

A Guide to Effective Instruction in Mathematics

Kindergarten to Grade 6

A Resource in Five Volumes
from the Ministry of Education

Volume Four
Assessment and
Home Connections

Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.

Une publication équivalente est disponible en français sous le titre suivant :
Guide d'enseignement efficace des mathématiques, de la maternelle à la 6^e année.

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Introduction

This is Volume Four of the five-volume reference guide *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6*. This volume contains Chapters 8 and 9.

Chapter 8: Evaluation and Assessment focuses on the critical role of assessment in making effective instructional decisions, and highlights the importance of observation as an assessment strategy in the primary or junior mathematics classroom.

A variety of other appropriate assessment strategies and tools are also described.

Chapter 9: Home Connections describes a variety of ways to promote positive communication with parents about the mathematics program – for example, by organizing a family math night, sharing information during parent-teacher conferences, and providing meaningful homework activities. (See the Introduction of Volume One for a summary of the organization and contents of the complete five-volume guide.)

A list of suggested professional resources for teachers and administrators is included in Volume One. It is meant to provide useful suggestions, but should not be considered comprehensive. A glossary of terms used throughout the guide is also provided at the end of Volume One. References are listed at the end of each individual volume.

This guide contains a wide variety of forms and blackline masters, often provided in appendices, that teachers can use in the classroom. Electronic versions of all of these materials can be found at www.eworkshop.on.ca. These electronic forms and blackline masters are in a Word format that can be modified by teachers to accommodate the needs of their students.

Go to www.eworkshop.on.ca for electronic versions of the forms and blackline masters provided throughout this guide. They can be modified to meet classroom needs.

Locating Information Specific to Kindergarten, Primary, and Junior Students in This Guide

An important feature of this guide is the inclusion of grade-related information and examples that help clarify the principles articulated. Such information is identified in the margins of this guide by means of icons referring to the relevant grades – K for Kindergarten, Grades 1–3 for primary, Grades 4–6 for junior. Examples and other materials that are appropriate for use at more than one level or are applicable to more than one level are identified by the appropriate combination of icons.

K – Kindergarten

1–3 – Primary

4–6 – Junior

Look for the following icons in the margins of this document:

K

1-3

4-6

8. Assessment and Evaluation

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Assessment and Evaluation

Successful teaching of mathematics involves more than providing engaging learning activities; it requires that teachers be “in tune” with their students’ learning needs, and that they know when and how a particular instructional strategy or approach will help students to consolidate or extend their understanding. Assessment, which is the gathering of evidence about students’ progress and achievement, provides the information that guides teachers in making decisions about next instructional steps to improve student learning.

“From their earliest school experiences, students draw life-shaping conclusions about themselves as learners on the basis of the information provided them as a result of classroom assessments.”

(Stiggins, 2001, p. 48)

Assessment is an ongoing *awareness* of students’ learning and their needs, rather than an occasional *event* in the program. Minute-by-minute observations of students, along with an understanding of how children learn, allow teachers to make valid decisions and judgements about:

- students’ strengths, weaknesses, and needs;
- students’ readiness to learn new concepts or procedures;
- the appropriateness and effectiveness of instructional approaches;
- students’ understanding of the big ideas in mathematics and students’ achievement of curriculum expectations;
- feedback and information that should be shared with parents, students, and other educators.



Assessment for Learning

The improvement of student learning is the most important focus of assessment. The first phase of assessment, often called *diagnostic assessment*, involves the gathering of information about the *prior knowledge* that students bring to a new learning situation. The second phase of assessment, often called *formative assessment*, involves the gathering of information about how well students are understanding the *new concepts and skills they are learning*.

ASSESSMENT OF PRIOR LEARNING

Students' success in understanding new mathematical concepts and skills depends on their having the prior knowledge and experience to which their learning can be connected. Before introducing a concept or skill, teachers can assess students' learning readiness through informal diagnostic activities such as conferencing, interviewing, and observing the students as they work and talk. Teachers can:

- ask students to explain what they already know about a concept or skill;
- have students use manipulative materials to represent and explain a mathematical idea;
- conduct brief, informal conversations with students about the new ideas;
- observe and talk with students as they solve a problem or complete a task that involves a concept or skill.

It is important for teachers to recognize that students new to Ontario may not have the prior knowledge and experience to make connections to new learning. Teachers need to ensure that these students are familiar with concepts and contexts that will allow them to understand new mathematical ideas.



Assessments of prior learning indicate whether students are ready to learn new concepts, whether they already have a firm grasp of these ideas, or whether a prerequisite understanding needs to be consolidated first.

Examples of Assessment of Prior Learning			K	1-3	4-6
Instructional Focus	Assessment Task	Assessment in Practice			
Kindergarten					
Students will learn number relationships of <i>more</i> , <i>fewer</i> , and <i>same</i> . Before this, the teacher assesses students' understanding of <i>same quantity</i> .	<p>The teacher displays a collection of dot cards to a group of students. The teacher selects a card, shows it to the students, and then asks them to find another card with the same number of dots.</p> <p>Next, the teacher shows dot cards, one at a time, and asks the students to place the same number of counters on their paper plates.</p>	<p>The teacher observes students to assess how well they:</p> <ul style="list-style-type: none"> • demonstrate an understanding of <i>same quantity</i> using dot cards and counters; • use counting strategies to find the same number of objects; • explain strategies for finding the same quantity. 			
Grade 1					
Students will learn about base ten relationships of two-digit numbers. Before this, the teacher assesses students' understanding of <i>tens</i> and <i>ones</i> .	<p>The teacher asks the class, "How can we show 42 fingers?" Standing in a line, four students hold up all 10 fingers and a fifth student shows 2 fingers. The teacher asks the students:</p> <ul style="list-style-type: none"> • "How can we find the total number of fingers?" • "What are different ways of counting the fingers?" • "How many sets of 10 are there and how many single fingers?" 	<p>The teacher observes and questions students to assess how well they:</p> <ul style="list-style-type: none"> • count by 10's and 1's; • explain that they can form two-digit numbers using groups of tens and ones. 			

(continued)

Examples of Assessment of Prior Learning (cont.)

Instructional Focus	Assessment Task	Assessment in Practice
Grade 2		
Students will learn about the concept of multiplication and its relationship to addition. Before this, the teacher assesses students' understanding of <i>equal groups</i> and <i>repeated addition</i> .	<p>The teacher poses a problem: There are 4 bowls. In each bowl, there are 3 apples. How many apples are there altogether?</p> <p>Using counters and empty margarine containers, students, working in pairs, represent and solve the problem. Students record how they solved the problem.</p>	<p>The teacher observes students to assess how well they:</p> <ul style="list-style-type: none"> • represent the problem using materials; • form equal groups; • find the total number of "apples" (e.g., counting by 1's, skip counting, adding, using basic facts); • explain the concept of multiplication
Grade 3		
Students will learn to measure area using centimetre grid paper. Before this, the teacher assesses students' understanding of <i>area</i> and <i>measurement</i> using square tiles.	<p>The teacher shows the class three different rectangles (A, B, and C) and asks students to predict the order from smallest to largest area. Some students are called upon to explain how they predicted the order.</p> <p>Next, the teacher asks students to explain ways to check the correct order from smallest to largest area.</p>	<p>The teacher observes and questions students to assess how well they:</p> <ul style="list-style-type: none"> • explain area concepts; • order the rectangles from smallest to largest area; • describe methods for measuring the area of rectangles; • recognize the need for a standard unit of measure.
Grade 4		
Students will learn to use titles and labels effectively when displaying data. Before this, the teacher assesses students' understanding of the functions of titles and labels in graphs.	The teacher shows the students a bar graph on which titles and labels have been deleted and asks students to explain a possible meaning for the graph. For each explanation, the teacher asks the students to state and explain the titles and labels that would help to clarify the meaning of the graph.	The teacher listens to the students and assesses how well they explain the function of titles and labels in displaying data effectively.

(continued)

Examples of Assessment of Prior Learning *(cont.)*

Instructional Focus	Assessment Task	Assessment in Practice
Grade 5		
Students will learn to construct nets of prisms and pyramids. Before this, the teacher assesses students' ability to identify the faces of three-dimensional figures.	The teacher asks students to select two different three-dimensional figures and to sketch their faces. The teacher then asks the students to identify the faces that the two figures have in common and the faces that differ.	The teacher observes the students and assesses how well they identify the faces of three-dimensional figures.
Grade 6		
Students will learn to solve simple rate problems. Before this, the teacher assesses students' understanding of expressions involving rate and of the use of rates in real-life situations.	<p>The teacher asks the students to explain the meaning of expressions such as:</p> <ul style="list-style-type: none"> • 50 km/h • 3 games for a loonie • 5¢ a minute <p>For each expression, the teacher asks students to explain how the rate might be used in real-life situations.</p>	<p>The teacher listens to the students and assesses how well they:</p> <ul style="list-style-type: none"> • explain notions about rate; • explain the use of rates in real-life situations.

ASSESSMENT OF NEW LEARNING

Observations of students who are completing learning tasks, conversations with students about their learning, and analyses of students' written work provide teachers with immediate information on how well students are learning new concepts and skills. Conducted throughout teaching and learning, assessment allows the teacher to:

- ascertain the progress that students are making;
- provide feedback to students on their learning;
- suggest next steps to extend learning;
- determine whether instructional approaches are working well, or whether different or modified teaching strategies would be more successful.

For assessment to be purposeful, teachers need to determine the kinds of student behaviours and demonstrations that indicate learning of new concepts and skills. Clear assessment criteria, based on curriculum expectations, allow teachers to gather evidence that learning is, or is not, occurring, and to make instructional decisions.

Instructional Focus	Assessment Task	Assessment in Practice
Kindergarten		
Students are learning to identify and describe common characteristics among three-dimensional figures.	The teacher shows 2 three-dimensional figures and asks students, "How are these two figures alike?" The students discuss a similarity (e.g., "They both roll", or "They both have squares on them"). With the teacher's guidance, the class continues to find figures with the same characteristic. Next, students, working in pairs, create sets of figures that are alike in some way. The teacher discusses shared characteristics of the three-dimensional figures with students.	The teacher plans subsequent learning tasks that provide opportunities for students to describe three-dimensional figures in different ways.
Grade 1		
Students are learning to identify pattern structures and represent patterns in various forms.	Students represent patterns given by the teacher in various forms (e.g., students express the teacher's oral pattern of "oink, oink, moo, oink, oink, moo, oink, oink, moo" as AAB AAB AAB or jump, jump, clap, jump, jump, clap, jump, jump, clap). Next, students use interlocking cubes to represent and describe patterns (actions, words, diagrams, sounds) given by the teacher or students.	After observing students, the teacher decides whether to modify the complexity of the patterns in subsequent learning tasks.
Grade 2		
Students are learning to add two-digit numbers.	Students are told they have 75¢ to spend and are shown items that cost between 10¢ and 30¢. Students record possible combinations of items they could buy, total costs, and change they would receive. Students explain how they find total costs and the change that is due.	The teacher uses the assessment information to design learning activities that will further develop students' addition skills.

Examples of Assessment of New Learning *(cont.)*

Instructional Focus

Assessment Task

Assessment in Practice

Grade 3

Students are learning about base ten relationships of three-digit numbers.

Students, working in pairs, are asked to show 327 with base ten blocks (3 flats, 2 rods, 7 ones). By trading among flats, rods, and ones, they find and record other ways of showing the same quantity.

Flats	Rods	Ones
3	1	17
3	0	27
2	12	7

The teacher considers the assessment information and provides other learning tasks that extend students' understanding of base ten relationships.

Grade 4

Students are learning to add 2 two-digit numbers mentally.

The teacher shows the students a price list from a souvenir shop:

Pencil – 43¢

Postcard – 38¢

Fridge magnet – 56¢

Bookmark – 24¢

The teacher asks the students to select any two items and to compute their total cost mentally. The teacher has the students explain their methods for adding 2 two-digit numbers mentally.

The teacher listens to the students' methods for adding 2 two-digit numbers mentally and assesses how well students:

- find accurate solutions;
- use effective mental computation strategies;
- explain their methods for adding numbers mentally.

The teacher uses the assessment information to design learning activities that will further develop students' mental addition skills.

(continued)

Examples of Assessment of New Learning *(cont.)*

Instructional Focus

Assessment Task

Assessment in Practice

Grade 5

Students are learning to determine the volume of rectangular prisms.

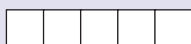
The teacher shows students a box and asks them to record a method for finding the number of cubes needed to fill the box if the cubes are arranged in layers.

The teacher reads the students' work and assesses how well students understand that the number of cubes needed to fill the box can be found by multiplying the number of cubes in one layer by the number of layers. By referring to their explanations, the teacher guides students in generalizing a method for finding the volume of a rectangular prism.

Grade 6

Students are learning to complete tables of values and to use tables of values to make predictions.

The teacher asks students to complete a table of values as they arrange square tiles in a row.



Number of Squares	Perimeter (Units)
1	4
2	6
3	8
4	10
5	12

After the students have completed the table of values for 1 to 5 squares, the teacher asks them to describe patterns in the table and to predict the perimeter of 6, 7, and 8 squares. The students check their predictions. Next, the teacher asks the students to explain how they can use the pattern they discovered to identify the perimeter of 13 squares.

The teacher assesses how well the students:

- complete a table of values;
- describe patterns in the table of values;
- make predictions using data from the table of values;
- explain a method for finding the perimeter, given the number of squares.

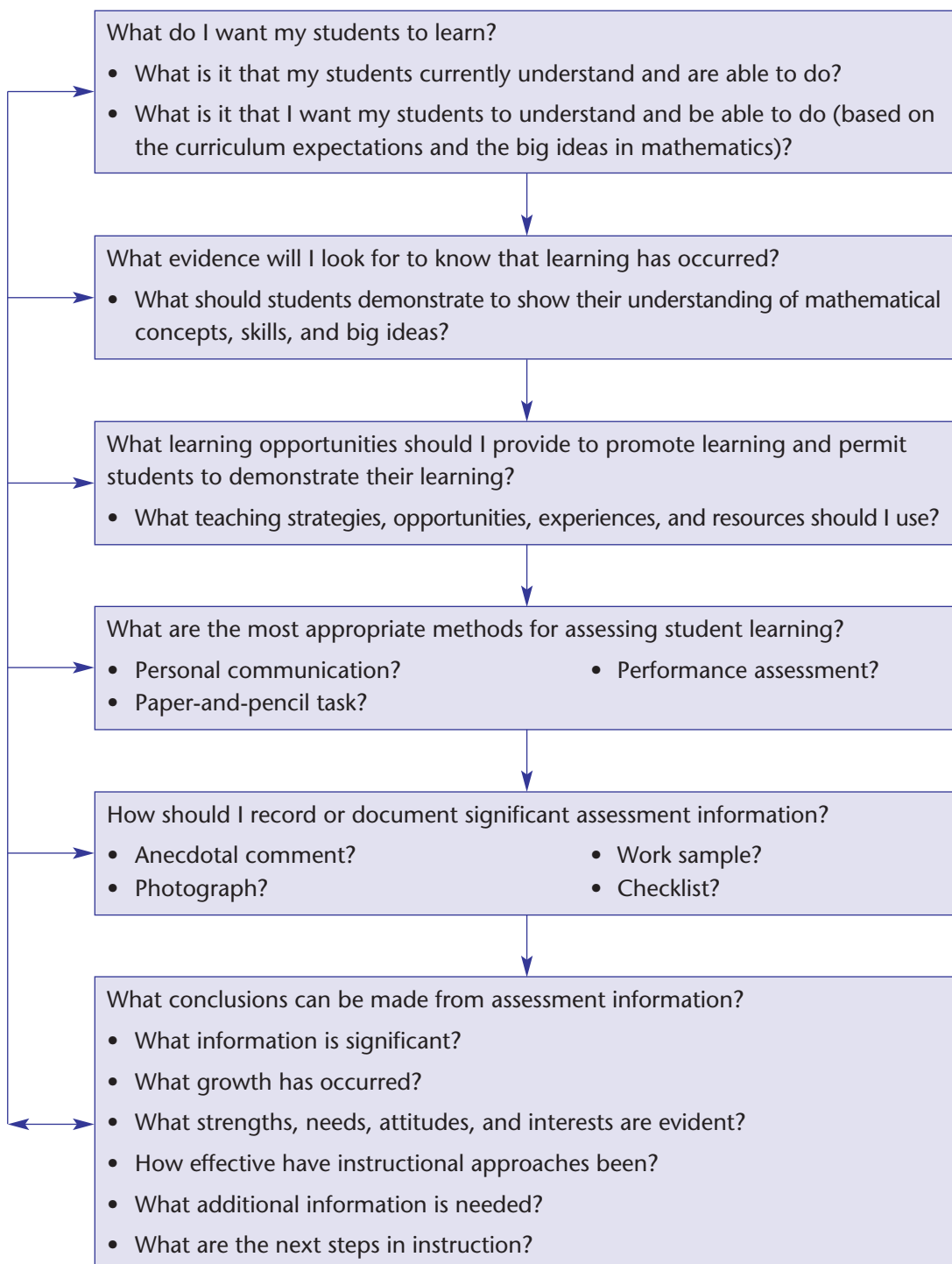
The teacher uses the assessment information to design learning tasks that will further develop students' ability to use tables of values.

Assessment and Instruction

Assessment is an integral part of teaching and learning. Quality instruction and assessment are not necessarily different activities and in fact should become nearly indistinguishable. The following questions help teachers connect instruction and assessment as they purposefully plan and conduct learning activities.

"The seam between assessment and instruction or learning is for the most part invisible."

(Stenmark & Bush, 2001, p. 25)



LINKING ASSESSMENT AND INSTRUCTION: MOMENT-BY-MOMENT DECISIONS

Teaching that is responsive to students' needs uses moment-by-moment assessment information to modify instruction as it is taking place. Teachers can observe students as they work, question them about what they are learning, and look for other indications of how well learning is proceeding. With this information, teachers can decide whether to continue the lesson or activity, to change it in some way, or to use a different approach. Often, these decisions need to be made immediately, when students' needs and readiness to learn are evident.

"Throughout the day, a student's overheard remark, an observed behavior, or a piece of student work may change our planned activities or approaches."

(Glanfield, Bush, & Stenmark, 2003, p. 52)

The following examples demonstrate how teachers can use assessment information to determine appropriate next steps in instruction.

K

Teacher's assessment

A Kindergarten student miscounts a set of objects by counting some objects in the set more than once.

Inference from assessment

The student may have difficulty recognizing one-to-one correspondence while counting and lacks a strategy for counting each object one time only.

Next instructional step

The teacher models counting a set of objects, touching and pushing aside each object as it is counted orally. The teacher then asks, "How many objects are there altogether?" to check whether the student understands the cardinality principle of counting (that the last counting word indicates the number of objects in the set).

