

Data Management and Probability, Grades 4 to 6

**A Guide to Effective Instruction
in Mathematics,
Kindergarten to Grade 6**

2008

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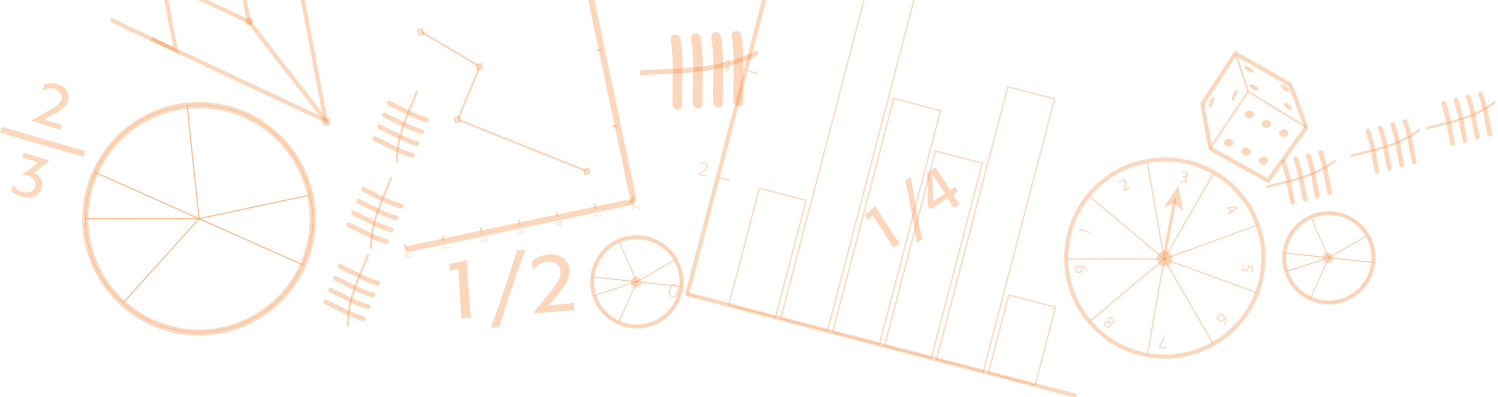
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Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.

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INTRODUCTION

Data Management and Probability, Grades 4 to 6 is a practical guide that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Data Management and Probability strand of *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. This guide provides teachers with practical applications of the principles and theories that are elaborated in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

The first part of the guide provides a detailed discussion of the three “big ideas”, or major mathematical themes, in Data Management and Probability, and provides a discussion of mathematical models and instructional strategies that have proved effective in helping students understand the mathematical concepts related to each big idea. The guide emphasizes the importance of focusing on the big ideas in mathematical instruction to achieve the goal of *helping students gain a deeper understanding of mathematical concepts*. At the end of the first part of the guide is a list of references cited.

The second part of the guide provides sample learning activities, for Grades 4, 5, and 6, that illustrate how a learning activity can be designed to:

- focus on an important curriculum topic;
- involve students in applying the seven mathematical processes described in the mathematics curriculum document;
- develop understanding of the big ideas in Data Management and Probability.

At the end of the second part of the guide is an appendix that discusses assessment strategies for teachers. There is also a glossary that includes mathematical and other terms that are used in the guide.

The Pleasure of Mathematical Surprise and Insight

Young children enter school mathematically curious, imaginative, and capable. They have to learn to be otherwise (Papert, 1980). The aim of this resource is to help consolidate and extend junior students’ mathematical capacity and their potential for mathematical growth by providing ideas and classroom activities that draw their attention to relationships embedded in the big ideas of the Data Management and Probability strand in the Ontario

mathematics curriculum and that offer them opportunities to experience the pleasure of mathematical surprise and insight (Gadanidis, 2004).

The activities in this resource incorporate the ideas and practice of classroom teachers. The activities have been field-tested in Ontario classrooms, and feedback from practising teachers has been used to create the final versions. The chapter “The ‘Big Ideas’ of Data Management and Probability” (pp. 11–19) discusses the big ideas on which the activities have been built and contains additional ideas for classroom activities.

The teaching of mathematics around big ideas offers students opportunities to develop a sophisticated understanding of mathematics concepts and processes, and helps them to maintain their interest in and excitement about doing and learning mathematics.

Working Towards Equitable Outcomes for Diverse Students

All students, whatever their socio-economic, ethnocultural, or linguistic background, must have opportunities to learn and to grow, both cognitively and socially. When students can make personal connections to their learning, and when they feel secure in their learning environment, their true capacity will be realized in their achievement. A commitment to equity and inclusive instruction in Ontario classrooms is therefore critical to enabling all students to succeed in school and, consequently, to become productive and contributing members of society.

To create effective conditions for learning, teachers must take care to avoid all forms of bias and stereotyping in resources and learning activities, which can quickly alienate students and limit their learning. Teachers should be aware of the need to provide a variety of experiences and to encourage multiple perspectives, so that the diversity of the class is recognized and all students feel respected and valued. Learning activities and resources for teaching mathematics should be inclusive, providing examples and illustrations and using approaches that recognize the range of experiences of students with diverse backgrounds, knowledge, skills, interests, and learning styles.

