- 1 large cube, 2 flats, 7 rods
- 2 large cubes, 6 rods

For each collection, have students identify the decimal number orally, and ask them to record the number using decimal notation. For example, 1 large cube, 2 flats, 7 rods represents one and twenty-seven hundredths, which can be recorded as "1.27".

Next, show a small cube, and ask students to discuss in their groups the value of the small cube if the large cube has a value of one. Discuss the idea that the small cube is one thousandth of the large cube, and that "one thousandth" can be recorded as "1/1000" and "0.001".

Show the following collections of base ten blocks, and ask students to identify and record each decimal number:

- 3 flats, 1 small cube ("three hundred one thousandths", 0.301)
- 1 large cube, 9 small cubes ("one and nine thousandths", 1.009)
- 2 large cubes, 3 rods, 1 small cube ("two and thirty-one thousandths", 2.031)

ACTIVITY 2: BACKPACK INVESTIGATION

Review the relationship between kilogram and gram. Provide each group of four students with a kilogram mass and a gram mass. Pose the following question: "If you place a kilogram mass in the ones column on the place-value mat, where would you place the gram mass?" Have students, in their groups, discuss their ideas. Then, review the concept that a gram is one thousandth of a kilogram (1/1000 or 0.001 of a kilogram).

Record the following on the board:

- 6 g = _______ kg
- 78 g = ________ kg
- 354 g = ________ kg

Have students work together in their groups to determine the missing decimal numbers in each number sentence. Discuss the solutions as a whole class.

Talk to the class about the health concerns related to the mass of students' backpacks – students who carry heavy backpacks risk back, shoulder, and neck injuries. Explain that studies have found that the maximum mass a backpack should be is 15% of the student's mass – about 5 kg for most Grade 6 students. Explain that a recent survey found that there was a wide range in the mass of students' backpacks, but that many students were carrying backpacks that exceeded the recommended maximum mass.

You might ask a few students to show their backpacks to the class, and have the class predict if the backpack has a mass of more than 5 kg. Use a bathroom scale to check the predictions.

WORKING ON IT

Arrange students in pairs. Provide each pair with a copy of **AddSub6.BLM1: Backpack Items** and a few sheets of blank paper. Provide access to base ten blocks, place-value mats, and mass sets, and encourage students to use the materials, if needed.

Explain that students will work with their partner to find combinations of items that come close to, but do not exceed, the recommended maximum mass of a backpack (5 kg). Encourage students to record combinations of different items and their total mass on the paper provided. Ask students to find more than one combination of items, challenging them to get as close to 5 kg as possible without going over.

As students work, observe their strategies, and ask the following questions:
- "How are you solving the problem?"
- "What strategy are you using to determine the total mass of the items?"
- "Can you calculate the total using another strategy?"
- "How are you using manipulatives/mass sets/drawings/computation to help you determine the total mass?"
- "Which numbers were easy to add? Why?"
- "Which combination of items is closest to 5 kg? How do you know?"

After students have had an opportunity to find different combinations of items, have them select the combination whose mass is closest to 5 kg. Provide each pair of students with markers and a sheet of chart paper or a large sheet of newsprint. Ask them to record the combination of items and the total mass. Have students record their strategies for adding the numbers in such a way that others will understand their thinking.

REFLECTING AND CONNECTING

Have pairs of students present their solutions and strategies to the class. Try to include two pairs who used different strategies (e.g., concrete materials, student-generated strategies, standard algorithms). Make positive comments about students' work, being careful not to infer that some approaches are better than others. Your goal is to have students determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Post students' work, and ask questions such as:
- "Which strategies are similar? How are they alike?"
- "Which strategy would you use if you solved a problem like this again?"
- "How would you change any of the strategies that were presented? Why?"
- "Which work clearly explains a solution? Why is the work clear and easy to understand?"

Discuss how determining the exact mass of backpack items is not practical in a real-life situation (although it provided a context for a math activity), and that estimating the mass of combinations of items might be a more appropriate approach.

Ask students to estimate the combined mass of two or three items on **AddSub6.BLM1: Backpack Items** (e.g., gym clothes and shoes; math textbook, binder, and agenda). Discuss students' estimation strategies. For example, to estimate the mass of a math textbook, a binder, and an agenda, students might recognize that the combined mass of the binder and agenda would be approximately 1 kg, and that the math textbook has a mass of approximately 1.5 kg. The estimated mass of the three items would be approximately 2.5 kg.

ADAPTATIONS/EXTENSIONS

Pair students who might have difficulty with a partner who can help them understand the problem and different strategies, including the use of concrete materials (e.g., base ten blocks, place-value mats).

To simply the activity for students experiencing difficulties, provide a list of fewer backpack items with masses given as tenths of a kilogram (e.g., math textbook: 1.4 kg).

Challenge students by asking them to determine the mass of an actual filled backpack using a scale and mass sets. Students could determine whether the mass of the filled backpack is less than 5 kg.

Ask students who understand percents to calculate 15% of their own mass and to determine whether the mass of their filled backpack is acceptable (less than 15% of their personal mass).