Next, the teacher and student count aloud together, as the student touches and slides each object aside so that it will not be counted twice. The teacher asks the student to count other sets of objects to determine whether he or she counts successfully.

Teacher's assessment

Grade 1 students are asked to show combinations for 6 on a part-part-whole mat. A student places 6 counters on each side of the mat.

Inference from assessment

The student fails to understand that he or she can use the part-part-whole mat to decompose a number.

Next instructional step

The teacher asks the student to make a train of 6 cubes using red and blue cubes. Next, the teacher has the student place the blue cubes on one side of the mat and the red cubes on the other side. The procedure is repeated for other combinations of 6 red and blue cubes. The teacher asks, "How does the mat help us know how we can break 6 into two parts?" Next, the teacher asks the student to take 7 counters and place them on the mat. "Look at the way you placed the 7 counters on the mat. What does the mat tell us about how 7 can be broken into two parts?"

Teacher's assessment

Several Grade 2 students give 13 as the answer to this problem: Luis had 9 stickers. He bought some more stickers. Now he has 13 stickers altogether. How many stickers did Luis buy?

Inference from assessment

The students have difficulty interpreting a problem in which the whole and one part are known but one part is unknown.

Next instructional step

The teacher guides students through a problem-solving strategy using counters:

- "How many stickers did Luis have at the beginning?" (9)
- "Is that how many he had altogether?" (No, he had 13.)
- "How many more would he need to have 10 altogether? To have 11 altogether? To have 12? To have 13?"
- "How many more stickers did Luis need to buy?"

The teacher guides students through similar problems, gradually reducing the questioning prompts until students are able to interpret the problem on their own.

Teacher's assessment

A Grade 3 student makes many errors on questions like $3 + \underline{\quad} = 12$ (giving 15 as an answer).

Inference from assessment

The student is having trouble interpreting relationships in number sentences. The student might not be ready to deal with symbolic representations for addition.

Next instructional step

The teacher guides the student in representing the question on a part-part-whole mat.

- "What number represents the total?" (12)
- "What is one way we could show 12 on the part-part-whole mat?"
- "What if we put 3 counters on this side of the mat? What number would we need to put on the other side to make 12?"
- "How is our part-part-whole mat like the addition question you tried to solve earlier?"

The teacher guides the student through other examples until the student demonstrates an understanding of the addition questions.

4-6

Teacher's assessment

A Grade 4 student makes errors in addition similar to the following:

$$\begin{array}{r} 45 \\ 67 \\ + 35 \\ \hline 1317 \end{array}$$

Inference from assessment

The student has a poor understanding of the standard addition algorithm and of the place value of numbers in addition questions.

Next instructional step

The teacher provides opportunities for the student to represent and solve addition questions concretely using base ten materials. The teacher encourages the student to represent the numbers on paper in a personally meaningful way before explaining the standard algorithm.

Teacher's assessment

A Grade 5 student explains that $\frac{7}{8}$ is greater than $\frac{3}{4}$ because 8 is greater than 4.

Inference from assessment

The student assumes, incorrectly, that eighths are larger parts than fourths because 8 is larger than 4.

Next instructional step

The teacher provides concrete experiences that allow the student to observe that the fractional parts of a whole divided into many pieces are smaller than the fractional parts of a whole divided into few pieces. For example, the teacher asks the student to construct a whole circle using thirds and another whole circle using sixths. The teacher and the student discuss why a third is larger than a sixth.

Teacher's assessment

A Grade 6 student calculates the perimeter of a rectangular yard by multiplying the length and the width of the yard.

Inference from assessment

The student attempts to find the perimeter of a rectangle by applying procedures for finding the area. The student may be unsure of the meaning of *perimeter* or may be confusing the procedures for finding perimeter and area.

Next instructional step

The teacher checks the student's understanding of perimeter by asking the student to trace along the perimeter on a sketch of a rectangle. After the meaning of perimeter has been clarified, the teacher asks the student to find the perimeter of the rectangle using a ruler. Next, the teacher guides the student in generalizing the idea that the perimeter of a rectangle can be found by adding the length and width of the rectangle, then doubling this amount.

USING ASSESSMENT TO PROMOTE LEARNING

Helping students set personal learning goals is an important aspect of the assessment-instruction link. Children are more likely to demonstrate what they know and can do, and to advance in their mathematical development, when teachers provide helpful feedback, make it clear to them what is expected of them in their work, and demonstrate characteristics of quality work. Sharing assessment information with parents also supports mathematical learning that occurs at home.

Providing Feedback

When students experience difficulties and receive no useful feedback, they are likely to attribute their problems to a lack of ability and give up. On the other hand, when students receive specific information about ways in which they can improve, and are given opportunities to revise their work, they have a clear message from the teacher that they are capable of learning and improving.

"Feedback can take many forms. Instead of saying, 'This is what you did wrong' or 'This is what you need to do,' we can ask questions: 'What do you think you need to do? What other strategy choices could you make? Have you thought about ...?'"

(Stenmark & Bush, 2001, p. 70)

Quality feedback goes beyond praise – it informs the student about successes, areas for improvement, and next steps to extend learning. Effective feedback helps students:

- recognize what they have done well, and how they might improve;
"You have used a lot of math language in your explanation. Can you draw a diagram that shows your ideas as well?"
- overcome obstacles by building understanding on what students already know;
"You know how to add using doubles ($4 + 4$, $5 + 5$, $7 + 7$). Let's use these ideas to help you with some other addition facts. How could you use $4 + 4$ to help you know $4 + 5$?"
- set improvement goals.
"You really spent a lot of time adding details to your picture. In the future, try to make simpler diagrams that show mathematical ideas without attention to details that are not important. We will have other opportunities to create pictures where detail is important."

Setting Clear Criteria

As teachers are preparing students for learning activities, they need to communicate clearly to students what is expected of them. Without clear guidelines, students might produce work that is not indicative of what they actually know or can do.

Demonstrating Characteristics of Quality Work

Students should know what quality work looks and sounds like.

- Teachers play an important role in modelling processes for learning mathematics: how to use materials to represent mathematical ideas, how to perform mathematical procedures, how to solve problems, how to communicate mathematical ideas orally and in writing. As teachers demonstrate these processes, they talk aloud, making their thoughts explicit to the students. Often, through guided mathematics experiences in which students and teacher are working collaboratively to solve a problem, investigate a concept, or write mathematical ideas, the teacher can focus students' attention on behaviours and characteristics that constitute quality work.
- Teachers and students share examples of quality work. Teachers can ask a student to show and talk about his or her work when some aspect of the work would help other students to improve.
- Teachers can explain characteristics of quality work to students using simple rubrics, classroom displays, and checklists (see the sample display on this page and the sample checklist on the next page). Teachers can discuss the characteristics of good work with students before, during, and after learning activities in order to clarify what students should strive for.

Sample Classroom Display



A classroom display can encourage students to apply effective problem-solving skills.

Sample Checklist

COMMUNICATION CHECKLIST

- ☐ I explained math ideas clearly.
- ☐ I used diagrams, words, and/or symbols to explain my ideas.
- ☐ I used math language wherever possible.
- ☐ I used math words and symbols correctly.
- ☐ Other people will understand my ideas.

A communication checklist helps students assess the quality of their work.

GAINING SUPPORT AT HOME

Communication with parents about assessment is critical to successful learning. Parents can play an important role in supporting their child's mathematical development when they are aware of assessment processes and of their own child's strengths and learning needs. Teachers can suggest ways in which parents can encourage and work with their child outside school. For an example of a letter to parents of children in the primary grades informing them of the importance of problem solving and communication in the mathematics program, and inviting them to promote learning at home, see Appendix 8-1.

Observation – A Powerful Lens on Learning

K 1-3 4-6

Primary and junior students demonstrate their mathematical knowledge and understanding through what they do, say, and show. Observation, therefore, is the most efficient and effective way for teachers to assess students' mathematical abilities and is an integral part of all assessment strategies. Teachers must be attentive observers of their students as they seek evidence of how well students are learning concepts and skills.

"By doing ongoing, nonstructured observations, we assess students in context."

(Stenmark & Bush, 2001, p. 62)

Observation is more than just looking. It involves:

- watching the way students approach learning tasks;
- listening to students' ideas and trying to understand their reasoning;
- discussing problems so that students reveal their way of thinking;
- asking questions that probe students' thinking;
- examining the products that students create.

THE FOCUS OF OBSERVATIONS

The understanding of concepts in mathematics is critical to students' present and future success in mathematics. It is important, therefore, that teachers focus their observation on behaviours and demonstrations that indicate the extent to which students understand fundamental concepts. Having a clear focus when observing students helps teachers watch and listen for evidence of learning, and guides them in providing feedback to students on their learning and on areas for improvement.

QUESTIONS GUIDING OBSERVATION OF STUDENTS' KNOWLEDGE AND UNDERSTANDING OF CONCEPTS

How well does the student:

- demonstrate knowledge of mathematical content (e.g., facts, terms, procedural skills, use of tools)?
- demonstrate understanding of mathematical concepts?
- give examples of a concept?
- show and explain relationships between and among concepts?

Related to students' understanding of important concepts are their competencies in three other categories: thinking and solving problems, communicating mathematically, and applying mathematical procedures.

QUESTIONS GUIDING OBSERVATION OF STUDENTS' THINKING AND PROBLEM-SOLVING SKILLS

How well does the student:

- understand the problem (e.g., retell it in his or her own words)?
- make a plan for solving the problem (e.g., select an appropriate problem-solving strategy)?
- carry out a plan for solving the problem (e.g., test ideas, revise strategies, form conclusions)?
- look back at the solution (e.g., evaluate reasonableness, explain and justify a solution, reflect on the solution)?
- use critical/creative thinking processes (e.g., problem solving, inquiry)?

QUESTIONS GUIDING OBSERVATION OF STUDENTS' MATHEMATICAL COMMUNICATION

How well does the student:

- express mathematical ideas and thinking?
- present ideas in oral, visual, and written forms (e.g., using concrete materials, diagrams, numbers, symbols)?
- communicate mathematical ideas for different audiences (e.g., peers, teachers)?
- communicate for different purposes (e.g., to explain a mathematical idea, to present a solution to a problem, to justify a solution)?
- use appropriate mathematical language and symbols?

QUESTIONS GUIDING OBSERVATION OF STUDENTS' APPLICATION OF MATHEMATICAL PROCEDURES

How well does the student:

- apply knowledge and skills in familiar contexts?
- transfer knowledge and skills to new contexts?
- make connections within and between various contexts (e.g., connections between concepts, connections between mathematics and other subjects, connections between mathematics and the real world)?

TYPES OF OBSERVATIONS: INFORMAL AND FORMAL

Significant observations of students' learning are often informal, occurring as teachers and students interact in the course of everyday classroom activities. Watching and listening to students engaged in learning tasks, and keeping the purpose of the tasks in mind, teachers focus their observations on evidence of students' mathematical learning. This form of observation allows the teacher to provide immediate feedback to learners and helps the teacher determine how learning can be reinforced and extended.

"Teachers must assess their own growth as well as children's progress in learning mathematics. To evaluate their teaching behaviors and effectiveness, teachers observe, listen, collect, and document children's learning and use this evidence to consider what is working and what is not."

(Copley, 2000, p. 25)

More formal observation sessions are beneficial when teachers require a focused assessment of students' understanding of concepts and skills. In a structured observation setting, the teacher asks individuals or a small group of students to complete a specific task or series of related tasks. The teacher then notes each student's demonstrations of understanding and any help that the student may require. (Examples of focused assessment tasks are provided on pages 27–28.)

To enhance their observations in formal or informal settings, teachers ask students questions to get a clearer picture of their thinking.

- "How did you do ...?"
- "How did you know ...?"
- "What happened when you ...?"
- "Why did you ...?"
- "How is this like ...?"
- "What patterns do you see?"

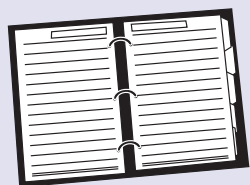
THE CONTENT OF OBSERVATIONS

It is impractical, if not impossible, for teachers to record all observations. Teachers should develop an accessible, efficient, and manageable system for recording significant observations. Teachers should:

- note behaviours that indicate students' thinking and learning (understanding of concepts and procedures);
- be brief and objective;
- record and date observations immediately, rather than trying to remember them later;
- focus on a small number of students at a time in order to gain insights into each student's learning;
- develop a system for identifying students who need more assistance (e.g., star, highlight).

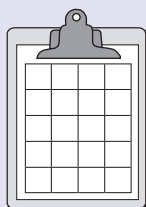
METHODS FOR RECORDING OBSERVATIONS

- binder notes



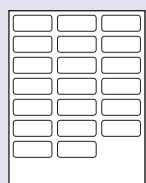
- Prepare a three-ring binder with section tabs for each student.
- Place several observation sheets in each student's section.
- Record observations directly into the book, dating each entry.

- clipboard chart



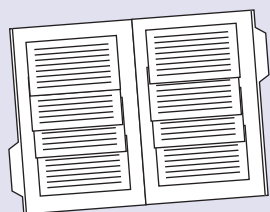
- Divide a sheet into sections, one for each student.
- Place a sheet for each day or week on a clipboard.
- At the end of the week, transfer comments to individual record pages kept for each student.

- peel-off notes



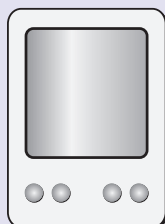
- Record comments on self-adhesive labels or on sticky notes. Include the date and the student's name.
- Later, attach the labels or notes to individual record pages kept for each student.

- flip file



- Laminate a file folder.
- Provide one index card for each child.
- Tape down the top edge of each card, overlapping the cards so that the bottom 2 cm of each card is exposed.
- Print the student's name on the exposed edge.
- Record observations on the card, dating each entry.

- personal digital assistant



- Record observations and other assessment data electronically into the PDA.

Assessment Methods

Teachers should use a variety of assessment strategies in order to assess students' mathematical development as completely as possible. The use of various assessment strategies also allows all students to demonstrate what they know and can do in ways that suit them. Assessment strategies can be categorized according to three main methods: personal communications, performance tasks, and paper-and-pencil tasks.

"A teacher who continually learns from children and about children can become the most effective assessor of the young child's mathematical understanding."

(Copley, 2000, p. 24)

PERSONAL COMMUNICATIONS

As teachers talk with students, they can gain insight into students' understanding, interests, ideas, opinions, and feelings. Teachers have many opportunities to assess their students' learning through dialogue and discussion, including instructional questions and answers, conferences or conversations, and interviews.

Instructional Questions and Answers

Much classroom instruction is characterized by the exchange of questions and answers between teachers and students. Teachers' questions are often the key to discovering students' understanding and encouraging further growth. Questions that reveal students' thinking:

- ask students to show what they know and are able to do;
"How can you use the cubes to show that 5 is one-half of 10?"
- prompt students to clarify their explanations;
"What did you mean when you said that you 'went to the next number' when estimating?"
- challenge students' curiosity;
"What would happen if you measured the length of the table using stir sticks rather than toothpicks?"
- ask students to make connections;
"How is your pattern like the one on the calendar?"
- encourage students to compare ideas;
"How is Jonathan's idea different from yours?"
- ask students to defend their thinking;
"How do you know that your solution is correct?"
- allow students to reflect on their learning.
"What did you find easy in this activity? Where did you have difficulties?"

When teachers assess student learning through questioning, they should strive to say as little as possible and to listen attentively to students' ideas. These ideas provide clues about how learning is progressing and what further instruction is needed.

More information about the use of questioning is found in the subsection "Questioning and Prompting Students" of Chapter 5: Problem Solving, in Volume Two.

Conferences or Conversations

Conferences (informal conversations) provide an opportunity for students to "talk math" while they are engaged in learning tasks. The teacher might initiate a conference with an individual or a group of students in order to probe their understanding more thoroughly, to help students clarify ideas, to challenge students' thinking if misconceptions are evident, or to direct attention to mathematical ideas that students are ready to explore.

"Insight into students' thinking is revealed by listening to their talk. Conversations with students can help us understand what they know and how they think."

(Glanfield et al., 2003, p. 53)

Questions similar to the following promote discussion with students during a conference:

- "What is this problem/task about? What do you have to do?"
- "How are you trying to solve the problem? Show me what you've done so far."
- "What is working well? Where are you having difficulties?"
- "Is there some part of the lesson we had earlier that would help you here?"
- "What is not clear? What don't you understand?"
- "What solution(s) did you find?"
- "How do you know your solution makes sense?"
- "Is there another way to solve this problem?"

Interviews

An interview usually involves a planned sequence of questions posed to an individual student. In an interview situation, the teacher asks the student to perform a task or a series of tasks and encourages the student to talk aloud during the activity.

Interviews with individual students provide valuable information about students' thinking processes but can be time consuming. It is wise, therefore, to use interviews as a format for assessing students' understanding of important concepts in mathematics.

An interview of a student who is having difficulties with a particular concept or skill can often help to identify the source of those difficulties. Careful observations and questions of clarification can help the teacher and student pinpoint where understanding is weak and identify ways to strengthen comprehension.

QUESTIONS THAT THE TEACHER MIGHT USE IN AN INTERVIEW

K**1-3****4-6**

Counting: "How many do you have there? Show me how you count them. What if I spread them apart – how many would there be?" "Which pile has more? Can you make this pile with one more? one less?"

Patterning: "How could you continue this pattern?" "Can you represent the same pattern using the tiles?" "What is the pattern rule?" "What patterns do you see in this table?"

Classifying: "Can you put these into groups so that the ones that are alike are together? Why did you put them into those groups?" "What name can we give to all the items in this group?"

Ordering: "Can you put them in order from shortest to longest?" "Here is a new thing. Can you place it in the correct order with these things that you have already ordered?" "How do you know that these numbers are ordered from smallest to greatest?"

Base ten relationships: "How could you show 315 using the base ten materials? What if I take 10 away? Add 20? What number is represented then?"

Estimating: "Estimate how many there are in all. How can you find how close you were? Did you estimate high or low? Were you close? How close?" "How did you make your estimate?" "How do you know that your estimate is reasonable?"

Measuring: "What are some ways in which we could measure this object? How tall (long, heavy) is it? What object in the classroom is about the same height (length, mass)?" "What tools could you use to find the length (area, volume, mass)?"

Number operations: "What number operations did you use to solve this problem? How could you solve this problem in a different way?" "How could you use materials to show how you added (subtracted, multiplied, divided)?"

SUGGESTIONS FOR MANAGING INTERVIEWS

K**1-3****4-6**

It is important that teachers prepare all students for times when interviews are conducted, so that the interviews can run smoothly without frequent interruptions.