The following are some strategies for creating a learning environment that acknowledges and values the diversity of students and enables them to participate fully in the learning experience:

- providing mathematics problems with situations and contexts that are meaningful to all students (e.g., problems that reflect students’ interests, home-life experiences, and cultural backgrounds and that arouse their curiosity and spirit of enquiry);
- using mathematics examples drawn from diverse cultures, including Aboriginal peoples’;
- using children’s literature that reflects various cultures and customs as a source of mathematical examples and situations;
- understanding and acknowledging customs and adjusting teaching strategies as necessary.

For example, a student may come from a culture in which it is considered inappropriate for a child to ask for help, express opinions openly, or make direct eye contact with an adult;

- considering the appropriateness of references to holidays, celebrations, and traditions;
- providing clarification if the context of a learning activity is unfamiliar to students (e.g., describing or showing a food item that may be new to some students);
- evaluating the content of mathematics textbooks, children's literature, and supplementary materials for cultural or gender bias;
- designing learning and assessment activities that allow students with various learning styles (e.g., auditory, visual, tactile/kinaesthetic) to participate meaningfully;
- providing opportunities for students to work both independently and interdependently with others;
- providing opportunities for students to communicate orally and in writing in their home language (e.g., pairing an English language learner with a first-language peer who also speaks English);
- using diagrams, pictures, manipulatives, sounds, and gestures to clarify mathematical vocabulary that may be new to English language learners.

For a full discussion of equity and diversity in the classroom, as well as a detailed checklist for providing inclusive mathematics instruction, see pages 34–40 in Volume 1 of *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

Accommodations and Modifications

The learning activities in this document have been designed for students with a range of learning needs. Instructional and assessment tasks are open-ended, allowing most students to participate fully in learning experiences. In some cases, individual students may require *accommodations* and/or *modifications*, in accordance with their Individual Education Plan (IEP), to support their participation in learning activities.

PROVIDING ACCOMMODATIONS

Students may require accommodations, including special strategies, support, and/or equipment to allow them to participate in learning activities. There are three types of accommodations:

- *Instructional accommodations* are adjustments in teaching strategies, including styles of presentation, methods of organization, or the use of technology or multimedia.

The term accommodations is used to refer to the special teaching and assessment strategies, human supports, and/or individualized equipment required to enable a student to learn and to demonstrate learning. Accommodations do not alter the provincial curriculum expectations for the grade.

Modifications are changes made in the age-appropriate grade-level expectations for a subject ... in order to meet a student's learning needs. These changes may involve developing expectations that reflect knowledge and skills required in the curriculum for a different grade level and/or increasing or decreasing the number and/or complexity of the regular grade-level curriculum expectations.

(Ontario Ministry of Education, 2004, pp. 25–26)

- *Environmental accommodations* are supports or changes that the student may require in the physical environment of the classroom and/or the school, such as preferential seating or special lighting.
- *Assessment accommodations* are adjustments in assessment activities and methods that enable the student to demonstrate learning, such as allowing additional time to complete tasks or permitting oral responses to test questions.

Some of the ways in which teachers can provide accommodations with respect to mathematics learning activities are listed in the following chart.

Instructional Accommodations

- Vary instructional strategies, using different manipulatives, examples, and visuals (e.g., concrete materials, pictures, diagrams) as necessary to aid understanding.
- Rephrase information and instructions to make them simpler and clearer.
- Use non-verbal signals and gesture cues to convey information.
- Teach mathematical vocabulary explicitly.
- Have students work with a peer.
- Structure activities by breaking them into smaller steps.
- Model concepts using concrete materials and computer software, and encourage students to use them when learning concepts or working on problems.
- Have students use calculators and/or addition and multiplication grids for computations.
- Format worksheets so that they are easy to understand (e.g., use large-size font; an uncluttered layout; spatial cues, such as arrows; colour cues).
- Encourage students to use graphic organizers and square grid paper to organize ideas and written work.
- Provide augmentative and alternative communications systems.
- Provide assistive technology, such as text-to-speech software.
- Provide time-management aids (e.g., checklists).
- Encourage students to verbalize as they work on mathematics problems.
- Provide access to computers.
- Reduce the number of tasks to be completed.
- Provide extra time to complete tasks.

Environmental Accommodations

- Provide an alternative workspace.
- Seat students strategically (e.g., near the front of the room; close to the teacher in group settings; with a classmate who can help them).
- Reduce visual distractions.
- Minimize background noise.
- Provide a quiet setting.
- Provide headphones to reduce audio distractions.
- Provide special lighting.
- Provide assistive devices or adaptive equipment.

Assessment Accommodations

- Have students demonstrate understanding using concrete materials, using computer software, or orally rather than in written form.
- Have students record oral responses on audiotape.
- Have students' responses on written tasks recorded by a scribe.
- Provide assistive technology, such as speech-to-text software.
- Provide an alternative setting.
- Provide assistive devices or adaptive equipment.
- Provide augmentative and alternative communications systems.
- Format tests so that they are easy to understand (e.g., use large-size font; an uncluttered layout; spatial cues, such as arrows; colour cues).
- Provide access to computers.
- Provide access to calculators and/or addition and multiplication grids.
- Provide visual cues (e.g., posters).
- Provide extra time to complete problems or tasks or answer questions.
- Reduce the number of tasks used to assess a concept or skill.