ASSESSMENT

Observe students to assess how well they:

- represent decimal numbers using materials (e.g., base ten blocks, place-value mats);
- read and record decimal numbers;
- explain concepts related to place value (e.g., 10 thousandths are 1 hundredth);
- use appropriate strategies to add decimal numbers;
- explain their strategies for adding decimal numbers;
- use appropriate estimation strategies.

HOME CONNECTION

Send home copies of **AddSub6.BLM2: Lessen the Load**. This Home Connection activity suggests that parents and students examine the contents of the student's backpack and remove any unneeded items. Parents and students are also encouraged to calculate the total mass of the student's backpack.

Before **AddSub6.BLM2: Lessen the Load** is sent home, students could fill in the blank spaces in the chart with the names and masses of other backpack items.

LEARNING CONNECTION 1
Weighty Names

MATERIALS

- **AddSub6.BLM3: Weighty Names** (1 per pair of students)

Ask students if they have ever thought about how "heavy" their name is. Tell them they will have a chance to figure out the "mass" of their name. Give each student a copy of **AddSub6.BLM3: Weighty Names**, and discuss the example. If necessary, do another example with the class. Allow students to work with a partner to determine the mass of their names. (Students can determine the mass of their first and last names.)

Allow students to share the masses of their names. Have students determine whose name is the heaviest, lightest, and closest to 1 kg. Challenge students to find a name that has a mass of exactly 1 kg.

Students could take home a copy of **AddSub6.BLM3: Weighty Names** and determine the mass of family members' names.

LEARNING CONNECTION 2
Missing the Point

Record the following number sentences on the board:

- 7.39 + 24.267 = 31657
- 25.398 + 9822 + 3.05 = 38.27
- 41562 – 33.257 = 8.305
- 40.03 – 1967 = 20.36

Explain that the decimal point is missing from one number in each number sentence. Have students work with a partner to copy the number sentences and to insert each missing decimal point. Explain to students that they are to use estimation, rather than computation with a paper and pencil or a calculator.

As a class, discuss strategies for determining the correct placement of the decimal points.

LEARNING CONNECTION 3
Surmising Sums

Record the following numbers on the board:

1.962	2.247	2.228	0.772
2.431	0.038	2.569	1.753

Have students work with a partner to determine which two decimal numbers have a sum of 2. Encourage students to use inspection, rather than paper-and-pencil calculations, to find the two numbers. Have students explain the strategies they used.

Repeat by having pairs of students find the two decimal numbers that have a sum of 3, 4, and 5. Have students explain their strategies.

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on addition and subtraction concepts. On the homepage, click "Toolkit". In the "Numeracy" section, find "Addition and Subtraction (4 to 6)", and then click the number to the right of it.

eworkshop.on.ca

Backpack Items

What combination of items could you put in a backpack so that the mass of the backpack comes close to, but does not exceed, the recommended maximum mass of 5 kg? You may include items more than once. Include the mass of the backpack (1.275 kg) in your total.

Find different combinations of items.

Backpack Item	Mass
math textbook	1.395 kg
binder	0.764 kg
workbook	0.102 kg
agenda	0.245 kg
paperback novel	0.140 kg
pencil	0.005 kg
calculator	0.075 kg
gym clothes	0.485 kg
shoes	0.598 kg
lunch	0.582 kg
pencil case and contents	0.302 kg
geometry set	0.109 kg

Lessen the Load

Dear Parent/Guardian:

Our class has been investigating the problem of overweight backpacks. Studies have shown that students should carry no more than 15 percent of their body weight or mass (about 5 kilograms for most Grade 6 students) to prevent injury to back, neck, and shoulders. We conducted an investigation in which we found the total mass of various backpack items. The activity provided an opportunity for students to add decimal numbers.

Find some time for you and your child to go through your child's backpack and to remove any unnecessary items. To reinforce an understanding about addition of decimal numbers, you and your child could use the following chart to find the total mass of your child's filled backpack.

Backpack Item	Mass
math textbook	1.395 kg
binder	0.764 kg
workbook	0.102 kg
agenda	0.245 kg
paperback novel	0.140 kg
pencil	0.005 kg
calculator	0.075 kg
gym clothes	0.485 kg
shoes	0.598 kg
lunch	0.582 kg
pencil case and contents	0.302 kg
geometry set	0.109 kg

Thank you for doing this activity with your child.

Weighty Names

How “heavy” is your name? Use the following key to determine the “mass” of your name.

Example: Alison = A + L + I + S + O + N

= 0.001 + 0.144 + 0.081 + 0.361 + 0.225 + 0.196

= 1.008 kg, or 1 kg and 8 g

MASS OF LETTERS IN KILOGRAMS

A	0.001	N	0.196
B	0.004	O	0.225
C	0.009	P	0.256
D	0.016	Q	0.289
E	0.025	R	0.324
F	0.036	S	0.361
G	0.049	T	0.400
H	0.064	U	0.441
I	0.081	V	0.484
J	0.100	W	0.529
K	0.121	X	0.576
L	0.144	Y	0.625
M	0.169	Z	0.676

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