Teachers should:

- provide familiar tasks for students not involved in the interview to lessen the need for questions about new activities;
- make sure that students know what an interview is and recognize the need for uninterrupted time;
- establish with students the procedures for solving problems without the teacher's help;
- establish acceptable reasons for interrupting an interview;
- meet in an area where the teacher and student can be comfortable and where the teacher can see the rest of the class.

PERFORMANCE TASKS

A performance assessment involves a meaningful and purposeful task in which students are required to perform, create, or produce something. Performance tasks allow the teacher to observe how well primary and junior students are able to apply new learning in a different situation. Such tasks allow for the examination of the process used as well as the solution, answer, or finished product. When performance assessments are used for evaluation purposes, teachers must ensure that students have had adequate experience in using the concepts and skills they need to complete the task.

"A good performance assessment task is often open ended, captures students' mathematical thinking, and encourages communication."

(Glanfield et al., 2003, p. 69)

Performance tasks should:

- address grade-level curriculum expectations;
- provide opportunities for students to solve problems, demonstrate their understanding of concepts, apply procedures, and/or communicate mathematical thinking;
- reflect an appropriate level of complexity;
- use language that is clear to students;
- be meaningful and engaging to students;
- be set in contexts that are familiar to students, including newcomers to Ontario;
- allow for participation by students with different levels of knowledge and skills;
- provide appropriate scaffolding.

EXAMPLES OF PERFORMANCE TASKS FOR PRIMARY STUDENTS

- Look around the room. Draw pictures of things that are taller than you, about the same height as you, and shorter than you.
- Cindy has 8 marbles in her hand. Each marble is either red or yellow. Show the different combinations of red and yellow marbles that Cindy might have.
- Darren wants to buy a snack that costs 75¢ from a vending machine. In his pocket, Darren has 7 nickels, 6 dimes, and 2 quarters. What are different ways in which Darren could pay for his snack?
- Aunt Denise has 14 carrot sticks. Hani, Trevor, Lili, and Amy get to eat the carrot sticks if they can share them equally among themselves. How could they share the carrot sticks equally? Show and explain your solution.
- A new student has joined our class. She has never seen pattern blocks. How would you describe each pattern block so that she would know what each one looked like?
- Solve this riddle: What figure has three rectangular faces and two triangular faces? Write some other riddles about the three-dimensional figures. Make sure that you give the answers.
- Imagine that you work in a chocolate bar factory. Your job is to put 6 chocolate bars into each box. Today your boss has given you 49 chocolate bars to put in boxes. How many boxes will you need? Explain your answer.
- There are 3 bicycles, 4 tricycles, and 2 wagons in the park. How many wheels are there altogether? Show and explain your solution.
- Erik earns \$2 every time he walks a dog. How much money did he earn if he walked 3 collies, 2 poodles, and 1 Dalmatian? Show and explain your solution.

EXAMPLES OF PERFORMANCE TASKS FOR JUNIOR STUDENTS

- Mr. and Mrs. Kamul took their 3 children to a concert. Adult tickets cost \$9.00. A child's ticket was half the price of an adult ticket. How much did the Kamul family pay to attend the concert?
- Hot dog buns come in packages of 8 and 12. Imagine that you are in charge of ordering 40 hot dog buns for a class lunch. What package combinations could you buy? What combinations would give the least amount of extra buns?
- Jeremy wants to frame 4 posters of the same size. Each poster measures 60 cm by 80 cm. The frame costs \$8 per metre. How much will it cost to frame the posters?
- Mohamed says he does not care whether he gets $\frac{3}{4}$ or $\frac{7}{8}$ of a chocolate bar, because a piece of chocolate is missing either way. Explain how you could help Mohamed better understand fractions.
- Corinne and Salma are playing a game with a number cube that has 1 to 6 on it. Corinne rolls the number cube 10 times. Every time there is an even number, Corinne earns a point. Next, Salma rolls the number cube 10 times. Every time there is an odd number, Salma earns a point. Is the game fair? Why or why not?
- What happens to the area of a rectangle if the lengths of its sides are doubled?
- In what ways are a rectangular prism and a hexagonal prism the same? How are they different?

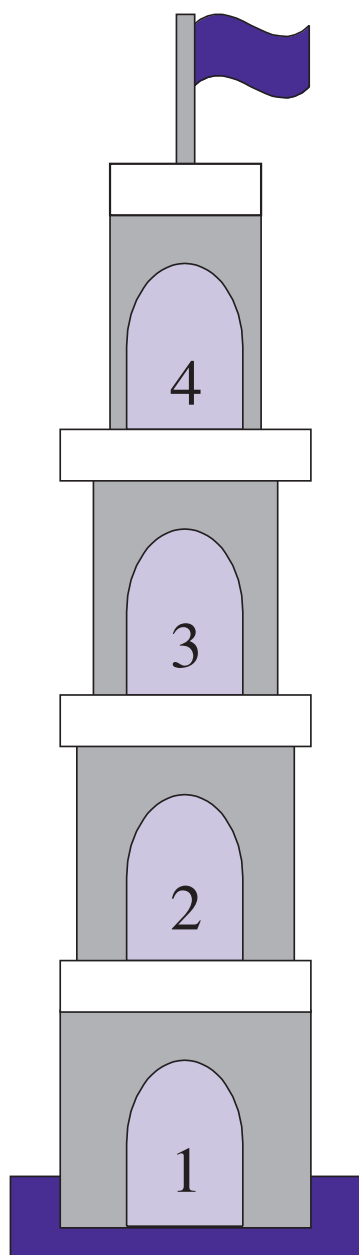
Rubrics

A rubric is often an appropriate tool for assessing a performance task. It identifies the criteria that describe performance at different levels of achievement.

Beginning in Grade 1, teachers and students can discuss the meaning and characteristics of the achievement levels shown in a rubric. Teachers can show and talk about the rubric before a performance task, so that students are aware of what they need to do to be successful. An example of a simple rubric that could be used to introduce students to levels of achievement is provided below.

Sample Problem-Solving Rubric for Primary Students

1-3



Level 4

You have a clever way to solve the problem.
You show clearly how you got your answer.
You use “math talk” to explain your ideas clearly.

Level 3

You have a good way to solve the problem.
You show how you got your answer.
You use a fair amount of “math talk”.

Level 2

You solve the problem but maybe have some difficulty.
You show a bit of how you got your answer.
You use some “math talk”.

Level 1

You could have solved the problem in a better way.
You have difficulty explaining what you did.
You use only a little bit of “math talk”.

PAPER-AND-PENCIL TASKS

Paper-and-pencil tasks contribute to the assessment program, but they do not define it. Students in the early grades are beginning to learn about expressing their ideas in written form, and paper-and-pencil work may not fully demonstrate what they know and are able to do. In the junior grades, students are able to express their ideas in written form with increasing skill and sophistication. Some junior students may still rely on pictorial or numeric representations to express their thinking.

DAILY WRITTEN WORK

Assessment of students' written work in journals and workbooks and on activity sheets involves more than checking answers. Teachers should analyse what students record in order to determine how well students:

- understand the mathematical investigation, problem-solving task, or exercise;
- represent their understanding and thinking processes in written form;
- employ various and appropriate forms of communication (drawings, words, numbers, symbols);
- express ideas using mathematical language;
- use classroom print and graphics in their own work;
- select and apply appropriate strategies to complete written tasks;
- carry out procedures symbolically;
- recognize their errors and correct them independently.

Teachers should emphasize with their students that written work is a means by which teachers and students can know what and how well students are learning. Teachers' brief written comments next to students' work can provide feedback, but conversations with students, especially if difficulties are apparent, can clarify the ideas students have attempted to express in their written work.

TESTS AND QUIZZES

Tests and quizzes can be useful for assessing key knowledge, facts, skills, and procedures. Carefully constructed, they can be used as well to assess more complex and challenging thinking skills. However, if young students are expected to read and write a test independently, or are asked to demonstrate their understanding at a symbolic level, they may not be able to express the depth of their understanding through a written response. For this reason, paper-and-pencil tests are usually an inappropriate method of assessment, especially in Kindergarten and the early grades. By Grades 3 and 4, most students have developed some skills in communicating their ideas on

K **1-3**

1-3 **4-6**

paper, and teachers may gradually use tests and quizzes as assessment tools. Teachers should, however, recognize the limitations of tests:

- A paper-and-pencil test may not reflect the kinds of learning opportunities given to students before the test (e.g., the use of manipulatives to explore concepts).
- Paper-and-pencil tests may not allow students to demonstrate fully their mathematical abilities.
- Students with difficulties in reading and writing may respond incorrectly or incompletely to test questions.
- Students may be anxious about taking tests.

A well-designed test can be used to assess more than factual and procedural knowledge (e.g., multiplication facts and computation); it can provide opportunities for students to demonstrate their understanding through problem-solving activities.

For an example of an appropriate paper-and-pencil test designed to assess the conceptual understanding of Grade 3 students, see Appendix 8-2.

1-3

Student Self-Assessment

Students should have frequent opportunities to reflect on their own learning, to identify their strengths and those areas requiring growth, and to set appropriate goals. Student self-assessment can be accomplished through the use of journals; rubrics, checklists, and rating scales; portfolios; and surveys and questionnaires.

JOURNALS (LEARNING LOGS)

Journal writing helps students think about what they are learning. Students can record in their journals using written work, pictures, diagrams, stamps, and charts. Through these forms students often communicate mathematical ideas that they cannot express orally. In their journals students also have opportunities to express how they feel about a particular learning activity or about mathematics in general.

Teachers can provide prompts to help students focus on what they did and learned in a mathematics activity:

- **Sentence stems:**
 - In math, I am learning . . .
 - I understand . . .
 - I don't understand . . .
 - I find it easy to . . .
 - I find it difficult to . . .
 - My favourite part of math is . . .
 - I do best in math when . . .

- **Other journal prompts:**

- Write everything you know about ... (e.g., 50; a cube; multiplication).
- Imagine that a classmate is absent today. Write a letter to explain what we did and what we learned in math class today.
- Explain how you could ... (e.g., find the most popular flavour of ice cream in the class; find the answer to 3×6 if you didn't know the answer by heart; find the area of the classroom floor).
- Write a story that ... (e.g., uses the fraction one-half or one-third; is about a symmetrical object; uses division).
- Write a math problem about ... (e.g., telling time; subtraction; the graph that the class created this morning; percent).
- Measurement is ...; Symmetry is ...; Division is ...

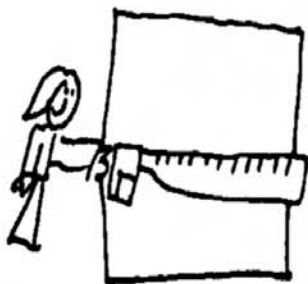
An example of a journal entry in response to a prompt is shown below.

Sample Response to a Prompt

Measurement is...

Measurement is When you measure the distance, length, weight, size, width, height, thickness, speed, area, perimeter or Volume.

I would use measurement to measure the length or size for example I would want to measure my wall to see how much wall paper rolls I needed.



This journal entry was completed by a student given the prompt "Measurement is ...".

(Used with the permission of the Kawartha Pine Ridge District School Board, 2000.)

STUDENT RUBRICS, CHECKLISTS, AND RATING SCALES

Following a problem-solving activity or an investigative task, students may use a rubric, checklist, or rating scale to help them reflect on a learning experience or on the quality of their work. An example of a checklist to help students reflect on a problem-solving experience is provided below.

Sample Checklist for Problem Solving

Problem-Solving Checklist	
I understood the problem.	<input type="checkbox"/> Not very well <input type="checkbox"/> A little bit <input type="checkbox"/> Pretty well <input type="checkbox"/> Really well
I knew how to solve the problem.	<input type="checkbox"/> Not very well <input type="checkbox"/> A little bit <input type="checkbox"/> Pretty well <input type="checkbox"/> Really well
I understood the math ideas I needed to solve this problem.	<input type="checkbox"/> Not very well <input type="checkbox"/> A little bit <input type="checkbox"/> Pretty well <input type="checkbox"/> Really well
I showed how I solved the problem.	<input type="checkbox"/> Not very well <input type="checkbox"/> A little bit <input type="checkbox"/> Pretty well <input type="checkbox"/> Really well

PORTFOLIOS AND COLLECTIONS OF WORK

A portfolio is a collection of student work that is compiled and maintained by the student with guidance from the teacher. Each student's portfolio contains pieces of work that represent his or her learning: paper-and-pencil tasks, drawings, journal entries, models, photographs of the student at work, computer printouts. Maintained over time, the portfolio can provide a record of the student's growth in mathematical understanding and development.

Various materials can be used to make a portfolio: a file folder, a plastic document holder, a large Manila envelope, or two sheets of Bristol board taped together on three sides to create a large envelope. Pieces of work are date stamped for reference later in the year.

Teachers use the portfolio as an effective assessment tool when they:

- model portfolio building by maintaining a class portfolio;
The teacher, in discussion with students, regularly adds pieces of work that have been done by the class collectively.
- explain the purpose of the portfolio to the students;
The teacher reminds students that the portfolio is a means by which they can show their good work as mathematicians.
- include a reflective component;

I chose this piece for my portfolio because . . .
When I did this work, I learned . . .
If I could do this work all over again, I would . . .

- help the student decide whether a piece of work should be included in the portfolio;
"Tanya, you are really proud of your project. Would you like to include it in your portfolio? Why would it be a good piece to include?"
- provide time for students to regularly review the contents of their portfolios;
The teacher meets with students to discuss their portfolio pieces and their growth in learning.
- have students select their best entry periodically throughout the year.
Teachers can ask students to write about the piece and explain why it is their best work. The written explanation is attached to the selected piece.

As a documentation of the student's mathematical growth over time, the portfolio provides a focus of discussion at teacher-parent and student-teacher-parent conferences throughout the year.

ATTITUDINAL SURVEYS AND QUESTIONNAIRES

An attitudinal survey can heighten teachers' awareness of and sensitivity to students' feelings about mathematics, learning activities, and the classroom environment.

With younger students, the teacher can read the survey/questionnaire statements aloud and have students respond individually. For examples of surveys in which students are asked about their attitudes towards mathematics, see Appendices 8-3 and 8-4.

A completed survey about attitudes and preferences is shown below.

Completed Sample Survey

1-3

Hannah's Math Survey	
1. I enjoy math activities that <u>I kn do on the</u> <u>Kn P le</u>	[I can do on the computer]
2. I am good at <u>Ka Ting</u> <u>To 100</u>	[counting to 100]
3. I need some help learning how to <u>WRK on MY</u> <u>Mht Book</u>	[work on my math book]
4. I like working alone when <u>I am STing</u> <u>out PaTRNS OF math</u>	[I am sorting out patterns of math]
5. Something interesting I've learned in math is <u>to +l +M</u>	[to tell time]
6. I like working in a group when <u>I a M Do: n to M</u> <u>Boo K</u>	[I am doing time book]
7. At home, I use math when <u>I KOT MY NKIS</u>	[I count my nickels]

This survey was completed by a Grade 1 student, over several sittings.

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Parent Observations

Parents' observations can assist teachers in promoting students' learning and positive attitudes about mathematics. Including parents in the assessment process on an ongoing basis is beneficial for the following reasons:

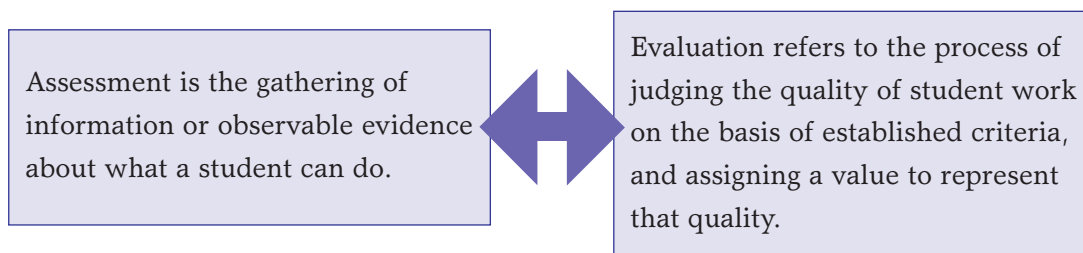
- Parents have the opportunity to observe their children demonstrating what they know and are able to do.
- Students and parents realize through experience that the mathematics learned in school is for use in the real world.
- Parents are able to provide information about personal situations or events that may have an impact on a child's learning or achievement at school.
- Parents have an opportunity to show interest in what their child is learning and to support that learning.

Translators can facilitate communication between teachers and parents of English language learners.

For an example of a checklist and comment form that involves parents in observing their child's mathematical abilities, see Appendix 8-5.

Evaluation

Assessment and evaluation, although interrelated, are different processes.



As teachers reflect on information that has been gathered through a variety of assessments, they are able to determine the degree to which a student's learning has progressed. This evaluation process allows teachers to consider student achievement of the curriculum expectations, focusing especially on students' understanding of mathematical concepts. As well, the evaluation of some assessment data indicates how well the student:

- solves mathematical problems;
- knows and applies mathematical procedures;
- communicates mathematical ideas.

The questions on the following page reflect some of the considerations teachers make as they evaluate student progress and achievement. Teachers should take the knowledge and skills categories (Knowledge and Understanding, Thinking, Communication, and Application) into account as they assess and evaluate in a balanced manner.

Knowledge and Understanding

- Does the student demonstrate limited / some / considerable / thorough knowledge of content (e.g., facts, terms, procedural skills, use of tools)?
- Does the student demonstrate limited / some / considerable / thorough understanding of mathematical concepts?

Thinking

- Does the student use planning skills (e.g., understanding the problem, making a plan for solving the problem) with limited / some / considerable / a high degree of effectiveness?
- Does the student use processing skills (e.g., carrying out a plan, looking back and evaluating solutions) with limited / some / considerable / a high degree of effectiveness?
- Does the student use critical/creative thinking processes (e.g., problem solving, inquiry) with limited / some / considerable / a high degree of effectiveness?

Communication

- Does the student express and organize ideas and mathematical thinking, using oral, visual, and written forms, with limited / some / considerable / a high degree of effectiveness?
- Does the student communicate for different audiences (e.g., peers, teachers) and purposes (e.g., to present data, to justify a solution) in oral, visual, and written forms with limited / some / considerable / a high degree of effectiveness?
- Does the student use mathematical conventions, vocabulary, and terminology in oral, visual, and written forms with limited / some / considerable / a high degree of effectiveness?

Application

- Does the student apply knowledge and skills in familiar contexts with limited / some / considerable / a high degree of effectiveness?
- Does the student transfer knowledge and skills to new contexts with limited / some / considerable / a high degree of effectiveness?
- Does the student make connections within and between various contexts with limited / some / considerable / a high degree of effectiveness?