MODIFYING CURRICULUM EXPECTATIONS

Students who have an IEP may require modified expectations, which differ from the regular grade-level curriculum expectations. When developing modified expectations, teachers make important decisions regarding the concepts and skills that students need to learn.

Most of the learning activities in this document can be adapted for students who require modified expectations. The following chart provides examples of how a teacher could deliver learning activities that incorporate individual students' modified expectations.

Modified Program	What It Means	Example
<i>Modified learning expectations, same activity, same materials</i>	The student with modified expectations works on the same or a similar activity, using the same materials.	The learning activity involves representing a probability as a fraction. Students with modified expectations represent the probability using an area model.
<i>Modified learning expectations, same activity, different materials</i>	The student with modified expectations engages in the same activity but uses different materials that enable him/her to remain an equal participant in the activity.	The activity involves creating a bar graph to display data. Students with modified expectations can use statistical software to create the bar graph. (continued)

Modified Program	What It Means	Example
<i>Modified learning expectations, different activity, different materials</i>	Students with modified expectations participate in different activities.	Students with modified expectations work on data management and probability activities that reflect their learning expectations, using a variety of materials and technological tools.

(Adapted from *Education for All: The Report of the Expert Panel on Literacy and Numeracy Instruction for Students With Special Education Needs, Kindergarten to Grade 6*, p. 119.)

It is important to note that some students may require both accommodations and modified expectations.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills. The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The learning activities in this guide demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful for students. The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

Problem Solving: Each of the learning activities is structured around a problem or an inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;

- work collaboratively and talk about mathematics;
- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;
- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

Reasoning and Proving: The learning activities described in this document provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The learning activities include questions that teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

Reflecting: Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine the mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

Selecting Tools and Computational Strategies: Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this document provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and to represent and communicate mathematical ideas at their own level of understanding.

Connecting: The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections among concrete, pictorial, and symbolic mathematical representations. Advice on helping students develop conceptual understanding is also provided. The problem-solving experience in many of the learning activities allows students to connect mathematics to real-life situations and meaningful contexts.

Representing: The learning activities provide opportunities for students to represent mathematical ideas by using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students' own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

Communicating: Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

Addressing the Needs of Junior Learners

Every day, teachers make many decisions about instruction in their classrooms. To make informed decisions about teaching mathematics, teachers need to have an understanding of the big ideas in mathematics, the mathematical concepts and skills outlined in the curriculum document, effective instructional approaches, and the characteristics and needs of learners.

The following chart outlines general characteristics of junior learners and describes some of the implications of these characteristics for teaching mathematics to students in Grades 4, 5, and 6.

Area of Development	Characteristics of Junior Learners	Implications for Teaching Mathematics
Intellectual Development	<p>Generally, students in the junior grades:</p> <ul style="list-style-type: none"> • prefer active learning experiences that allow them to interact with their peers; • are curious about the world around them; • are at a concrete, operational stage of development, and are often not ready to think abstractly; • enjoy and understand the subtleties of humour. 	<p>The mathematics program should provide:</p> <ul style="list-style-type: none"> • learning experiences that allow students to actively explore and construct mathematical ideas; • learning situations that involve the use of concrete materials; • opportunities for students to see that mathematics is practical and important in their daily lives; • enjoyable activities that stimulate curiosity and interest; • tasks that challenge students to reason and think deeply about mathematical ideas. <p style="text-align: right;"><i>(continued)</i></p>

Area of Development	Characteristics of Junior Learners	Implications for Teaching Mathematics
Physical Development	<p>Generally, students in the junior grades:</p> <ul style="list-style-type: none"> • experience a growth spurt before puberty (usually at age 9–10 for girls, at age 10–11 for boys); • are concerned about body image; • are active and energetic; • display wide variations in physical development and maturity. 	<p>The mathematics program should provide:</p> <ul style="list-style-type: none"> • opportunities for physical movement and hands-on learning; • a classroom that is safe and physically appealing.
Psychological Development	<p>Generally, students in the junior grades:</p> <ul style="list-style-type: none"> • are less reliant on praise, but still respond well to positive feedback; • accept greater responsibility for their actions and work; • are influenced by their peer groups. 	<p>The mathematics program should provide:</p> <ul style="list-style-type: none"> • ongoing feedback on students' learning and progress; • an environment in which students can take risks without fear of ridicule; • opportunities for students to accept responsibility for their work; • a classroom climate that supports diversity and encourages all members to work cooperatively.
Social Development	<p>Generally, students in the junior grades:</p> <ul style="list-style-type: none"> • are less egocentric, yet require individual attention; • can be volatile and changeable in regard to friendship, yet want to be part of a social group; • can be talkative; • are more tentative and unsure of themselves; • mature socially at different rates. 	<p>The mathematics program should provide:</p> <ul style="list-style-type: none"> • opportunities to work with others in a variety of groupings (pairs, small groups, large group); • opportunities to discuss mathematical ideas; • clear expectations of what is acceptable social behaviour; • learning activities that involve all students regardless of ability.
Moral and Ethical Development	<p>Generally, students in the junior grades:</p> <ul style="list-style-type: none"> • develop a strong sense of justice and fairness; • experiment with challenging the norm and ask "why" questions; • begin to consider others' points of view. 	<p>The mathematics program should provide:</p> <ul style="list-style-type: none"> • learning experiences that provide equitable opportunities for participation by all students; • an environment in which all ideas are valued; • opportunities for students to share their own ideas, and evaluate the ideas of others.

(Adapted, with permission, from *Making Math Happen in the Junior Years*. Elementary Teachers' Federation of Ontario, 2004.)

Learning About Data Management and Probability in the Junior Grades

The development of an understanding of data management and probability concepts and relationships is a gradual one, moving from experiential and physical learning to theoretical and inferential learning. Data management and probability thinking in the junior years begins to bridge the two.

PRIOR LEARNING

In the primary grades, students learn to collect and organize discrete primary data and to display data using concrete representations, as well as charts and bar graphs. They learn to read, describe, and interpret primary data presented in charts and graphs. They predict and investigate the frequency of a specific outcome in a simple probability experiment. Learning about data management and probability allows students to develop the concepts and language they need for understanding data and chance in the world around them.

Experiences in the primary classroom include sorting and classifying objects into categories; conducting simple surveys; organizing and displaying data in charts, tables, and graphs; reading, describing, and drawing conclusions from primary data; predicting the frequency of an outcome in a simple probability experiment; and investigating the concept of fairness in games.

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES

In the junior grades, students continue to collect, organize, display, and interpret primary data, and explore simple probability. They also learn to deal with secondary data and to display and read data presented in a greater variety of forms, such as double bar graphs and stem-and-leaf plots. They extend their understanding of mode and develop an understanding of median and mean. They also extend their knowledge of probability. Junior students develop a more sophisticated and abstract understanding of probability and start developing their knowledge of theoretical probability. They use websites, such as E-Stat or Census at Schools; spreadsheets; and dynamic statistical software.

Junior students extend their understanding of data management and probability relationships through investigation. They conduct surveys and probability experiments and solve related problems. For example, “Would the results of a survey of primary students about their favourite television shows represent the favourite shows of students in the entire school? Why or why not? How might the survey be improved?” Or “Suppose you roll two number cubes and calculate the sum of the numbers rolled. Which sum is more likely: 2 or 8? Conduct a probability experiment to test your hypothesis. Determine the theoretical probability for each sum.”

Such problems offer junior students opportunities to use and extend their knowledge of data management and probability. Data management and probability problems are often situated in real-life settings. Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to data management and probability.

Students are better able to see the connections in mathematics, and thus to *learn* mathematics, when it is organized in big, coherent “chunks”. In organizing a mathematics program, teachers should concentrate on the big ideas and view the expectations in the curriculum policy documents for Grades 4 to 6 as being clustered around those big ideas.

The clustering of expectations around big ideas provides a focus for student learning and for teacher professional development in mathematics. Teachers will find that investigating and discussing effective teaching strategies for a big idea is much more valuable than trying to determine specific strategies and approaches to help students achieve individual expectations. In fact, using big ideas as a focus helps teachers to see that the concepts presented in the curriculum expectations should not be taught as isolated bits of information but rather as a network of interrelated concepts.

In building a program, teachers need a sound understanding of the key mathematical concepts for their students’ grade level, as well as an understanding of how those concepts connect with students’ prior and future learning (Ma, 1999). Such knowledge includes an understanding of the “conceptual structure and basic attitudes of mathematics inherent in the elementary curriculum” (p. xxiv), as well as an understanding of how best to teach the concepts to children. Concentrating on developing this knowledge will enhance teaching and provide teachers with the tools to differentiate instruction.

Focusing on the big ideas provides teachers with a global view of the concepts represented in the strand. The big ideas also act as a “lens” for:

- making instructional decisions (e.g., choosing an emphasis for a lesson or set of lessons);
- identifying prior learning;
- looking at students’ thinking and understanding in relation to the mathematical concepts addressed in the curriculum (e.g., making note of the ways in which a student solves a division problem);
- collecting observations and making anecdotal records;
- providing feedback to students;
- determining next steps;
- communicating concepts and providing feedback on students’ achievement to parents¹ (e.g., in report card comments).

Focusing on the big ideas also means that teachers use strategies for advancing all students’ mathematical thinking (Fraivillig, 2001) by:

- eliciting from students a variety of solution methods through appropriate prompts, collaborative learning, and a positive, supportive classroom environment;

1. In this document, *parent(s)* refers to parent(s) and guardian(s).

- helping students develop conceptual understanding by attending to relationships among concepts;
- extending students' mathematical thinking by (a) encouraging them to try alternative ways of finding solutions and to generalize, and (b) setting high standards of mathematical performance for all students.

About the Teaching and Learning of Data Management and Probability

The related topics of data management and probability are highly relevant to everyday life. Graphs and statistics bombard the public in advertising, opinion polls, population trends, reliability estimates, descriptions of discoveries by scientists, and estimates of health risks, to name a few.