Evaluating student achievement results in an understanding of the degree to which a student has been successful in learning. The purpose of evaluation, however, is not to label the student and his or her accomplishments, but to gain a stronger awareness of the learning that has occurred and of further measures to improve learning.

The Provincial Report Card is one among several ways of communicating the evaluation of student achievement to parents and students. Communication about the evaluation of student achievement should be ongoing throughout the year and should include, in addition to the report card, parent-student-teacher conferences, samples of student work that are sent home, interviews, telephone calls, and informal reports. Through these different forms of communication, teachers can clarify for parents and students the evaluation processes that have been used (i.e., the methods used for determining a grade, level, or mark), and the reasons for a student's achievement of a particular grade, level, or mark. Importantly, teachers need to suggest "next steps" – ways in which the student can improve achievement and possible strategies for parents to use in supporting their child's learning.

[Date]

Dear Parents,

Learning math is an important part of your child's education. In class, we will be doing many activities that will help your child understand and use math to solve problems and to explain mathematical ideas.

You can help your child be successful by doing math at home!

- Praise your child's efforts in mathematics. Recognize when your child uses mathematics in everyday life situations.
- Be positive about learning mathematics. Let your child know that everyone can learn mathematics.
- Make math fun. Spend time with your child playing games and doing activities that encourage better attitudes and stronger math skills.
- Talk about real-world examples of math. Point out ways that people use math every day (e.g., in the kitchen, in the grocery store, on the highway, on television).
- Encourage your child to solve problems. Provide assistance if it is needed, but let your child figure out the problem by himself or herself. Ask your child to explain how he or she solved the problem.

Sincerely,

Multiplication Test, Grade 3 Name: _____

1. Draw diagrams to show these multiplication facts.

$$4 \times 3 = 12$$

$$2 \times 7 = 14$$

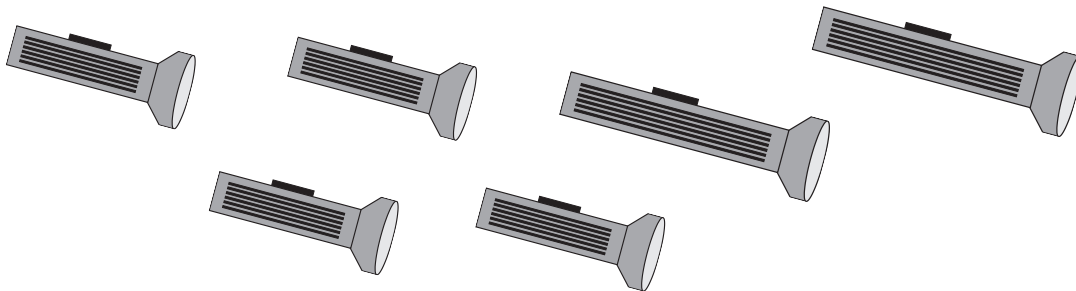
2. Write a multiplication fact for each problem.

A bee has 6 legs. How many legs are there on 3 bees? _____

A car has 4 tires. How many tires are there on 6 cars? _____

A week has 7 days. How many days are there in 3 weeks? _____


























3. A short flashlight takes 2 batteries. A long flashlight takes 3 batteries.



Use multiplication to find the number of batteries for all the flashlights.

Math Survey

Circle the face that shows how you feel.

1. I like solving math problems.			
2. I like doing patterns.			
3. I like working with shapes.			
4. I like measuring.			
5. I like doing number work.			
6. I like working with a partner.			
7. I like working in a small group.			
8. I like working by myself.			
9. I am good at math.			

Appendix 8-4: Sample Survey for Older Primary or Junior Students

1-3 **4-6**

How do you feel about math? Circle Yes, Sometimes, or No.

I like to work on really hard math problems.	Yes	Sometimes	No
Math is my favourite subject at school.	Yes	Sometimes	No
Math class makes me feel nervous.	Yes	Sometimes	No
On a math paper, I write down anything just to get it done.	Yes	Sometimes	No
I will work a long time on a problem until I think I've solved it.	Yes	Sometimes	No
I wish I were better at math.	Yes	Sometimes	No
If I get stuck on a problem, I just quit.	Yes	Sometimes	No
I know what I am doing in math.	Yes	Sometimes	No

Appendix 8-5: Checklist and Comment Form for Parents

[Date]

Dear Family,

I hope that you enjoy solving the problem attached. Please fill out the information below and return it to me along with the completed problem.

When working on a problem, can your child:

- ☐ explain the problem?
- ☐ decide on a plan for solving the problem?
- ☐ try other ways if something doesn't work?
- ☐ stick to the task?
- ☐ explain his or her thinking while working?

Comments

9 • Home Connections

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Home Connections

Introduction

As the changing vision of mathematics education continues to have an impact on the way mathematics is taught and students' learning of mathematics is assessed, the need for teachers to share this vision becomes increasingly important. Strong connections between home and school are needed to ensure that parents and teachers are working together to support the mathematical development of all children. These connections can be established through effective and varied communications with parents. Many teachers use interviews, newsletters, and telephone calls as their principal methods for communicating with parents. Today, information can also be distributed through websites and e-mail. Family math nights, presentations, curriculum nights, math career displays, math career nights, and celebrations of student performance in mathematics can also provide opportunities for sharing important information and viewpoints related to mathematics education.



Effective communication with parents needs to be friendly, informative, ongoing, and consistent. Teachers should begin communicating about the mathematics program early in the school year. They will want to let parents know that they are looking forward to teaching the children, and they can share some of the highlights of the upcoming mathematics program. Teachers can open the channels of communication early in the year by inviting parents to share information or observations related to their child's mathematical development, along with any specific concerns parents might have. Teachers will help to keep the channels open throughout the year by reiterating their desire to work with each family to support the child's growth in mathematical understanding.

There are a number of ways in which parents can support their child's developing understanding of mathematics. Becoming familiar with mathematical ideas that their child will be learning is an important first step. Parents also need to know which skills their child is expected to be proficient in by the end of the school year. Familiarity with the curriculum will help parents to make math connections for their child at home. Parents who find ways to help their children see mathematics in their daily lives will make learning mathematics a more meaningful experience for them. Understanding how best to help children complete non-traditional homework assignments is also important, especially for those parents who expect to see a great deal of drill and practice homework rather than homework that provides more meaningful learning.

This chapter provides information that will help teachers in making connections between the classroom mathematics program and the home, by considering the changing face of mathematics, parent-teacher math connections, and parent-teacher-student math connections.

The Changing Face of Mathematics

The National Council of Teachers of Mathematics (2000) has articulated a vision that has been incorporated into the mathematics curricula of provinces throughout Canada. Sharing this vision with parents can help them to understand why the mathematics they see their children learning may look different from what they themselves experienced as students.

"Adults who were taught mathematics primarily through the use of abstract symbols need the opportunity to make a connection between the mathematical ideas they encountered in school and the materials their children will be using."

(Litton, 1998, p. 35)

Key points that might be communicated to parents about this vision are listed below.

The vision of mathematics:

- Mathematics can be viewed as the study of relationships or the science of patterns.
- Mathematics requires critical-thinking skills that are used to solve a broad range of problems encountered every day.
- Mathematics is an activity that people do, one that is broad in content and encompasses many fields.
- Mathematics can and must be at the command of all in our technological society.

The vision of *learning* mathematics:

- To learn mathematics, students must develop skills in reasoning and communicating mathematically.
- Students must have experiences that help them value mathematics.
- Students must be encouraged to become problem solvers who are confident in their mathematical abilities in classrooms where teachers facilitate a risk-taking and supportive mathematical environment.
- Everyone can learn mathematics.

The vision of *teaching* mathematics:

- Teaching mathematics involves providing opportunities for students to conjecture, invent, and problem solve rather than merely find correct answers.
- Mathematical ideas and applications must be presented in ways that highlight connections. No longer is it sufficient to treat mathematics as a body of isolated concepts and skills.
- Teachers must strive to establish classrooms as mathematical communities rather than as collections of individuals.
- Developing mathematical reasoning becomes more important than the memorization of procedures.
- Instead of acting as the sole authority on how to get correct answers, the teacher must encourage students to use a variety of strategies that they are able to explain and provide mathematical evidence for.

In this vision, inquiry and problem solving are central to the process of learning mathematics. The Ontario curriculum in mathematics highlights the skills that students develop through problem solving as follows:

- Students learn to solve problems using a variety of different strategies.
- Students learn to examine their own thinking skills.

Problem solving helps students in their use of technology, promotes in students the ability to work with others, and facilitates in students the ability to talk about and reflect on mathematics.

Parent-Teacher Math Connections

Teachers have the important responsibility of keeping parents informed about current mathematical practices and what they look like in their child's classroom. Challenges arise when parents hold a traditional vision of mathematics that focuses on arithmetic proficiency rather than mathematical understanding. These parents, who tend to see their role as limited to supporting the memorization of number facts and the drill and practice of arithmetic procedures, can be helped to an expanded role when teachers inform them about the changing vision of the elementary mathematics classroom and assist them in different ways in their ongoing efforts to help their child at home.

"By being upbeat, informative, and open about my approach to mathematics I'm likely to end up with informed and supportive parents who work for and with my students – and me."

(Litton, 1998, p. 134)

Teachers may use various methods to establish and maintain open and productive communication between the home and the school. Among the most effective methods of providing the information that parents need in order to be partners in their child's learning of mathematics are newsletters, school or classroom websites, and parent-teacher conferences.

NEWSLETTERS

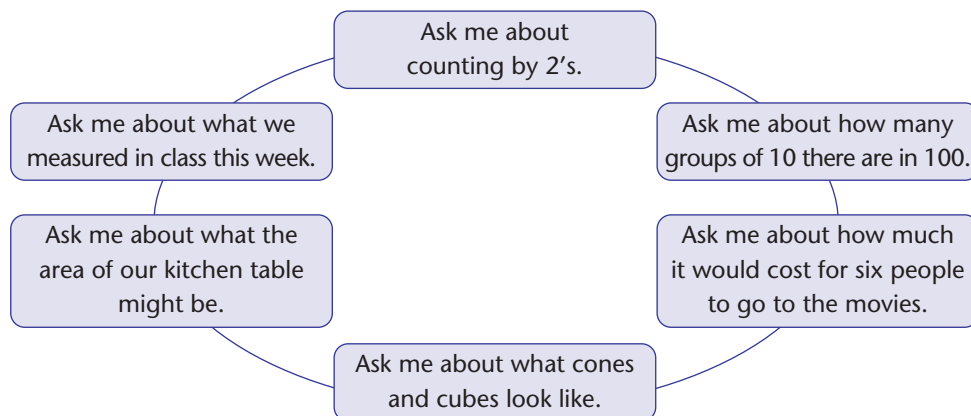
A weekly, biweekly, or monthly newsletter is a valuable medium for informing parents of what is happening in their child's classroom, providing them with a description of the mathematical content presented in class, and suggesting ways in which they can help their child experience mathematics daily.

Newsletters can have different formats, depending on the preference of the teacher or the grade team. Among the appropriate formats are flyers, letters, and calendars that include information about the classroom.

For examples of flyer-type newsletters for the primary and junior grades, see Appendices 9-1 and 9-2. For sample letter-type newsletters for the primary and junior grades, see Appendices 9-3 and 9-4.

One way for teachers to communicate with parents in a newsletter is to use an "Ask Me About . . ." format (Baskwill, 1992), as shown in the example at the top of the opposite page:

Ask Me About ...



Several "Ask Me About ..." boxes or bubbles provide parents with specific suggestions for things to ask their child. Students will be more likely to recall and relate what they have explored in mathematics class if they are provided with focused prompts such as these.

SCHOOL OR CLASSROOM WEBSITES

A website is a vehicle that gives teachers wonderful opportunities to share with the community what students have been exploring in mathematics and to explain the mathematics program to parents. Some schools may have created their own websites for parents, students, and the community. Some classrooms have their own web pages within these websites.

Before including any photographs or names on a website or web page, teachers are advised to check their school district policy regarding restrictions on school and student information that can be posted online.

The following are possible ideas for web pages to post on a school or classroom website:

- student work and projects that have been photographed, scanned in, and/or linked (if created with a computer application);
- photographs of students working on mathematics investigations or explorations. Teachers must check for permission from parents to use students' photographs;
- examples of children's literature that supports the early mathematics program, and recommendations for ways in which parents and their children can use the literature as starting points for mathematics activities;
- a math problem for parents and students to solve together at home;

- links to mathematics curriculum websites such as ministry websites for information on curriculum expectations;
- links to websites that offer mathematical activities or problems and homework help for students and parents.

PARENT-TEACHER CONFERENCES

A parent-teacher conference:

- is a meeting between the teacher and parents to review the student's progress and to set goals for the student's further learning;
- can be initiated by the teacher or the parent;
- may be formally or informally structured;
- may include the student in a role to share his or her learning and ideas about goals for further learning.

"A conference gives me a chance to show parents that I'm knowledgeable about their children and that I'm open to any ideas and information the parents have."

(Litton, 1998, p. 49)

Teachers and parents can cooperatively use conferences to:

- exchange information and share ideas about the student;
- explore the student's growth and plan next steps in learning for the student;
- review and clarify what the student has already completed and learned, and plan how the student's previous learning can be extended;
- provide feedback that will help the student to progress more successfully towards achieving specific curriculum expectations.

Parent conferences are an opportunity for the teacher to share any knowledge that he or she has about the student and to be open to any ideas and information that parents may have. During conferences, teachers and parents share their deep care for the child and their responsibilities for the child's success both as a student and as a person (Litton, 1998). It is important that this valuable opportunity be used to focus on the specific needs and abilities of the child.

Teachers or parents may have a variety of reasons to schedule conferences throughout the year. Parents may want more information about the mathematics program in the classroom, about the curriculum and how it is being implemented, or about tips and strategies they may use at home to help their child. This time could also be used to discuss specific individual goals for the student or strategies to support the student's mathematical learning.

For some helpful suggestions that teachers can use when preparing for and conducting a conference, see Appendix 9-5. For a sample letter that the teacher can send to parents

before a conference, see Appendix 9-6. For examples of forms to use for recording the issues and outcomes of a conference, see Appendices 9-7 and 9-8. For a sample letter that the teacher can send to parents after a conference, see Appendix 9-9.

Parent-Teacher-Student Math Connections

Classroom teachers model positive attitudes towards math by promoting family awareness of mathematics, especially by encouraging families to engage in rich mathematical activities at home. Teachers need to support and promote activities that stimulate mathematical thinking and mathematical language. Learning experiences should focus on a student's problem-solving abilities and be created with fun in mind. Ideas and activities can be shared in class, in school newsletters, or during parent-teacher conferences.

This section will provide suggestions, templates, and sample activities that can be used to share ideas and strategies related to the following categories:

- mathematics and the child's daily experiences
- take-home math programs
- agendas
- homework: meaningful follow-up to classroom mathematics
- mathematics and literature
- mathematical learning for English language learners and their parents
- math curriculum nights
- family math nights, family math series, or math career nights



MATHEMATICS AND THE STUDENT'S DAILY EXPERIENCES

Working with children to help them discover mathematics in their daily lives is an excellent way for families to enjoy mathematics together. The following are ideas that teachers may present to parents in a weekly or monthly newsletter, particularly as a way of following up on concepts that are being taught in the classroom.

"Children's and parents' understanding in mathematics improves when they are able to make connections between school mathematics and real world mathematics."

(Adams, Waters, Chapple, & Onslow, 2002, p. ii)

Around the home:

- Encourage explorative math play at home (e.g., invite your child to build, measure, and compare structures made with different-sized cardboard boxes; build and describe structures in a sandbox; play jump-rope games or games like hopscotch, catch, and hide-and-seek).
- Explore opportunities for your child to make connections with mathematics in daily routines (e.g., estimating the distance to a nearby location, measuring the time it takes to complete a chore). Be sure to talk with your child about the math connections in these activities.
- Work along with your child in creating designs from toothpicks, straws, paper shapes, paper towel rolls, and other found materials. Invite your child to describe his or her design(s) using numbers, words, and pictures.
- Play board games, number cube (dice) games, card games, and dominoes, and solve puzzles together. In conversation, ask your child to tell you what he or she did to try to win the game or solve the puzzle. Then ask your child to tell you whether he or she would do the same thing next time and give reasons why or why not.
- Invite your child to save his or her change in a piggy bank or other suitable container. Identify coins and bills, and estimate and count money.
- Encourage your child to use a simple spreadsheet to keep an account of his or her saving and spending of money.
- Engage in role-play games such as "store" or "restaurant" with your child. One person can be the owner of the store or restaurant and the other person can be the customer. Be sure to pose grade-level and age-appropriate math problems for your child to solve as you play.
- Children love to play "school". Try being the student and let your child be the math teacher.
- Bake or cook together and follow directions for favourite recipes.
- Make a third, fourth, or half of a simple recipe or try doubling it.
- Create and describe a pattern together while you frost a cake.
- While baking cookies, let your child make an array of columns and rows of cookies and practise multiplication.

- Invite your child to place 3 eggs in an egg carton. Ask, "How many more eggs do we need to fill the carton?" Then arrange the same 3 eggs differently, using different sections of the carton, and ask, "How many eggs are in the whole carton?" Always have your child explain his or her reasoning by asking, "How do you know?" Try again, this time using a different number of eggs.
- Have your child use anything with columns and rows, such as muffin tins or egg cartons, to practise multiplication and early division concepts.
- Ask your child to show you a third, fourth, or half a cookie.
- Cut an apple into thirds, fourths, and/or halves and have your child put the apple back together to make a whole.
- Read numbers in newspapers, in telephone books, on addressed envelopes, on bills, on household thermostats, on measuring tapes, and so forth.
- Help your child find items in your home that are shaped like cubes, pyramids, cones, spheres, cylinders, octagons, hexagons, and rectangular and triangular prisms.
- In the kitchen, have your child compare large cans with small cans and large boxes with small boxes. You can ask, "Which is the big one?" "Which is the small one?"
- Have your child use a measuring tape to measure dimensions of containers found in the kitchen cupboards. Encourage your child to estimate the dimensions before measuring them.
- Have your child estimate and count! Count everything (e.g., books, chairs, compact discs, towels, steps, tiles on a floor).
- Count by 1's, 2's, 3's, 4's, 5's, 6's, and 7's, and remember to consider the ability and grade level of your child to know where to begin and when to stop.
- Count forward and count backwards, starting with different numbers.
- Sort a variety of items at home (e.g., toys, utensils, dishes, socks, mail, shoes, colouring tools, fabric, recyclables). Describe the sorting rule. Try sorting the same item(s) again using a new rule.
- Talk about math experiences in daily events (e.g., measuring laundry detergent, packing a suitcase, creating a grocery list, setting an alarm clock).
- Have your child read through the newspaper to find math-related material (e.g., advertisements for retail sales, stock market quotes, sports box scores and statistics, classified ads for mathematics-related career opportunities).
- Help your child think of different kinds of data that people or organizations collect (e.g., data about consumer preferences from telephone opinion surveys, data about political preferences from exit polls, numbers of hits at a website) and why they collect the data.

- Keep a family yearly calendar. Record upcoming events and count the number of days, weeks, and months up to the event. Use words like *days*, *weeks*, *months*, and *year*.

In the neighbourhood:

- Play I Spy, looking for and describing shapes in a playground, on a farm, in a town, or in a city.
- Estimate and count things in a grocery store.
- Ask, "How are the foods grouped in the grocery store?"
- Talk about what is heavy and what is light in a grocery store.
- Estimate and measure produce in a grocery store.
- When you are waiting in line to pay for your groceries, ask your child to estimate how much the bill will be. This activity can be done at the end of a meal at a restaurant as well.
- Find the shortest and the longest checkout line in a grocery store.
- Find and describe patterns in a section of a garden.
- Look for and identify sets during a nature walk (e.g., each maple leaf has three parts; insects have six legs).
- Estimate and count the number of footsteps between two trees as you walk.
- Use non-standard and/or standard measurement units to estimate and measure the distance between two fence posts or other objects.
- Look for, identify, and describe patterns in a landscape.
- Ask your child for examples of perimeter and area in his or her daily life.
- Collect some rocks with your child and ask your child to sort them: heavy and light, shiny and dull, big and small, rough and smooth. How many of each are there? Ask your child to use the different-shaped rocks to create an image.
- Ask, "How far is a kilometre?" Walk a kilometre together.
- Predict and measure the length of time of your walks.
- Calculate and graph temperatures, and describe differences between the morning, afternoon, and evening temperatures.
- Have your child record and share directions to a final destination.

On the road:

- When you are travelling in a car or sitting on a bench, take turns with your child calling out licence plate numbers (driver excluded for safety reasons).
- Have your child record the licence plate numbers.
- Make the largest three-digit number possible from the plate numbers.

- Ask younger children to name the largest single-digit number and the smallest single-digit number and to call out the numbers in order from smallest to largest. Some children may just name each number.
- Have your child record any palindromes that he or she might see on licence plates.
- Read the numbers on a licence plate as number words.
- Compare the value of a licence plate number with the value of another licence plate number. Is the second number greater than or less than the first number?
- Find the sum of the numbers in a licence plate.
- Ask children to find ways to make a target number between 1 and 20 (depending on ability and the requirements of the curriculum expectations) with any of the numbers in a licence plate, using addition and/or subtraction.
- Ask young children to double each digit in the licence plate number.
- The next time you fill up your car with gas, let your child read the different numbers on the pump. Ask him or her to estimate the distance between where you are and where you are going. Have your child estimate how much gas will be needed to travel that distance.
- Have older children choose any number as a factor. Every time they see their number, they multiply it. For example, if they choose 6 and spot a number 6 on a licence plate, they multiply the two 6's and get 36. The next time they see 6, they multiply 36×6 and get a new answer of 216. They continue until you reach your destination. You may want to keep a calculator in the car. As a treat, have someone open the car door for the person who scores the highest number.

In the garden:

- Help your child estimate how much space will be needed for planting.
- Together, plan a shape for your garden.
- Create a list of the types of plants that you would like to buy and list how many of each.
- Estimate how many plants of each kind will fit into the garden.
- Help your child estimate how many flowers and/or vegetables you will need to buy.
- Discuss and plan the arrangement of the plants in the garden.
- Estimate how much time it will take to plant the garden. Plant the garden together and measure the time that it takes to plant one plant, two plants, and the whole garden.
- Have your child look through seed catalogues or garden-center flyers to help estimate the cost of the seeds, plants, or supplies needed to set up the garden. Comparison shopping develops estimation skills and money-value sense.

- Create a watering schedule.
- Depending on your child's interest and ability level, ask such questions as, "How many fewer cucumber plants do we have than tomato plants? How many more tomato plants do we have than cucumber plants?" "Which are the shortest and tallest plants?" "What is the largest number of plants of one kind in our garden? What is the smallest number?"
- Help your child estimate how big the plants will grow, then measure the plants when they are full grown.

TAKE-HOME MATH PROGRAMS

A take-home math program that reinforces the home-school connection may be established in the classroom, across the division, or throughout the school. A take-home math program allows the teacher to establish a series of learning experiences that supplement or enhance the daily classroom program. These activities may be designed to extend a concept taught in class, foster communication in mathematics, reinforce computational proficiencies through game playing, and enhance the student's enjoyment of mathematics.

"Parents and children can enjoy mathematics together. With the proper resources and information, parents, families, and the community can become a teacher's greatest asset and support system."

(Sutton & Krueger, 2002, p. 95)

The take-home math program should contain activities that help the student to meet Ontario curriculum expectations for the specific grade. Activities may be developed by classroom teachers, obtained from textbook programs, or taken from professional resources devoted to the development of essential skills and problem-solving strategies in mathematics. Although the activities should be designed to create interest and provide fun, they should also be appropriate homework for students, by virtue of introducing, developing, or reviewing important mathematical concepts and skills. All activities chosen for the take-home math program should involve both the child and the parent. An atmosphere of fun and discovery will encourage reluctant children to participate and create a relaxed environment in which the whole family can enjoy mathematics.

An open-ended approach will encourage the kind of discussion that will enable both children and their parents to come to a better understanding of the mathematics curriculum. In an open-ended approach, the activity provides a problem that does not present an immediate answer. There may be multiple answers to the problem, or multiple strategies for reaching one answer. Where the activity is open-ended, and children work with their partners, parents, or someone else, they learn about different mathematical paths by which to reach answers, and they develop a greater understanding of the mathematics being presented.

The Role of the Teacher

For a take-home math program to be successful, the teacher must select activities that enhance the classroom program. Traditional paper-and-pencil homework focused on computation can often heighten the anxiety that students feel about their success in mathematics. The take-home math program should provide students with opportunities to engage actively in problem solving, to apply mathematical procedures where appropriate, to demonstrate their understanding of concepts, and to communicate their thinking clearly and precisely. To this end, the teacher should select as take-home math activities either rich problem-solving tasks, which may or may not involve the use of manipulatives, or games that strengthen number fluency or logical-thinking skills.

Involvement of the Parents

Most parents want to be involved in their children's mathematics education, but some may be reluctant to do so for a variety of reasons. They realize that the way mathematics is taught today has changed, and they are not sure of how best to help their child at home. Also, they may experience a level of anxiety with respect to mathematics. A take-home math program can help to relieve some of the fears that parents may have if it is well designed, that is, if the instructions given with the activities are clear and precise, and if the activities themselves provide parents with easy access to the mathematics involved. It is important to have a program of take-home activities that parallel the activities occurring in the classroom.

The take-home math program may be organized in any of the following ways:

- A. Each week during the year, activities can be sent home. (At the primary level, each activity is sent in a resealable plastic bag that contains all the materials needed to engage in the activity.) Each take-home math activity is accompanied by a feedback sheet or small journal (half notebook) in which the parents or caregiver can comment on the activity. The use of a feedback sheet or a journal is one way to track parents' participation in the take-home math program.
- B. The school council can be approached to help set up the take-home math program. Teachers and parents in the community can work together to assemble the activities. If the parents in the community are fluent in both English and a second language, they could assist the teachers in translating the activities into different languages to serve the needs of the community.
- C. Take-home math activities may be developed and implemented for specific units of study. For example, if the classroom mathematics program is focusing on probability, several games or activities could be included in the take-home math program to supplement the learning experience in this area.

D. Another possibility would be to establish a math buddy system. A time could be established within the routine of the school day to accommodate a meeting between a student and his or her math buddy for the purpose of utilizing the take-home math activities. Alternatively, the teacher may want to arrange for a volunteer to work with those students who have not had the opportunity to participate at home.

See Appendix 9-10 for a sample letter that teachers at the Kindergarten and primary levels may use to introduce their take-home math program. See Appendix 9-11 for a sample letter that teachers at the junior level may use to introduce their take-home math program. See Appendices 9-12 through 9-21 for sample primary-level and junior-level activities that can be used as part of a take-home math program.

AGENDAS

A daily agenda may be one that the school provides, that students purchase, or that the teacher creates. The agenda is used to provide daily communication among students, parents, and teachers about a variety of topics, including mathematics. Students can record important information, make a note of materials needed for upcoming events or units, or write a reminder about something to ask their parents. Teachers and parents can also use the agenda as a means of sharing information with each other or asking a question.

HOMEWORK: MEANINGFUL FOLLOW-UP TO CLASSROOM LEARNING

Homework is defined as out-of-class tasks assigned to the students to prepare them for classroom work or to have them practise or extend classroom work. Homework tasks must provide meaningful experiences for both children and parents.

Homework activities should be engaging mathematical experiences that highlight mathematics in the students' environment. Whether students are playing a game requiring strategy with family members or friends or completing an assigned task, their mathematics homework should be meaningful, enjoyable, and productive.

The most important thing to consider when assigning mathematics homework to students is fostering a love of mathematics through engaging, positive activity. A positive homework experience in mathematics will leave students open and eager for more mathematics.

In order to establish and nurture a strong home-school connection, teachers must communicate to parents the need to set certain standards for their children as the children undertake homework tasks. Parents need to be informed of the role they

"Children and families have different needs, so one size does not fit all. Working out a useful and well-received homework program is tricky but ultimately worthwhile."

(Litton, 1998, p. 91)

play in fostering a positive educational home environment. The following sections are designed to help teachers communicate to parents some of the key messages about homework, including the purpose of homework, the parameters for parental involvement, and the types and amounts of homework recommended. Teachers may present these ideas at a math curriculum night, during parent-teacher interviews, in a class newsletter, or informally with parents as the need arises.

The Purpose of Mathematics Homework

Homework may be viewed as having three distinct purposes: practice, preparation, and extension. Homework for practice, the most common type assigned, should require the students to practise a skill or understanding that they have gained during a classroom investigation or activity. Homework for practice may take the form of games, counting activities, measuring activities, or problem-solving situations.

Homework for preparation allows students the opportunity to think about and observe certain concepts before they are formally explored in a classroom activity. Homework for preparation could involve having students investigate the surface area of yard spaces, look for and name shapes at home, compare statistics of professional sports teams, make lists of objects that are bigger or smaller than a given object, or investigate patterns before participating in the classroom investigation.

Homework for extension should provide students with opportunities to extend their understanding of a concept explored in class, to further develop their skills in a specific area, or to further develop a mathematical concept or problem-solving situation. Homework for extension may involve games or problem-solving activities that allow the student to explore at home a scenario that is similar to but slightly different from a scenario developed in class.

Parental Involvement With Mathematics Homework

Parents may want to know how much they should help their child with homework. Teachers should let parents know that the extent of their involvement will depend on the activity. For example, it is perfectly acceptable for the whole family to be involved in game playing and discussions about mathematical discoveries. On the other hand, homework designed to reinforce a procedure that has been explored in class should be largely an independent activity. What is expected of students in their homework should be made clear to parents, who will then be able to help their children effectively with their learning. For example, for a student who comes home with a game in a take-home math bag, a letter could be included that invites the whole family to play the game. For a student who is given independent homework, mention of the homework might be made in the student's agenda or in an informational letter. Parents should be encouraged to communicate with the teacher when

their child seems to be requiring extensive parental help with activities that are designed to be completed independently.

Parents can further support their child by ensuring that there is a quiet place for study, one that has adequate furniture and lighting. Children should have appropriate resources, tools, and materials readily available when doing homework (e.g., pencils, ruler, calculator, coloured pencils).

Also, parents should set high standards for their children by letting them know that homework activities should be completed to the best of the children's ability. Parents may find it useful to keep track of their child's mathematics homework with a simple homework log. This log may simply be a calendar with a check mark to indicate that the student has homework on a particular day and a smiley face next to the check mark to show that the homework has been completed.

While students are engaged in mathematics homework activities, parents should ask probing questions rather than provide answers. They may ask questions such as:

- "What do you know?"
- "What do you need to know?"
- "What do you need to find out?"
- "What can you try to do to solve this problem?"
- "How can you show your work?"
- "Do you think that your plan is working?"
- "Is there anything else that you need to do?"
- "Do you think that your answer makes sense?"
- "Is there any other way to solve this problem?"

Parents should encourage their child to use objects or manipulatives, use numbers, draw diagrams, and/or write down his or her ideas while exploring a problem. Parents should have their child talk to them about math while solving a problem. Questioning can provide a prompt for the child and give the child a starting point for exploring an idea or justifying his or her thinking. Writing and talking help children to clarify ideas and think about their next steps and can give parents clues about when to start helping or when to extend their child's thinking.

Appropriate Mathematics Homework

Many school boards have guidelines specifying the appropriate length of time for students to spend on homework activities. It is important to recognize that there is more inherent value in engaging a student in family mathematics activities than in having the student complete worksheets for homework every day.

Games for Homework

Students need ample opportunities to practise basic number facts. Games provide an enjoyable context for the practice of facts. Children naturally engage with games. Using games as a context for practising number facts also reinforces students' social skills and communication skills, and develops their strategic thinking and logical reasoning. Board and card games can be used to reinforce basic facts and number recognition.

Mathematics Workbooks

Many parents feel that worksheets focusing on fact recall help students become "better" at mathematics. Only if worksheets and workbooks are used in connection with what is happening in the classroom can they be an appropriate source of homework for students. Students may become frustrated and suffer the beginnings of math anxiety if they are given sheets of workbook drill and fact practice that are not related to the learning taking place in the classroom or not connected with a strategy being explored in the classroom. There must be a balance in the mathematics homework that children are given – a balance among games, problems to solve, math activities within the home such as those mentioned earlier, and paper-and-pencil homework.

Students benefit greatly from interactive experiences that promote the recall of math facts and procedures. Many card, number cube (dice), and board games allow students to practise these facts in a meaningful and engaging way. Moreover, a game might be connected with a certain strategy that is being explored in the classroom, such as doubles, making tens, or one more than. Students and parents can have fun playing these games and reduce the math anxiety that often accompanies paper-and-pencil worksheets.

MATHEMATICS AND LITERATURE

Students love literature. They like to listen to and read certain stories over and over again, to the point where they can remember and retell the stories. They enjoy answering the *who*, *what*, *when*, *where*, and *why* questions about a story. In the process of answering these questions, they are actively involved in listening and learning. Stories that engage students in such a way can also be used to develop mathematical understanding and knowledge, with the important provision that students first be

given the opportunity to read the story (or listen as it is read to them) *for enjoyment*. It is also essential that students understand the story as a whole before examining or exploring it for specific mathematical concepts.

The use of picture books to help younger as well as older students develop math concepts is a valuable strategy available to both parents and teachers. Through literature, mathematics becomes more meaningful and more closely connected with students' experiences. Literature helps to connect mathematical concepts with the world that the student experiences when he or she reads a story.

Through literature, parents and children can explore concepts relating to the five strands of the Ontario mathematics curriculum for Kindergarten and Grades 1 to 6. For example:

- Geometric shapes can often be found in a book and related to real-world objects (e.g., circular wheels, rectangular windows).
- Measurement concepts can be explored through references to money, size, time, and so forth.
- Number sense concepts may be illustrated in literature (e.g., counting on, skip counting, and counting backwards, and pattern recognition in the development of addition, subtraction, multiplication, and division facts).
- Patterns may be found in the way a story or poem is written or illustrated.
- Probability may be developed by posing questions – for example, “What are the chances of . . . ?” “What is the likelihood of . . . ?” “Do you think this will happen?”

Children's literature can also serve as a focus for problem solving, which is at the heart of mathematics, and can provide a context in which students try out and justify strategies used to solve problems. Parents and children can often work together to generate problems while they are reading and talking about a book.

See Appendix 9-22 for some ways in which parents can make connections between mathematics and literature for their child. See also the section on children's literature and mathematics in Chapter 7: Classroom Resources and Management and the list of titles given in Appendix 7-11, both in Volume Three.

MATHEMATICAL LEARNING FOR ENGLISH LANGUAGE LEARNERS AND THEIR PARENTS

Schools and classrooms today may comprise a diverse range of cultures and languages. Ensuring that all children and their parents feel themselves to be a part of the everyday happenings in such schools and classrooms is an important responsibility for teachers. The following are a few examples of ideas that value the diversity within the classroom

and provide connections between the classroom and English language learners and their parents. Teachers can:

- have available in the classroom some fairy tales or fables in English and in the student's home language, whenever possible (e.g., "The Little Red Hen" in English and in Spanish). When the student borrows a book, both the English and the other-language versions can be sent home at the same time. A mathematical inquiry or activity about the story should accompany the story. Also, there are many commercially assembled tote bags that include a piece of literature, an activity card, support materials, and usually a journal. These tote bags can be modified by inserting the text of the piece of literature in the student's home language, with the math question or activity translated;
- buddy their class with a class in a junior or an intermediate grade, and buddy the English language learner(s) with a student(s) who can translate the story. The older student buddy can also work on developing a mathematical problem-solving activity to be completed at home;
- when possible, send a letter home to parents in the language spoken at home, asking that they write a description, in the language spoken in the home, of a neighbourhood walk, a visit to the grocery store, or a visit to the theatre that they took with their child. The information discovered on their excursion may then be translated by the older student buddy, and the child can share it with the class. Such sharing of information from the student's home culture or environment can foster genuine discussion among students;
- have available books that reflect, describe, or illustrate the students' cultures, for example, books that talk about a certain food that relates to a particular culture (e.g., sushi – Japanese, curry – Indian, tamale – Mexican, paella – Spanish, poi – Hawaiian). The child can make the food, using the recipe from the book or a recipe provided by the teacher or parents. The child can be asked to compare the ingredients visually with those given in the book (e.g., for appearance, quantity), to estimate how much would fill a teaspoon or cup, and then actually to measure the ingredients needed. During this process, the parent can engage the child in mathematical discussion, using mathematical language – for example, asking, "How much?" "Is it about _____?" or "Do we need more or less of this ingredient?";
- have family math nights that include parents of English language learners by providing an interpreter or by pairing the parents with those who are able to communicate with them. High school students could also be asked to volunteer their community service hours to help act as interpreters for parents and children during a family math night;
- have a multicultural math night in which parents bring in math games or inquiry-based problems that are particular to their country of origin. (An inquiry-based problem is one that allows parents and children to ask questions about the

mathematics. A wide variety of resources for multicultural problems exist in print and on the Internet.) During the evening, activities can be shared with parents, and one or two activities that are particular to the math program used in the school can be added.