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005, p. 9)

The National Council of Teachers of Mathematics (NCTM, 2000) suggests that junior students need frequent experiences with investigations, ranging from “quick class surveys to projects that take several days” (p. 177).

Frequent work with brief surveys (How many brothers and sisters do people in our class have? What’s the farthest you have been from your home?) can acquaint students with particular aspects of collecting, representing, summarizing, comparing, and interpreting data. More extended projects can engage students in a cycle of data analysis – formulating questions, collecting and representing the data, and considering whether their data are giving them the information they need to answer their question (NCTM, 2000, p. 177).

In the junior grades, students are becoming more aware of and interested in the world around them. By studying data management and probability in contexts provided by other subject areas, such as social studies, science, and health and physical education, they have the opportunity to connect with the world.

It is not uncommon to encounter forms of data representation that give a misleading interpretation of the data. It is also not uncommon to find that adults often have misconceptions about probability. Junior students need a solid understanding of probability and need help to develop critical data analysis skills. Meaningful hands-on investigations are the ideal starting point.

Big Ideas and Tiered Instruction²

How students experience a big idea, and how big it becomes, depends greatly on how it is developed pedagogically in the classroom. It is not enough to label a mathematical concept as a big idea. Big ideas must be coupled with a pedagogy that offers students opportunities to attend deeply to mathematical concepts and relationships.

Big ideas, and a pedagogy that supports student learning of big ideas, naturally provide opportunities for meeting the needs of students who are working at different levels of mathematical performance. The reason for this is that teaching around big ideas means teaching around ideas that incorporate a variety of levels of mathematical sophistication. For example, consider the problem of *finding the probabilities associated with tossing two coins* and the tiers at which the problem can be approached or extended:

Tier 1: Students predict which of the outcomes – 2 heads, 2 tails, or 1 head and 1 tail – is more likely (or whether they are all equally likely) and conduct a probability experiment to test their predictions.

Tier 2: Students also consider the effect of the number of trials on experimental probability. They describe their reasoning in the context of typical misconceptions, such as “previous probability trials affect future trials” (for example, if a coin is tossed and comes up heads 5 times in a row, the probability of getting a head on the 6th toss decreases).

Tier 3: Students may also consider all possible outcomes (HH, HT, TH, TT) in order to explain why the probability of 1 head and 1 tail is $\frac{1}{2}$, while the probability of 2 heads is $\frac{1}{4}$. This is a first, informal step towards the development of an understanding of theoretical probability.

A probability activity such as *finding the probabilities associated with tossing two coins* addresses Grade 5 expectations within the Data Management and Probability strand of *The Ontario Curriculum, Grades 1–8: Mathematics*, 2005 (p. 85). It is also a problem that students encounter in earlier grades. For instance, a Grade 2 expectation (p. 52) includes the following sample of student thinking: “I tossed 2 coins at the same time, to see how often I would get 2 heads. I found that getting a head and a tail was more likely than getting 2 heads.”

The problem of *finding the probabilities associated with tossing two coins* can be extended to 3 coins and 4 coins. When students do this, teachers can show them a connection to Pascal’s Triangle (Figures 1 and 2). When students toss 2 coins, there is one way of getting 2 heads (HH), two ways of getting 1 head and 1 tail (HT or TH), and one way of getting

2. A tiered approach to instruction is suggested in *Education for All – The Report of the Expert Panel on Literacy and Numeracy Instruction for Students With Special Education Needs, Kindergarten to Grade 6*, 2005, pp. 60, 120, 123.

2 tails. When students toss 3 coins, there is one way of getting 3 heads (HHH), three ways of getting 2 heads and 1 tail (HHT, HTH, THH), three ways of getting 1 head and 2 tails (HTT, THT, TTH), one way of getting 3 tails (TTT), and so forth.

Students can use either Figure 1 or Figure 2 to determine theoretical probabilities. For example, the theoretical probability of tossing 3 heads (HHH) is $1/8$. The theoretical probability of tossing 2 heads and 1 tail is $3/8$.



Figure 1. Tossing coins

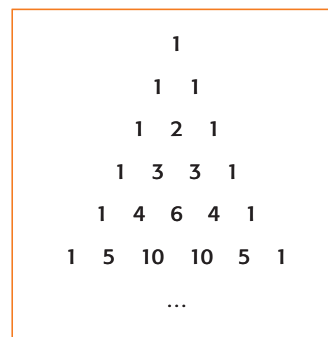


Figure 2. Pascal's triangle

When students are studying algebra in Grades 9 and 10, they can extend this understanding to expressions, such as $(x + y)^2$ or $(x + y)^3$.

$$\begin{aligned}
 (x + y)^0 &= 1 \\
 (x + y)^1 &= 1x + 1y \\
 (x + y)^2 &= 1x^2 + 2xy + 1y^2 \\
 (x + y)^3 &= 1x^3 + 3x^2y + 3xy^2 + 1y^3 \\
 (x + y)^4 &= 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4 \\
 (x + y)^5 &= 1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5 \\
 &\dots
 \end{aligned}$$

Figure 3. Expanding binomial products

Notice the similarity of the patterns in Figures 1, 2, and 3.

Underlying all of this is the binomial theorem, which students encounter in Grades 11 and 12 and at the university level in the study of algebra, probability, and statistics.

Big ideas, big problems, and a pedagogy that supports them at the classroom level provide opportunities for students to engage with *the same mathematical situation* at different levels of sophistication. They also help students discover the connections among, and the beauty of, mathematical ideas.

The Big Ideas of Data Management and Probability in Grades 4 to 6

The goal of teaching and learning mathematics through big ideas is an integral component of *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. In each of the strands and in each of the grades, the specific expectations have been organized around big ideas in mathematics.

The big ideas in Data Management and Probability in Grades 4 to 6 are:

- collection and organization of data
- data relationships
- probability

The tables that follow show how the expectations for each of these big ideas progress through the junior grades. The sections that follow offer teachers strategies and content knowledge to address these expectations in the junior grades while helping students develop an understanding of data management and probability. Teachers can facilitate this understanding by engaging students in meaningful activities where:

- they collect and organize primary and secondary data;
- they interpret and use charts and graphs, including continuous line graphs;
- they explain relationships between data sets and draw inferences from data;
- they determine the theoretical probability of an outcome in a probability experiment.

Curriculum Expectations Related to Collection and Organization of Data, Grades 4, 5, and 6

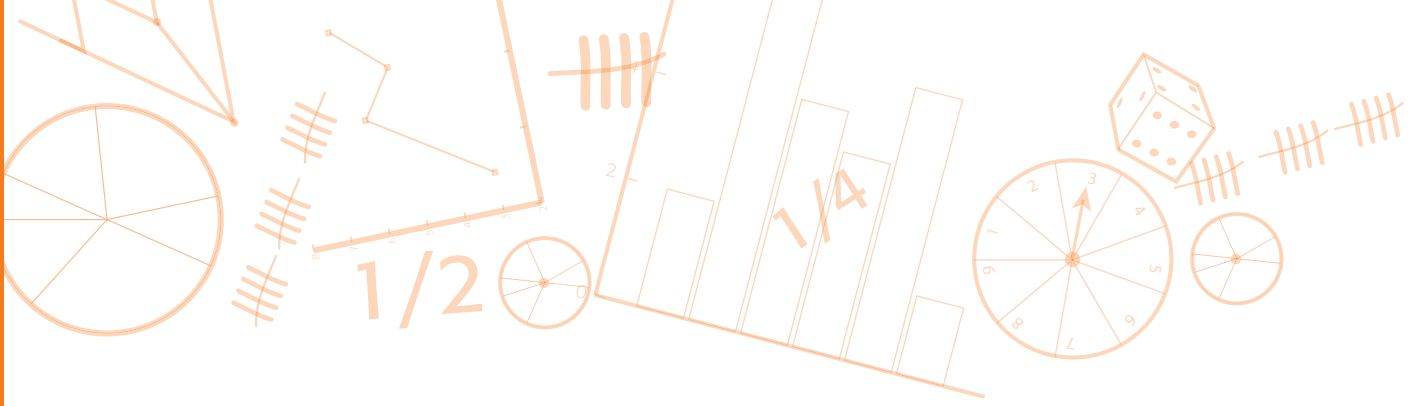
By the end of Grade 4, students will:	By the end of Grade 5, students will:	By the end of Grade 6, students will:
<p>Overall Expectations</p> <ul style="list-style-type: none"> collect and organize discrete primary data and display the data using charts and graphs, including stem-and-leaf plots and double bar graphs. <p>Specific Expectations</p> <ul style="list-style-type: none"> collect data by conducting a survey or an experiment to do with themselves, their environment, issues in their school or the community, or content from another subject, and record observations or measurements; collect and organize discrete primary data and display the data in charts, tables, and graphs (including stem-and-leaf plots and double bar graphs) that have appropriate titles, labels, and scales that suit the range and distribution of the data, using a variety of tools. 	<p>Overall Expectations</p> <ul style="list-style-type: none"> collect and organize discrete or continuous primary data and secondary data and display the data using charts and graphs, including broken-line graphs. <p>Specific Expectations</p> <ul style="list-style-type: none"> distinguish between discrete data (i.e., data organized using numbers that have gaps between them, such as whole numbers, and often used to represent a count, such as the number of times a word is used) and continuous data (i.e., data organized using all numbers on a number line that fall within the range of the data, and used to represent measurements such as heights or ages of trees); collect data by conducting a survey or an experiment to do with themselves, their environment, issues in their school or community, or content from another subject, and record observations or measurements; collect and organize discrete or continuous primary data and secondary data and display the data in charts, tables, and graphs (including broken-line graphs) that have appropriate titles, labels, and scales that suit the range and distribution of the data, using a variety of tools; demonstrate an understanding that sets of data can be samples of larger populations; describe, through investigation, how a set of data is collected and explain whether the collection method is appropriate. 	<p>Overall Expectations</p> <ul style="list-style-type: none"> collect and organize discrete or continuous primary data and secondary data and display the data using charts and graphs, including continuous line graphs. <p>Specific Expectations</p> <ul style="list-style-type: none"> collect data by conducting a survey or an experiment to do with themselves, their environment, issues in their school or community, or content from another subject, and record observations or measurements; collect and organize discrete or continuous primary data and secondary data and display the data in charts, tables, and graphs (including continuous line graphs) that have appropriate titles, labels, and scales that suit the range and distribution of the data, using a variety of tools; select an appropriate type of graph to represent a set of data, graph the data using technology, and justify the choice of graph (i.e., from types of graphs already studied, such as pictographs, horizontal or vertical bar graphs, stem-and-leaf plots, double bar graphs, broken-line graphs, and continuous line graphs); determine, through investigation, how well a set of data represents a population, on the basis of the method that was used to collect the data.