MATH CURRICULUM NIGHTS

A math curriculum night will focus on informing parents. In particular, math curriculum nights help to develop a parent's understanding of the changes that have taken place in the way teachers teach mathematics and the impact of these changes on parents in their efforts to support their children in learning mathematics. The following suggestions may help principals and teachers who are organizing a math curriculum night for their school. Teachers can:

- **select a format**

Formats for curriculum-focused evenings include a general overall presentation in the gym, division or grade presentations in classrooms, a carousel of teacher-led mathematics activities for parents, or perhaps a math walk through the school to highlight the mathematical applications studied by the students. (In the carousel format, teachers are set up at various stations. Each teacher works with a small group on a particular activity. Then that group moves on to the next teacher, who works with them on a different activity, and so on.) No matter which format is used, the focus is always on sharing key messages related to mathematics education.

- **tap into memories of mathematics**

Most parents have memories of themselves as students in a mathematics class. For some parents, these memories may include experiences that fostered anxiety about mathematics or experiences that left them wondering when they would ever use the mathematics they were learning. For these parents, change is welcome. Other parents who experienced success when mathematics was taught primarily as a series of procedures to be memorized may have trouble understanding why anything needs to be changed.

After eliciting parents' experiences, the teacher may explain that, in the past, elementary mathematics focused on basic arithmetic, a very narrow part of what mathematics is. The emphasis was on procedural knowledge, rather than on thinking about and understanding the important underlying mathematical concepts and relationships. Such an emphasis often presented a significant challenge for many children who needed to understand in order to succeed in mathematics. Students were often not expected to make sense of the mathematics they were learning, and this approach was reflected in the teaching and assessment strategies used in the classroom.

"Giving parents a personal glimpse into your program will build their understanding of the math instruction you're offering their child. And building their understanding is the best way to build their confidence."

(Burns & Silbey, 2000, p. 93)

- **share the vision of mathematics**

Parents who share their past experiences in mathematics during curriculum nights can be invited to discuss why they believe educators are changing the way mathematics is taught. It is important that teachers share the current vision of what mathematics is, how it is taught, and how educators and parents can know that children have learned it. Parents need to understand the Ontario curriculum and the strands outlined in the curriculum. They also need to understand how and why mathematics is taught as it is taught today.

- **provide a problem-solving experience**

Engaging parents in a non-threatening problem-solving activity or an inquiry-based activity can highlight the importance of problem solving as well as touch on and explore the problem-solving strategies that parents are likely to observe their children using. An example of an inquiry-based activity in which parents could be engaged is determining how many coins they could have if they have \$2.74 in their pocket. Such a problem has a wide variety of solutions that can be explored and discussed. The problem can also be modified (e.g., at least three ways of reaching the total must be found, the greatest/least number of coins must be used, only coins of certain denominations are allowed).

The focus should be on the big ideas addressed by the curriculum expectations – the main concepts that students learn in mathematics each year. Teachers should help parents to see through the lists of expectations to the key concepts that students will be expected to understand and apply. Teachers should demonstrate to parents that students may explore a concept one year and then investigate and consolidate the concept the next year. Students may not become proficient with a concept when they are initially introduced to it and will need time to understand and explore it.

- **address the achievement chart**

Parents benefit from learning more about the achievement chart on pages 22–23 of the 2005 Ontario mathematics curriculum document. Parents can provide better support for their children if they understand the foundation on which teachers develop their instructional and assessment strategies. It is important for parents to know that teachers are working on building and developing knowledge and skills that relate to the four categories of the achievement chart: Knowledge and Understanding, Thinking, Communication, and Application.

- **demonstrate how current text programs are designed**

Parents benefit from an opportunity to examine current text programs and how they have been designed. Providing parents with the opportunity to explore other components besides the student book (e.g., the teacher manual or activity cards used in learning centres) can also be very helpful. In this process teachers can engage parents in a discussion of the role that various learning resources play in the delivery of an effective mathematics program.

- **provide some helpful tips for parents**

Parents usually want to leave a mathematics presentation with a few tips on how they can best help their children learn mathematics. Many articles have been written that list ways parents can help. Some suggestions can be found in the publication entitled *Helping Your Child Learn Math: A Parent's Guide* (Ontario Ministry of Education, 2003b). Another set of suggestions, compiled by Jan Mokros in her book *Beyond Facts and Flashcards* (1996), is included in this guide and can be used as a handout (see Appendix 9-23).

FAMILY MATH NIGHT OR FAMILY MATH SERIES

Description of Family Math Night

Family math night is a event designed to bring parents and children together to engage in mathematics activities and to help parents learn more about the Ontario mathematics curriculum. There are many important reasons for teachers to assist parents in becoming familiar with the Ontario mathematics curriculum and to help parents see how the curriculum links with the developmental stage of their child. Many parents already provide some form of program support, either by helping their child with homework or by providing supplementary resources. Parents who have a good general understanding of the mathematics curriculum can better align the support they provide with the mathematics program delivered in the classroom.

"Family Math sessions educate parents to work and play with their children in order to develop positive attitudes towards mathematics."

(Adams et al., 2002, p. i)

Research has shown how important family involvement is in promoting student success. The more parents become involved, the greater the chances are that their child will succeed. Besides leading to more focused support of the mathematics program at home, a family math night can provide an environment for positive parent and child interaction and increased parent-teacher communication.

There are several other excellent reasons for holding family math nights. They can be used to:

- provide a comfortable, unthreatening environment in which parents and children can experience math together;
- demonstrate that math is more than arithmetic;
- provide opportunities for children to see parents valuing mathematics;
- help parents learn more about the mathematics program and the various strands of the mathematics curriculum;
- highlight the importance of concrete materials and communication when learning mathematics;

- demonstrate that math is used everywhere – at home, at work, in recreational activities, in shops and stores – and not just studied in isolation at school;
- emphasize that math can be fun and that success in mathematics is not determined genetically.

This section provides suggestions for organizing a family math night or a series of family math nights.

Family Math Night Formats

Family math nights can be held at any time throughout the year. They can be held once a year or, to be most effective, can be held in a series, once each term. The evenings can be organized as one big event for the whole school or for specific divisions in the school, or even for families with children in a specific grade. The sessions are usually about one and a half hours in duration. Various themes can be incorporated into the sessions (e.g., a celebration of the hundredth day of school; sporting events such as the Olympics, the World Series, the Stanley Cup playoffs, or the NBA/WNBA playoffs), to provide rich mathematics experiences that families can share.

A family math night could showcase or highlight math activities for families to explore; it could also include a formal presentation or discussion to focus on developing a better understanding of the mathematics curriculum. As part of their community service, students from the local high school may be willing to assist with conducting activities with younger students while parents participate in a short mathematics presentation. If no formal presentation is planned, each math activity should be preceded or followed by an explanation of the related mathematics concepts and skills.

Organization of a Family Math Night

There are numerous things to be considered when organizing a family math night, from the personnel involved to the format adopted. A checklist of points for consideration is included in Appendix 9-24, and a variety of suggested activities for a family math night have been included in Appendices 9-25 to 9-27. A family math night is an excellent event for a school team to plan and carry out with the support and help of their administrator(s).

Additional resource materials are available commercially and on the Internet to help in the planning and conducting of family math nights. One well-known set of resources, developed by the University of California at Berkeley, has been published and distributed under the following titles: *Family Math* (Stenmark, Thompson, & Cossey, 1986), *Family Math for Young Children: Comparing* (Coates & Stenmark, 1997), and *Family Math: The Middle School Years – Algebraic Reasoning and Number Sense* (Thompson & Mayfield-Ingram, 1998). Berkeley also hosts a website,

<http://equals.lhs.berkeley.edu>, that posts useful information and sample activities. Another program, the Esso Family Math Program, is designed for families in Ontario. A resource package for this program can be downloaded from the website <http://www.edu.uwo.ca/essofamilymath>. This website provides detailed instructions and suggested activities for family math nights. In addition to the resources above, student text programs from educational publishers also offer suggestions for games and activities that can easily be incorporated into a family math night.

The Wonderful World of Grade 2 Math!

What's New in Grade 2 Math?

- * Our math centres for fractions and two-dimensional polygons were a wonderful learning experience. Check the "Ask Me About ..." corner for details.
- * We are continuing our investigation of three-dimensional figures.
- * Our focus on strategies for the recall of basic facts is also continuing with the strategy of "doubles plus one more".
- * We have been searching the school for different mechanisms and investigating how they move and how they help structures move.

"Ask Me About..." Corner

Ask me what strategies I can use when I come to a word that I don't know when I am reading!

Ask me to explain how I can divide something exactly in half or into fourths.

Ask me to name three of the polygons that I learned about and find examples of them at home.

Ask me the different ways that I can use to count to one hundred!

Hundreds Day!

Our class will be celebrating the hundredth day of school on Monday, February 11. We are asking that the children each bring in 100 items in a labelled bag. The items should be a collection of things that are the same - for example, toothpicks, pennies, paper clips, pasta, toy cars. Be as creative as you wish!!! We will be using these items to explore measurement and numeration activities.

Connecting Math With Home

- * Use fraction language in the kitchen when cooking or preparing snacks (we have been working on *whole, half, fourth, third*).
- * Play board games with number cubes (dice) to reinforce the recall of basic facts. (Watch for strategies your child is using to add, and notice how fast your child's recall is becoming!)
- * Play I Spy for shapes, in a room or on a drive.
- * We are in need of boxes of various sizes and shapes that we can use in identifying the geometric properties of three-dimensional figures and in classifying the figures. Please send in any you may have.
- * If you have any beautiful "scraps" that you no longer have use for, please send them in, as we will make use of them in building during our geometry and structures units. Pieces we could use include wallpaper, paper rolls, yarn, small pieces of wood, and fabric scraps.

Appendix 9-2: Sample Newsletter in a Flyer Format for Junior Grades

4-6

The Wonderful World of Grade 5 Math!

What's New in Grade 5 Math?

- * Our math centres for measurement and fractions provided wonderful learning experiences. Check the "Ask Me About ..." corner for details.
- * We are continuing our investigation of three-dimensional figures.
- * Our focus on strategies for the recall of basic facts is also continuing, with the strategy of recognizing multiplication patterns in the hundreds chart.
- * We have been searching the classified ads in the newspaper to find careers that are related to mathematics.

World Series Baseball!

Our class will be keeping statistics of our favourite baseball players as the season ends. We will use these statistics to help us develop skills in gathering and interpreting data.

"Ask Me About..." Corner

Ask me what strategies I can use when I skim the newspaper for specific information.

Ask me to explain how I can divide a group of things into thirds.

Ask me to describe polygons that I see at home according to the number of their sides, angles, and vertices.

Ask me to show you how I can use a string to measure the circumference of a can.

Connecting Math With Home

- * Use fraction language in the kitchen when cooking or preparing snacks (we have been working on *whole, half, fourth, third*).
- * Play card games with your child to reinforce the recall of basic facts. (Watch for strategies your child is using to multiply, and notice how fast your child's recall is becoming!)
- * We are in need of containers of various sizes and shapes that we can use in identifying the geometric properties of three-dimensional figures and calculating surface area. Please send in any containers you may have.

Appendix 9-3: Sample Newsletter in a Letter Format (Primary Level)

1-3

[Date]

Dear Parent,

The Grade 3 students have just begun a new unit that focuses on three-dimensional figures. Over the next four weeks your child will be engaged in many activities that allow him or her to examine the environment for three-dimensional figures, identify the figures by name, explore all the two-dimensional shapes that make up the three-dimensional figures, create nets of these three-dimensional figures, and discuss their characteristics using mathematical language.

In order to explore the three-dimensional figures in the environment, we are asking the students to bring in samples of empty three-dimensional containers, such as tissue boxes, laundry detergent boxes, cereal boxes, and so forth. Please help your child discover the three-dimensional figures at home and allow him or her to bring one or more to class to share with his or her classmates.

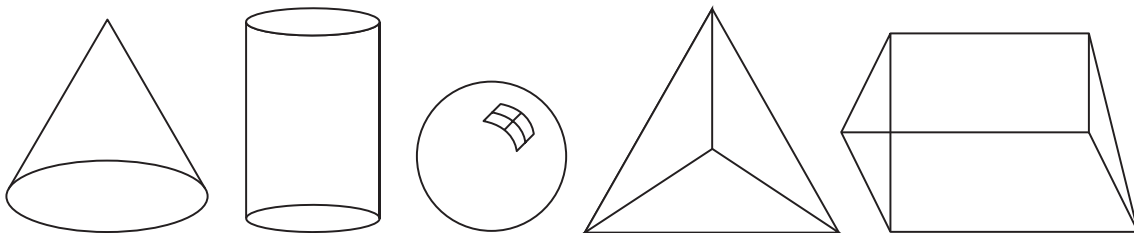
This unit on three-dimensional figures will also provide experience with ideas in the strands of Measurement and Number Sense and Numeration.

From time to time, the students will be asked to teach someone at home one of the activities or games they have learned in class. Instructions will be sent home, so that while your child is teaching you the game or activity, you can verify the rules or instructions. Allow your child to explain the instructions to the best of his or her ability. This homework activity will give you first-hand experience with the unit.

A game should be played at least three times. After each completed game, discuss with your child any discoveries that he or she made. Ask questions such as, "Will you change your strategy the next time you play?" "What numbers are rolled most frequently?" "How many moves did you take to get to the other side of the game board?"

If you have any questions, please do not hesitate to send a note with your child and I will get back to you as soon as possible.

Sincerely,



Appendix 9-4: Sample Newsletter in a Letter Format (Junior Level)

4-6

[Date]

Dear Parent,

The Grade 4 students have just begun a new unit in the Geometry and Spatial Sense strand of the mathematics curriculum. This unit focuses on three-dimensional figures. Over the next four weeks your child will be engaged in many activities that allow him or her to solve problems using geometric models, to investigate the geometric properties of three-dimensional figures and two-dimensional shapes using concrete materials and drawings, to draw and build three-dimensional objects and models, and to explore transformations of geometric figures.

In order to explore the three-dimensional figures in the environment, we are asking the students to bring in samples of empty three-dimensional containers, such as tissue boxes, laundry detergent boxes, cereal boxes, and so forth.

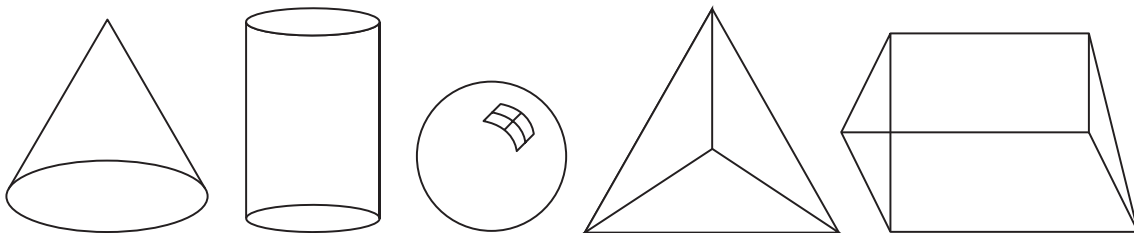
This unit on three-dimensional figures will also provide experience with ideas in the strands of Measurement and Number Sense and Numeration.

From time to time, the students will be asked to teach someone at home one of the activities or games they have learned in class. Instructions will be sent home, so that while your child is teaching you the game or activity, you can verify the rules or instructions. Allow your child to explain the instructions to the best of his or her ability. This homework activity will give you first-hand experience with the unit.

A game should be played at least three times. After each completed game, discuss with your child any discoveries that he or she made. Ask questions such as, "Will you change your strategy the next time you play?" "What numbers are rolled most frequently?" "How many moves did you take to get to the other side of the game board?"

If you have any questions, please do not hesitate to send a note with your child and I will get back to you as soon as possible.

Sincerely,



Appendix 9-5: Parent-Teacher Conferences: Suggestions for the Teacher

K **1-3** **4-6**

The following are helpful suggestions for the teacher to consider when preparing for, conducting, and following up on a parent-teacher conference:

Before the parent-teacher conference you may want to:

- **encourage parents to be active participants;**

Send a letter to parents recognizing them as their child's first and most important teacher. In the letter, ask the parents to be prepared to share something about their child's interests and strengths, to outline any problems they see that relate to the child's development, and to express their goals for their child in the current school year. (See Appendix 9-6 for a sample letter.)

At the beginning of the school year or at any time during the year, invite parents to send you a letter describing their child and providing any additional information that may benefit the growth and development of the child. Return any such letters to the parents at the end of the school year to be placed in family archives.

- **schedule conference times;**

Communicate the purpose of the conference. Discuss and decide on a time and place for the conference. When scheduling conferences, try to accommodate parents. Develop a formal schedule to ensure that all parents have an opportunity to meet with you.

- **compile samples of student work and individual assessments;**

You may want to collect student work and individual assessments before the parent-teacher conference. Be sure that all samples are dated. During the conference place samples in three piles: work that shows evidence of strong understanding, work that can be categorized as being at grade level, and work that reflects any confusions or shows difficulties in problem solving. You can then look for patterns of consistent strength or weakness. This process should provide a clear picture of how the student is developing mathematically and help you to identify which student work samples will offer the most information to parents. The samples chosen should reflect a balanced program, with evidence of the student's learning across the mathematical strands and the categories of the achievement chart.

You will find it helpful to have available some notes reviewing the student's work, along with a record of observational notes and individual assessments that will help you in sharing what you have learned about the student's growth in mathematics.

Sharing information gathered from a performance-based assessment task completed during a teacher-student interview will also be helpful (e.g., strategies that you observed the student use for adding, subtracting, multiplying, or dividing numbers). You may want to present all this information in an individual student math portfolio.

Finally, you should prepare all assessment records (e.g., checklists, anecdotal notes, records of marks) to share during the conference.

- **be prepared to respond to specific parent questions;**

Questions that parents might ask include the following:

- “What mathematical skills and knowledge will my child be expected to master this year?”
- “How will my child be evaluated?”
- “What can I do as a parent to stay more informed about my child’s academic progress?”
- “How do you accommodate differences in learning?”
- “What can I do as a parent at home to complement what is happening in the classroom?”

- **prepare a welcoming environment for the conference.**

Arrange for comfortable adult-sized chairs and a circular table that can accommodate those at the interview. Ensure that a clock is visible, so that participants can keep track of time casually. Organize the materials to be used during the conference according to the schedule of interviews. Spread the stacks of student work across a table or counter so that each student’s work is readily found.

During the conference you may want to:

- **begin every conference on a positive note;**

You may want to mention some recent success that the child has experienced at school or some special quality that she or he has exhibited (e.g., likes to ask questions about the mathematics that he or she is exploring; readily helps other students; works cooperatively with others to discover solutions to a problem; is able to communicate his or her learning effectively).

- **invite the parents to tell you about their child;**

- **listen attentively to the messages that parents/guardians convey;**

- **provide feedback;**

Keep feedback balanced and positive by praising areas of strength, offering suggestions, discussing any areas of concern, and clarifying any misconceptions. In the process, share the student’s work samples and individual assessments, and explain what the work and the assessments tell you about the student’s growth.