Curriculum Expectations Related to Data Relationships, Grades 4, 5 and 6

By the end of Grade 4, students will:	By the end of Grade 5, students will:	By the end of Grade 6, students will:
<p>Overall Expectations</p> <ul style="list-style-type: none"> • read, describe, and interpret primary data and secondary data presented in charts and graphs, including stem-and-leaf plots and double bar graphs. <p>Specific Expectations</p> <ul style="list-style-type: none"> • read, interpret, and draw conclusions from primary data and from secondary data, presented in charts, tables, and graphs (including stem-and-leaf plots and double bar graphs); • demonstrate, through investigation, an understanding of median, and determine the median of a set of data; • describe the shape of a set of data across its range of values, using charts, tables, and graphs; • compare similarities and differences between two related sets of data, using a variety of strategies. 	<p>Overall Expectations</p> <ul style="list-style-type: none"> • read, describe, and interpret primary data and secondary data presented in charts and graphs, including broken-line graphs. <p>Specific Expectations</p> <ul style="list-style-type: none"> • read, interpret, and draw conclusions from primary data and from secondary data, presented in charts, tables, and graphs (including broken-line graphs); • calculate the mean for a small set of data and use it to describe the shape of the data set across its range of values, using charts, tables, and graphs; • compare similarities and differences between two related sets of data, using a variety of strategies. 	<p>Overall Expectations</p> <ul style="list-style-type: none"> • read, describe, and interpret data, and explain relationships between sets of data. <p>Specific Expectations</p> <ul style="list-style-type: none"> • read, interpret, and draw conclusions from primary data and from secondary data, presented in charts, tables, and graphs (including continuous line graphs); • compare, through investigation, different graphical representations of the same data; • explain how different scales used on graphs can influence conclusions drawn from the data; • demonstrate an understanding of mean, and use the mean to compare two sets of related data, with and without the use of technology; • demonstrate, through investigation, an understanding of how data from charts, tables, and graphs can be used to make inferences and convincing arguments.

Curriculum Expectations Related to Probability, Grades 4, 5, and 6

By the end of Grade 4, students will:	By the end of Grade 5, students will:	By the end of Grade 6, students will:
Overall Expectations <ul style="list-style-type: none"> predict the results of a simple probability experiment, then conduct the experiment and compare the prediction to the results. 	Overall Expectations <ul style="list-style-type: none"> represent as a fraction the probability that a specific outcome will occur in a simple probability experiment, using systematic lists and area models. 	Overall Expectations <ul style="list-style-type: none"> determine the theoretical probability of an outcome in a probability experiment, and use it to predict the frequency of the outcome.
Specific Expectations <ul style="list-style-type: none"> predict the frequency of an outcome in a simple probability experiment, explaining their reasoning; conduct the experiment; and compare the result with the prediction; determine, through investigation, how the number of repetitions of a probability experiment can affect the conclusions drawn. 	Specific Expectations <ul style="list-style-type: none"> determine and represent all the possible outcomes in a simple probability experiment, using systematic lists and area models; represent, using a common fraction, the probability that an event will occur in simple games and probability experiments; pose and solve simple probability problems, and solve them by conducting probability experiments and selecting appropriate methods of recording the results. 	Specific Expectations <ul style="list-style-type: none"> express theoretical probability as a ratio of the number of favourable outcomes to the total number of possible outcomes, where all outcomes are equally likely; represent the probability of an event (i.e., the likelihood that the event will occur), using a value from the range of 0 (never happens or impossible) to 1 (always happens or certain); predict the frequency of an outcome of a simple probability experiment or game, by calculating and using the theoretical probability of that outcome.



COLLECTION AND ORGANIZATION OF DATA

Overview

Students enter the junior grades with experience in and knowledge of collecting and organizing data and displaying the data “using charts and graphs, including vertical and horizontal bar graphs, with labels ordered appropriately along horizontal axes, as needed” (Ontario Ministry of Education, 2005, p. 63). As they move through the junior grades, students consolidate the following skills:

- dealing with discrete and continuous data;
- using a variety of data collection techniques (surveys, experiments, observations, measurements);
- using a variety of data representation tools (charts, tables, graphs, spreadsheets, statistical software);
- describing and explaining their data collection and organization methods.

Example: Favourite TV Shows

Let’s see how the big idea of “collection and organization of data” could be developed during a survey of favourite TV shows, a sample problem listed among the expectations for Grade 6 in *The Ontario Curriculum, Grades 1–8: Mathematics, 2005* (p. 95).