Describe both the mathematical information that you have gathered and the observations that have helped to inform you about the student's organizational skills and confidence level. Always try to help parents understand the complexity of the mathematics involved from the student's point of view.

- **develop learning goals for the student collaboratively and a plan of action for each learning goal;**
- **suggest games and activities that parents can do at home to support mathematics learning;**
- **summarize and record any valuable information obtained during or immediately after the conference;**
- **discuss whether another conference should take place and suggest a time for it;**
- **invite parents to keep in touch and to feel free to contact you when they want to share information or when they have any concerns;**
- **end on a positive note.**

Let the parents know that you appreciate what they have to say and that you enjoy working with their child.

After the parent conference, you may want to:

- **review and complete any notes taken during the conference**
See Appendix 9-7 for an example of a form to use for recording your notes on a conference.
- **make a copy of the conference notes for your files and let the parents keep the originals;**
- **send parents a follow-up thank-you letter that promotes cooperation and reviews the suggestions that were made at the conference;**
See Appendix 9-9 for an example of a follow-up letter.
- **let students know that you have enjoyed meeting with their parents. Share any information from the conference that would benefit the student. Keep a friendly approach;**
- **monitor student progress in achieving identified goals.**

Appendix 9-6: Sample Letter to Be Sent to Parents Before a Parent-Teacher Conference

K

1-3

4-6

[Date]

Dear Families,

As the end of the first term approaches, I am looking forward to meeting with each of you at our upcoming parent-teacher conferences. These conferences provide us with an opportunity to celebrate your child's successes and discuss next steps. You will be able to provide helpful information about how you feel your child is doing.

During the conference, I'll be sharing a portfolio of work that your child has completed throughout the term. In many cases, your child will have been involved in selecting the samples for the portfolio, choosing things that he or she is proud of and things from which he or she learned lots. We'll also have a chance to discuss the observations that I have made about your child's growth both academically and socially this term.

I look forward to learning from your knowledge of your child as well, and we'll have a chance to talk about your child's strengths and weaknesses, as well as his or her interests outside school. I've had a few months to get to know your child, but you have a deep understanding of your child that I can learn from. The conference will provide us with an opportunity to set some goals and discuss any concerns that you may have.

I look forward to meeting you at our parent-teacher conference.

Sincerely,

Appendix 9-7: Sample Form for Recording Notes About a Parent-Teacher Conference

K **1-3** **4-6**

Parent-Teacher Conference Guide

Name of Student _____ Conference Date _____

In Attendance _____

Topics the teacher wants to discuss
(*areas of strength, areas for improvement*)

Issues, questions, and concerns raised by parents

Recommendations and goals

Next steps

Adapted, with permission, from the Toronto Catholic District School Board, *Assessment of Student Achievement in Catholic Schools Resource Document*, 2001, p. 23.

Appendix 9-8: Sample Form for Recording Notes About a Parent-Teacher-Student Conference

K **1-3** **4-6**

Parent-Teacher-Student Conference Guide		
Name of Student _____		
Date of Conference _____ Time of Conference _____		
Areas of strength		Areas for improvement
Goals		Action plan
Student will:	Teacher will:	Parents will:
Further notes		

Adapted, with permission, from the Toronto Catholic District School Board, *Assessment of Student Achievement in Catholic Schools Resource Document*, 2001, p. 22.

Appendix 9-9: Sample Letter to Be Sent to Parents After a Conference

K **1-3** **4-6**

[Date]

Dear _____,

Thank you very much for attending our conference on _____ [date of conference]. It was very good to meet you and share information about _____ [student's name].

We discussed suggestions and actions at our conference, and I have taken the liberty of outlining these as a reminder for you at home, and for me here at school.

We decided on the following:

Again, thank you for your support, cooperation, and understanding. If you have any further questions, please do not hesitate to contact me.

Sincerely,

Appendix 9-10: Sample Letter to Introduce the Take-Home Math Program to Parents (Kindergarten or Primary Level)

K 1-3

[Date]

Dear Family,

Our class is beginning a mathematics program called take-home math. This program includes many meaningful and enjoyable mathematics activities that reflect current research and provincial curriculum expectations.

Your child will bring home, in a resealable bag, activities for you to explore as a family. Some activities will require things that can be found around the home; for other activities, what is needed will be inside the bag. Your child can explore these activities with the whole family or with individual members, such as a parent, brother, or sister. When children participate in the activities with family members, their self-confidence increases and their attitude towards mathematics becomes more positive.

[Include a paragraph establishing the time lines for your program. For example: "A math activity for family participation will be sent home with your child every Wednesday. Please have your child return the resealable bag to the class by the following Wednesday. That day, your child will be provided with a new activity to take home."]

The activities chosen for this project provide opportunities for children and adults to solve problems, to use logical thinking, to develop strategies, and to communicate their mathematical thinking and understanding. The activities are designed to help students meet provincial curriculum expectations in mathematics through sharing literature, playing games, or completing open-ended problems. Included with each activity are some questions that you may use to help your child gain a clear understanding of the math concepts being explored. Some of the activities review concepts that have already been presented in class, some extend your child's understanding, and some may prepare your child for concepts to be explored in class in the future.

Included in the bag is a reflection book, which will travel with the activity when other students in our class take it to their homes. After you and your child have completed an activity, please take a few minutes to record your discoveries in the reflection book. You may do the recording or your child may want to do it. Writing about the activity will give you a chance to think about the discoveries that you and your child have made together and an opportunity to express some thoughts and feelings about math that can be shared with other families in our class.

Thank you very much for your continued support of your child's learning. I welcome any ideas or suggestions for the take-home math program.

Sincerely,

Appendix 9-11: Sample Letter to Introduce the Take-Home Math Program to Parents (Junior Level)

4-6

[Date]

Dear Family,

Our class is beginning a mathematics program called take-home math. This program includes many meaningful and enjoyable mathematics activities that reflect current research and provincial curriculum expectations.

Your child will bring home problem-solving activities for you to explore as a family. Some activities will require things that can be found around the home. Your child can explore these activities with the whole family or with individual members, such as a parent, brother, or sister. When children participate in the activities with family members, their self-confidence increases and their attitude towards mathematics becomes more positive.

[Include a paragraph establishing the time lines for your program. For example: "A math activity for family participation will be sent home with your child every Wednesday. Please have your child return the activity to the class by the following Wednesday. That day, your child will be provided with a new activity to take home."]

The activities chosen for this project provide opportunities for children and adults to solve problems, to use logical thinking, to develop strategies, and to communicate their mathematical thinking and understanding. The activities are designed to help students meet provincial curriculum expectations in mathematics through sharing literature, playing games, or completing open-ended problems. Included with each activity are some questions that you may use to help your child gain a clear understanding of the math concepts being explored. Some of the activities review concepts that have already been presented in class, some extend your child's understanding, and some may prepare your child for concepts to be explored in class in the future.

Included with each activity is a reflection book, which will travel with the activity when other students in our class take it to their homes. Your child has been asked to record his or her discoveries in this reflection book after an activity has been completed. Writing about the activity will give your child an opportunity to think about the discoveries that you and he or she have made together and an opportunity to express some thoughts and feelings about math that can be shared with other families in our class.

Thank you very much for your continued support of your child's learning. I welcome any ideas or suggestions for the take-home math program.

Sincerely,

Appendix 9-12: Take-Home Math Sample Activity I (Primary Level)

1-3

Going Shopping!

Where's the Math?

Children will explore quantity relationships through the manipulation of money, by budgeting and making change.

Materials

grocery flyers, calculator, pencil, paper, scissors, glue

Activity Steps

1. Collect some grocery flyers to have on hand.
2. Decide on a meal for which to plan and "shop". Remember to include all four food groups (vegetables and fruits, grain products, milk products, meats and alternatives).
3. Decide on a budget – up to \$10.00 – with which to go "shopping" through the flyers.
4. Use your calculator to keep track of how much money you are spending while you are "shopping" through the flyers. The amounts could be recorded on a piece of paper, or the ads could be cut out and glued onto a piece of paper.
5. Look back at the budget amount you decided on and figure out how much change you would receive from the cashier.

Math Talk

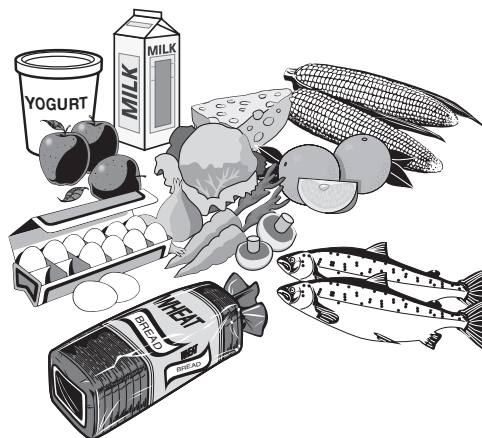
What did you discover about having a budget? Did you shop over or under your budget total?
Was it difficult to go shopping? Why or why not?

Hints for Parents

Allow your child to make mistakes and buy more than the budget calls for. Going over budget will provide a good opportunity for your child to problem solve.

Extensions

As an extension, you could double the number of items or you could shop for a menu for the day, a week's lunches, a party, or a family barbecue.



From *Home Connections: Primary Grades*, by Halton District School Board Numeracy Team, 2001. Adapted with permission.

Appendix 9-13: Take-Home Math Sample Activity II (Primary Level)

1-3

A Pocketful of Change

Where's the Math?

Children will explore a variety of coins, estimating their value and determining totals.

Materials

a handful of change, pencil, paper, calculator (if needed)

Activity Steps

1. Take a handful of change with a variety of coins in it.
2. Quickly estimate how much money is in a handful.
3. Sort the coins into their appropriate values.
4. Add up the amount of money you have.
5. How close were you to your estimate? What could you buy with your handful of change?

Math Talk

What is on each coin? What is the value?

Why did you estimate the amount you did?

Would you change your estimate? Why?

Hints for Parents

Younger children may just sort the coins, then count the total number of coins.

You can also alter what is in the handful of change, depending on your child's ability level.

Extensions

Make trades for coins. For example:

I have 10 pennies. I can exchange them for a dime.

I have 4 quarters. I can exchange them for a dollar.



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Appendix 9-14: Take-Home Math Sample Activity II (Junior Level)

4-6

Coin Tosses

Where's the Math?

Students will attempt to organize a set of given information in different ways.

Materials

a coin, paper, pencil

Activity Steps

1. A coin was tossed 20 times. This is how it landed and how the information was recorded:

Toss	Result	Toss	Result
1	heads	11	heads
2	heads	12	heads
3	heads	13	heads
4	tails	14	tails
5	heads	15	tails
6	heads	16	heads
7	heads	17	tails
8	tails	18	heads
9	heads	19	tails
10	heads	20	heads

2. Discuss some of the ways in which you might reorganize, sort, illustrate, or record this information.
3. Go ahead and try these different ways.
4. Make comparisons of the different ways in which you organized the information. In which way is the information best organized? Why?

Math Talk

Why did you organize the data in the way you did?

Hints for Parents

Perhaps there are ways of collecting and organizing data in your lives that your child might not have thought of but would benefit from knowing. Gather some other data and record them in different ways. Look in newspapers, in magazines, and on the Internet for ways in which data can be organized.

Extensions

Take a survey of your friends or family, using a question you are interested in – for example, “What is your favourite pizza topping?” or “Do you like chocolate or vanilla ice cream?” Then, organize the data you collect.

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Appendix 9-15: Take-Home Math Sample Activity III (Primary Level)

1-3

Shape Search

Where's the Math?

Children will explore and investigate two-dimensional shapes and three-dimensional figures in the world around them.

Materials

pencil, paper, keen eyes

Activity Steps

1. Go on a shape search around your home.
2. Look for two-dimensional shapes (e.g., circle, triangle, square) and three-dimensional figures (e.g., cube, sphere, cylinder).
3. Which shape or figure do you see the most?
4. Why do you think that shape or figure is the most common?

Math Talk

What shapes do you see most in nature? Why do you think that is so?

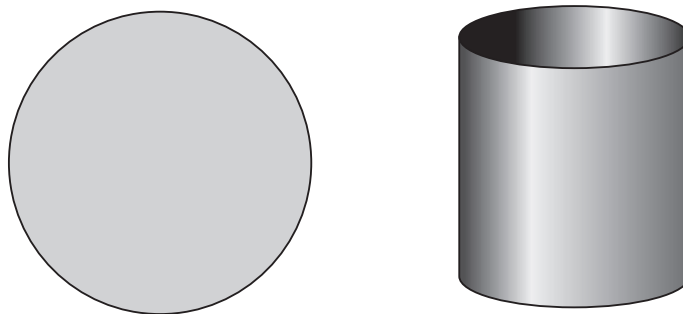
What shapes do you see most in things that people make? Why do you think that is so?

Hints for Parents

This activity can be done while driving in the car, walking to the park, sitting in the arena, and so forth.

Extensions

Make a chart of the different shapes and solids that you see. Tally the number of two-dimensional shapes and three-dimensional figures.



From *Home Connections: Primary Grades*, by Halton District School Board Numeracy Team, 2001. Adapted with permission.

Appendix 9-16: Take-Home Math Sample Activity III (Junior Level)

4-6

Crazy Quilt

Where's the Math?

In this game for two players, the players will be looking for geometric figures and developing logical strategies to create them.

Materials

- 2 different-coloured pencils or markers (1 for each player)
- a copy of the Crazy Quilt Game Sheet, which is shown at the end of this activity description

Activity Steps

Play with a friend or a family member.

1. The object of the game is to score more points than your opponent by completing more four-piece shapes.
2. Each player chooses a colour.
3. The first player colours any single triangle on the outer part of the board (the twelve outside squares).
4. The second player colours a single triangle in the inner part of the board (the four inner squares).
5. Players take turns colouring a triangle anywhere on the grid. They count points as they go.
6. The game ends when the grid is completely filled or when neither player can score further.

Math Talk

What strategies did you use to earn the most points? Which shape was the most difficult to create and why?

Hints for Parents

Many interesting problems arise during this game. Try to articulate what your options are and how you are deciding on your actual moves. Encourage your child to do the same. There are many shapes within shapes, and they all count. Help your child to find the shapes that are hiding.

Extensions

Change the rules to give different values to the shapes. How does this change of rules affect your strategies? How else can you modify the game?

From *Home Connections: Primary Grades*, by Halton District School Board Numeracy Team, 2001. Adapted with permission.

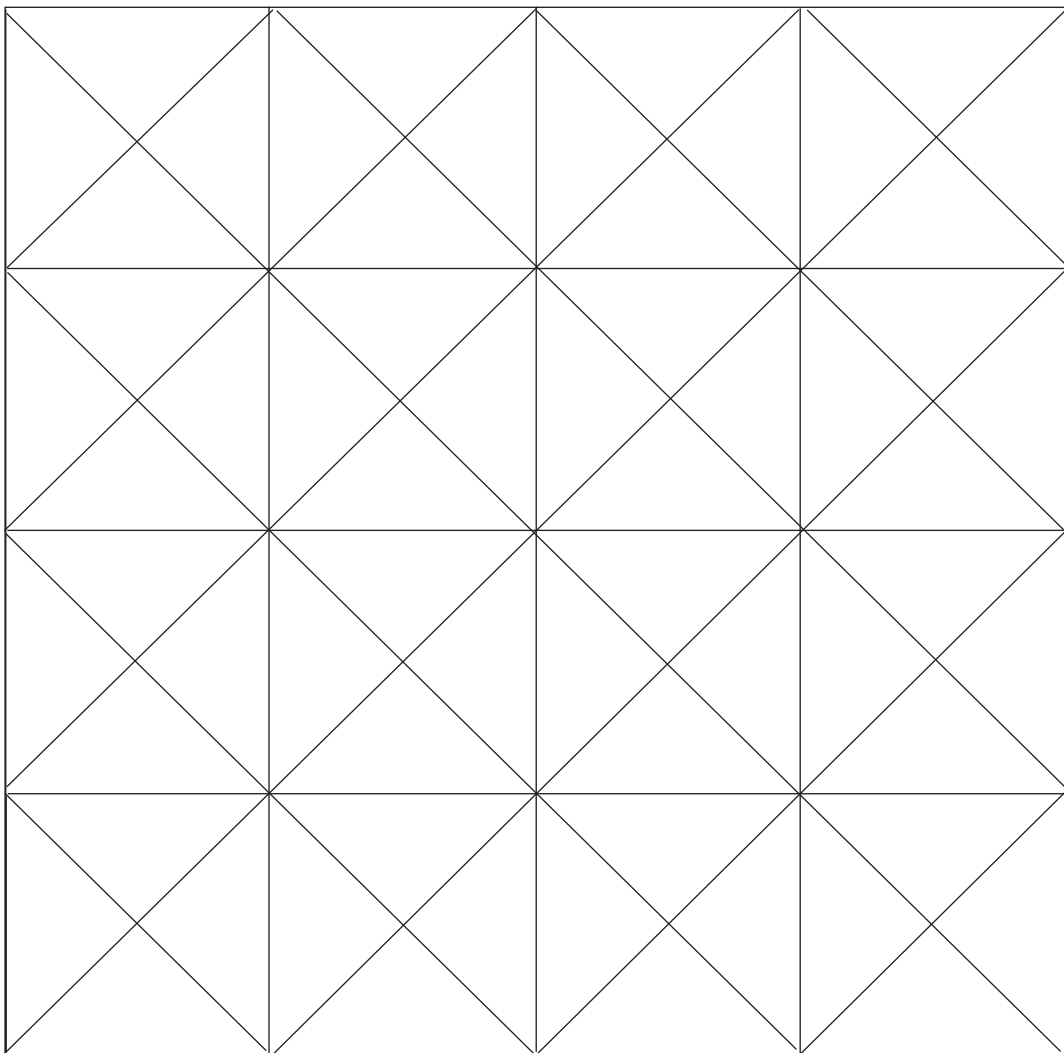
Crazy Quilt Game Sheet

Parallelogram = 1 point

Rectangle = 2 points

Triangle = 3 points

Square = 4 points



Appendix 9-17: Take-Home Math Sample Activity IV (Primary Level)

1-3

Patterns Are Everywhere

Where's the Math?

Children will explore patterns in their environment, describing the patterns and making models of them.

Materials

magazines, scissors, glue, paper

Activity Steps

1. Look everywhere around you (home, car, store, playground, or neighbourhood) for patterns that repeat.
2. You can look in magazines for different patterns and designs or draw patterns yourself.
3. Try to find patterns that are interesting and different from one another.

Math Talk

Why do you think nature has the patterns that it does? Do you see any differences between the patterns that are created in nature and the patterns that you find in things that people make?

Hints for Parents

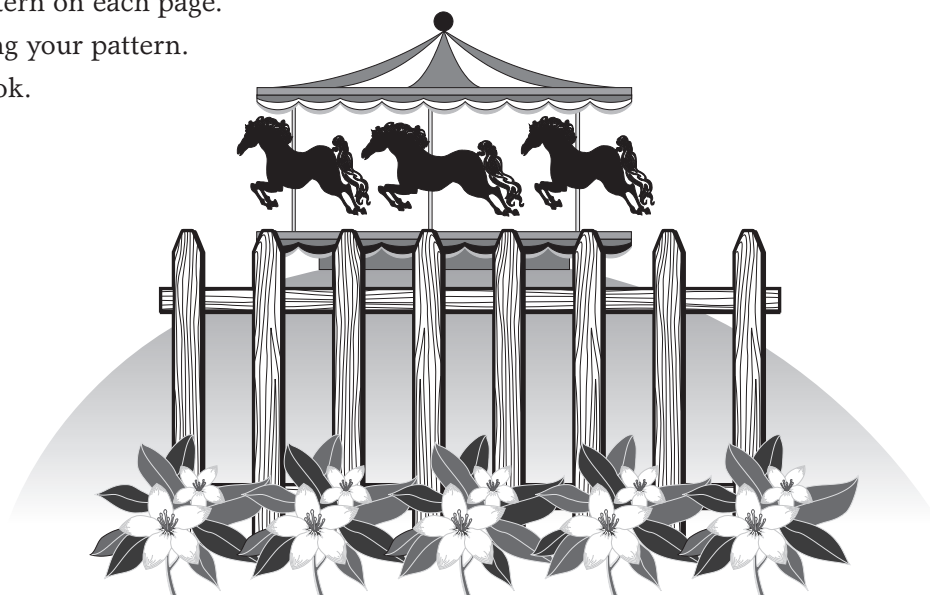
Look for as many different patterns as you can in every part of your family's environment.

Extensions

Make a book. Put one pattern on each page.

Write a sentence describing your pattern.

Make a cover for your book.



From *Home Connections: Primary Grades*, by Halton District School Board Numeracy Team, 2001. Adapted with permission.

Appendix 9-18: Take-Home Math Sample Activity V (Primary Level)

1-3

Toothpicking

Where's the Math?

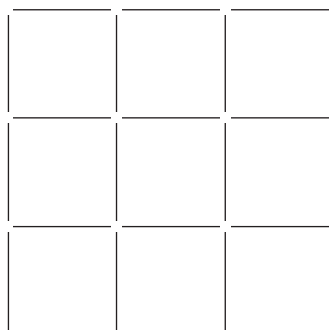
Children will explore and describe patterns in numbers and will use logical reasoning.

Materials

a box of toothpicks

Activity Steps

1. Choose 24 toothpicks from the box and arrange them like this:



2. How many squares do these toothpicks make? Keep looking until you find 14 squares.
3. Take away just 8 toothpicks in order to have just 2 squares left.

Math Talk

How can you be sure you have found all the squares? How do you know that you haven't counted a square more than once?

Hints for Parents

Once your child has discovered that squares of different sizes are acceptable, he or she will have unlocked the secret to this problem. Encourage your child to build other shapes and explore the hidden shapes within them. Organizing the counting of squares is an important skill.

From *Home Connections: Primary Grades*, by Halton District School Board Numeracy Team, 2001. Adapted with permission.

Extensions

What is the least number of toothpicks you need to build 1 square? What about 2 squares? 3 squares? And so on. Can you find a pattern in the numbers?

toothpicks	squares
4	1
7	2
	3

Use 12 toothpicks. Place them in any way you want. How many different numbers of squares can you make?

Appendix 9-19: Take-Home Math Sample Activity V (Junior Level)

4-6

Calculator Quiz

Where's the Math?

Using a calculator, children will solve riddles by exploring operations.

Materials

pencil, paper, calculator

Activity Steps

1. Use a calculator to solve the number sentence. Then turn the calculator upside down to see the word answer to the riddle.
 - a. It's the outside of an egg. $50\,045 + 25\,309 - 9 + 2000 = ?$
 - b. Mountains usually are this. $4300 + 400 - 100 + 14 = ?$
 - c. Bubble and steam! $28\,432 \div 4 = ?$
 - d. It's what a salesperson does. $3849 \times 16 - 3849 = ?$
 - e. It's good for a car. $1600 \div 2 - 90 = ?$
 - f. This person is in charge. $4 \times 9 \times 9 \times 17 = ?$
 - g. To cry out loud $13 \times 13 \times 5 - 40 = ?$
 - h. Every garden should have one. $0.002\,415 \div 0.007 = ?$
2. Make your own quiz.

(0 = O or D, 1 = I, 3 = E, 4 = h, 5 = S, 6 = g, 7 = L, 8 = B)

 - a. Make up a word from the letters.
 - b. Now make up a riddle.
 - c. Last, make a number sentence that gives the riddle answer you want.
 - d. Bring your riddle and number sentence to class.

Math Talk

What was the most challenging step in creating your riddle and clues?

Hints for Parents

Experimenting with whole and decimal numbers as well as the four operations (addition, subtraction, multiplication, division) allows your child to develop a deeper understanding of number. Challenge your child by asking questions like the following: "What happens to the total (word spelling) when you add 1000? Subtract 300? Divide by 20?"

From *Esso Family Math Resources for Grades 2-5 Children and Their Parents*, by B. Onslow and N. Chapple, 2004, pp. 71-72. Adapted with permission from the Esso Family Math Centre.

Appendix 9-20: Take-Home Math Sample Activity VI (Primary Level)

1-3

Treasure Hunt

Where's the Math?

Children will use a map and instructions in appropriate language to describe the specific location of an object.

Materials

something special to hide, paper, pencil

Activity Steps

1. Find a secret spot in your home or backyard to hide your special treasure.
2. Don't tell your family member where you hid your special treasure.
3. Start in your bedroom and plan the route to your hidden treasure.
4. Remember to use good mapping words on your map (e.g., *left, right, past, in front of, behind, below, above*).
5. Try out your map first. Then make any changes that you need to make.
6. Give your map to a family member and watch that person hunt for your treasure.

Math Talk

What math words helped you to describe where your special treasure was? Are there other words that you discovered?

Hints for Parents

You could make a map for your child to follow. Use good mapping words to continue to build on mapping concepts.

From *Home Connections: Primary Grades*, by Halton District School Board Numeracy Team, 2001. Adapted with permission.

Extensions

Here are some suggestions:

- Plot out your map on graph paper.
- Change the route to your treasure, and describe the new route.
- Make a map from your home to the grocery store, to a friend's home, or to the school.
- Pull out a real map and try to find how to get to a good vacation spot.

The possibilities are endless!



Appendix 9-21: Take-Home Math Sample Activity VI (Junior Level)

4-6

Licence Plate Riddles

Where's the Math?

By arranging digits from car licence plates, children will further develop their understanding of the place-value system.

Materials

licence plates, paper, pencil

Activity Steps

1. The object of this activity is to use the digits on a licence plate to make the largest three-digit number possible.
2. Each person travelling in a car or riding on a bus is given a five-minute period in which to choose a licence plate.
3. During the five-minute period, each player chooses a licence plate and arranges the numerals on it to make a three-digit number.
4. At the end of the period, the players call out the three-digit numbers they have made. The person with the largest number wins the round.
5. In the next round, the winner will be the person with the smallest three-digit number.

For younger children, this activity can be simplified by having them find the largest single digit or double digit on the licence plate, having them add all the numbers on it, or having them just recognize digits on it.

Math Talk

How do you know that you have arranged the digits to make the largest (smallest) number? Which number can be divided by 10? By 5? By 2? How do you know?

Hints for Parents

This activity can be done with house numbers or numbers obtained by rolling a number cube several times. Have your child record each three-digit number. After several rounds of making numbers, have your child put the list of numbers in order from smallest to largest or largest to smallest.

Extensions

Let each letter on a licence plate be worth the value of its position in the alphabet – for example, A=1, M=13, Z=26. Each person chooses a licence plate and adds the value of the letters on it. The person with the lowest or the highest value wins the round.

Appendix 9-22: Mathematics and Literature: Ways in Which Parents Can Make Connections for Their Child

K **1-3** **4-6**

Look at the pictures in the book.

- Play I Spy: "I spy something that is round."
- Ask questions that describe size, quantity, and shape.
- Describe the various geometric shapes.
- Make comparisons that involve measurement, sorting, or looking at different attributes. Compare people with other people, objects with other objects, or symbols with other symbols.
- Look for numbers; count objects in a picture.

Read the text.

- Use the number concepts in the story to count in various ways (backwards, forward, by 2's, by 5's, by 10's).
- Connect mathematical concepts with your child's daily life.
- Count similar or different objects or pictures.
- Ask problem-solving questions – for example, "How many times do you think the sun rose from the beginning of the story to the end? Explain how you calculated."
- Sequence the appearance of characters and events in time, using mathematical language – for example, "Which character appeared first?" "What happened before ____ and after ____?"
- When words like *over*, *under*, *below*, *above*, *backwards*, or *forward* arise, you may have an opportunity to discuss temperature, time, number lines, or patterns.
- Extend the story to your child's world.
- Look for opportunities to estimate or predict.

Look at the book itself.

- Estimate the number of pages in the book.
- Describe the shape of the book.
- Measure the perimeter and area of the book using standard or non-standard units.
- Turn the story into a word problem.

Appendix 9-23: More Tips for Helping Your Child Learn Mathematics

K**1-3****4-6**

- **Do some math with your child every day.**

Reading, cooking, or playing basketball can provide opportunities to improve mathematical understanding and provide practice for children, rather than just doing a half-hour of drill practice of basic facts. Families should look for opportunities to do math that connects to meaningful activities for the child (e.g., How can a family figure out a fair way to do all the chores?).

- **Let your child see and hear you doing math.**

Even when your child is too young to understand your mathematical reasoning, just seeing you do math will promote the idea that mathematics is important business. Occasionally, talk aloud when you are doing math. When you use a calculator, talk about what you are doing and how you are entering the numbers. When you use different proportions for a recipe, share your thinking with your child. For example, "I'll need to double everything, so instead of 3 eggs, I will need 6 eggs."

- **Let your child lead the process.**

Make a point of trying to do a little math with your child each day, but don't overdo it. Children should be stimulated by the math that you do with them. If they show signs of being tired or disinterested, wait for another opportunity to spend more time working on a mathematics problem together. A primary goal should be to build confidence, skills, and an enjoyment of math, so that your child will want to keep doing mathematics throughout his or her life.

- **Keep doing math as your child grows.**

As your child grows, continue to do math with him or her. You will probably find there are some math concepts you may have forgotten and other concepts may have evolved and taken on a new language. Here is an opportunity to show your child that you never stop learning. Have your child explain a concept to you. You will quickly be able to tell how well your child understands the mathematics being taught.

- **Value mistakes.**

Mistakes are an opportunity for learning and everyone young or old makes them. Figuring out what the mistake is and looking at the different ways the problem can be solved are essential to learning mathematics. When a child announces an answer to a problem, regardless of whether the answer is right or wrong, try to remember to ask questions like: "Does your answer make sense? How do you know it is right? How did you get your answer?" A child who may have made an error will begin to see and reverse his or her faulty reasoning.

From *Beyond Facts and Flashcards: Exploring Math With Your Kids*, by J. Mokros, 1996. Used with permission.

Appendix 9-24: A Guide to Planning Family Math Sessions for the School Community

K 1-3 4-6

✓	POINTS FOR CONSIDERATION	NOTES
Before the event		
	How many family math nights will be held this year?	
	Who will attend each session? <i>(Will it be open to all families or to families of certain grades or divisions?)</i>	
	Where will the sessions be held? <i>(In the gymnasium, library, or individual classrooms, or in a combination of these?)</i>	
	How long will each session be?	
	Will all families and children work on the same activity simultaneously, or will stations or centres be set up for families and children to rotate through? <i>(If all participants are working on the same activity at the same time, usually one or two people can serve as leaders, but additional personnel will be needed to walk around and answer questions during the activities. If families will be rotating through stations or centres, then a person will be required at each station to lead the activity.)</i>	
	Who will plan the activities for each session? <i>(Teachers, school board resource staff, retired teachers, and parent volunteers can assist with the planning of the activities.)</i>	
	Who will organize all materials needed for each session? <i>(Will manipulatives, calculators, paper, pencils, crayons, scissors, tape, and so forth, be needed? Will a computer lab be needed?)</i>	
	Who will assist with activities during the session? <i>(Teachers, school board resource staff, retired teachers, parent volunteers, high school students, students from older grades in the school?)</i>	

✓	POINTS FOR CONSIDERATION	NOTES
Before the event <i>(continued)</i>		
	Will language support be required? <i>(Are there students who can translate general instructions for families? Perhaps having a few adult translators will be more beneficial. If adult translators are unavailable, consider inviting older students from the school, or high school students fluent in English to assist and translate during activities and instructions.)</i>	
	Will refreshments be provided? <i>(If so, where will they be, how much will you need? Remember to consider students with allergies.)</i>	
	If a separate presentation or discussion for parents is being considered, who will engage the children? <i>(Consider asking high school students to help with the event as part of their community service credit.)</i>	
	How will the room(s) be organized? <i>(How will the tables and chairs need to be set up to allow for group work and discussion?)</i>	
	How will the event be advertised? <i>(Will school newsletters, flyers, posters, or morning announcements be used? Get students involved in advertising the event, to encourage their families to attend.)</i>	
During the event		
	What will the introductory activities be? <i>(Consider generating a large graph in which participants could add family data relating to something of interest, perhaps birthday months. Invite estimation by asking families to guess how many jelly beans are in a jar. Provide on tables some activities that families who show up early can explore before the evening formally begins.)</i>	
	When will the key messages about mathematical concepts and skills be explained? <i>(Consider delivering key messages either before or after each individual activity or in a separate information session at the beginning or end of the evening.)</i>	

✓	POINTS FOR CONSIDERATION	NOTES
After the event		
	<p>Will there be any package or information distributed to take home and/or follow up? <i>(Consider providing a package with copies of some of the activities/games that can be continued at home.)</i></p>	
	<p>How will feedback from the session be gathered? <i>(A variety of methods can be used to provide feedback on the session – survey forms for participants to complete; an informal question/answer period or discussion at the conclusion of the session; a large chart at each individual activity [if centres are used] on which participants record what they enjoyed about the activity.)</i></p>	
	<p>How will information about the results of the event be communicated to the school community? <i>(Provide all teachers in the school community with an outline of activities conducted during the family math night. Teachers may want to refer to these activities during class discussions. Celebrate the family math session. Create a display with pictures and sample work from the event. Advertise when the next session will take place.)</i></p>	

Appendix 9-25: Family Math Night Sample Activities I

What's Your Name Worth?

The letter A is worth 1 point, B is worth 2 points, C is worth 3 points, and so on.

What is your name worth?

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

Whose name in the family has the most points?

Who in the family can make the most valuable word?

Can anyone in the family make a word worth 100 points exactly?

One Step at a Time

Estimate how many giant steps it will take to get from one end of the room to the other end.

Check your estimate. How many baby steps will it take? Check your estimate.

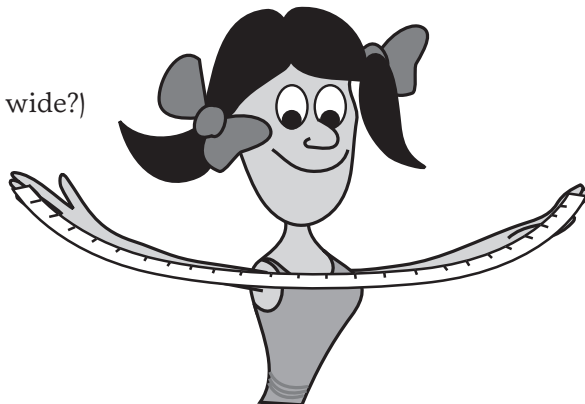


Appendix 9-26: Family Math Night Sample Activities II

What Shape Are You?

Are you a "square" person? (Are you as tall as you are wide?)

Tear a strip of adding-machine tape (or cut a piece of twine) equal to your arm span (both arms stretched out). Is it equal to your height, or are you a "rectangular" person?

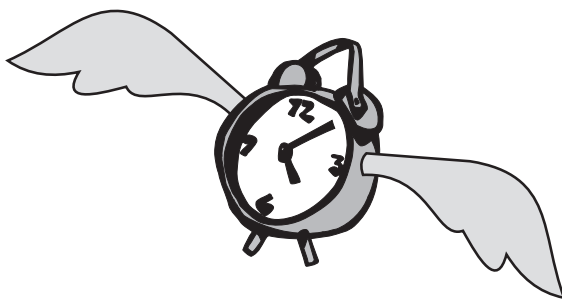


How many times will your arm span fit around your head?

Is the fit of the arm span the same for adults and children?



Does Time Fly?



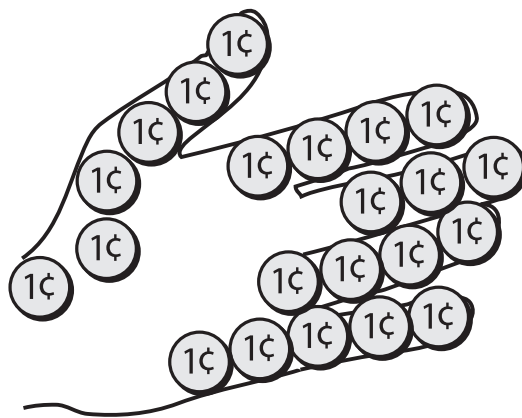
How long is a minute? Close your eyes and raise your hand when you think one minute has passed.

How many times do you think you can print or write your name in one minute?
Check.

Appendix 9-27: Family Math Night Sample Activities III

Lend a Hand

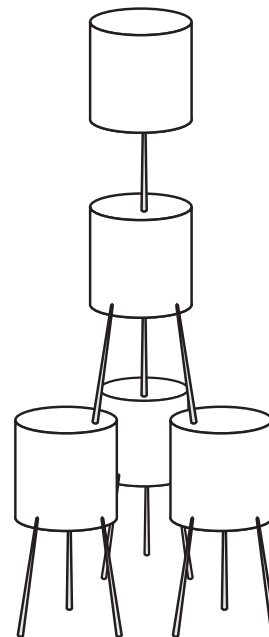
Trace your hand, with fingers close together, on graph paper. Estimate how many pennies it will take to cover the area of your hand. Check how much your hand is worth.



Do you think your foot is worth more or less than your hand?
Check.

Architect for a Day

Use marshmallows and toothpicks to create the tallest structure you can.
Who in the family can create the tallest structure?





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