Sample problem: Would the results of a survey of primary students about their favourite television shows represent the favourite shows of students in the entire school? Why or why not?

This problem presents students with the opportunity to describe and explain the appropriateness of a data collection method (in this case, a survey of primary students) in relation to a question asked (What are the favourite shows of students in the school?).

The problem can be presented to students as part of a larger group of similar problems (see Figure 4) that they discuss with partners or other members of small groups, afterwards

sharing their ideas with the whole class (as in Method 1 below). The problem can also be extended to engage students in designing their own questions and methods of data collection and organization (see Method 2, p. 24).

Method 1: Matching Questions With Methods of Data Collection and Organization

Questions such as those in Figure 4 help students to focus their attention on the relationship between (1) the question asked and the data collection method used, and (2) the data collected and the method use to organize and display it.

WHAT DO YOU THINK?

1. Would the results of a survey of primary students about their favourite television shows represent the favourite shows of students in the entire school? Why or why not?
2. Would the results of a survey of students in the school lunch room about how often they go home for lunch be representative? Why or why not?
3. Would the mean height of your friends be representative of the mean height of students in your grade? Why or why not?
4. Would a bar graph showing the types of cars parked in the school parking lot be representative of the cars in your neighbourhood?
5. Would a broken-line graph be an appropriate method for displaying the data from question 1?
6. Would a stem-and-leaf plot be an appropriate method for displaying the data from question 2?
7. Would a circle graph be an appropriate method for displaying the data from question 3?

Figure 4. Questions to get students thinking

Students in the junior grades are in the process of developing an understanding of methods of data collection and organization.

In terms of data collection students are learning to do the following:

- distinguish between discrete and continuous data
 - some examples of discrete data are shoe sizes and number of cars
 - discrete data occur in cases where there are only a limited number of values or when students are counting something
 - discrete data typically involve the use of whole numbers (0, 1, 2, 3, ...)
 - some examples of continuous data are temperature, time, and height
 - continuous data typically occur with physical measurement
 - continuous data typically involve the use of all numbers on the number line (whole numbers and all the decimals between them)

- design surveys to answer questions
 - *develop good survey questions*
 - *distinguish between a population to be surveyed and a sample of the population*
 - *ensure that the sample is representative of the population*
- design experiments to answer questions
 - *relate the design of the experiment to the scientific method studied in science*
- use observation or measurement to answer questions
 - *use strategies to observe or measure*
 - *decide how accurate observations or measurements need to be*
- describe, explain, and justify their data collection methods

In terms of data organization students are learning to:

- distinguish among the different methods of data organization (tally, chart, pictograph, stem-and-leaf plot, bar graph, circle graph, line graph);
- select methods of data organization that are appropriate for the data collected;
- describe, explain, and justify their methods of data organization.

By using technology, students can experience some of these methods of data organization in a dynamic fashion. For example, when using a spreadsheet, they can enter a set of data, quickly generate a variety of graphs based on the data, and then consider which graph is the most appropriate for the data and for the question asked. With technology students can quickly change features, such as range and scale, thus allowing them to see and consider the effect of the change on data representation. Although it is important for students to have experience in drawing graphs by hand, so that they come to understand the features that make up a graph, most people in the real world today use technology to generate graphs.

It is important to create opportunities for students to experience the full range of data organization methods, from the concrete to the visual to the symbolic. Consider the following sequence of classroom activities, designed to help students understand circle graphs:

- Students are organized into groups based on the predominant colour of their shirt.
- They form one circle, standing beside other students in their shirt-colour group (as in Figure 5).
- The teacher stands in the centre of the circle.

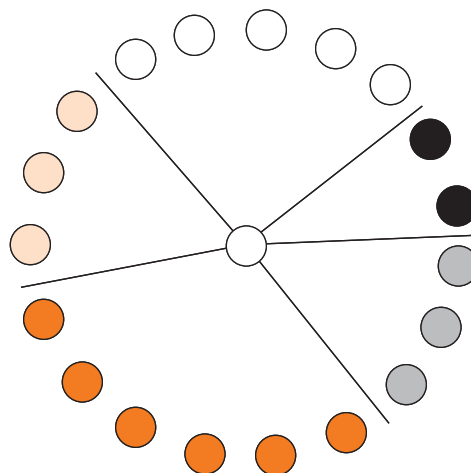


Figure 5. Creating a concrete circle graph

- Ropes are stretched from the teacher to the boundaries between the colour groups.
- The teacher explains that together they have formed a physical circle graph.

Another way to develop this concept is to cut square grid paper into strips, taping strips together until there are enough squares to represent all students in the class. Students colour the squares (as in Figure 6) to represent the data. The strip can then be shaped into a circle, which could be used as the basis for drawing a circle graph.



Figure 6. Creating an outline of a circle graph

Likewise, when modelling bar graphs – for instance, of the seasons in which students were born – the teacher could write the names of the seasons across the board and students could line up in front of the appropriate seasons, forming a physical bar graph. Alternatively, students could place a yellow sticky note above the appropriate seasons, forming a bar graph on the board.

Such concrete experiences anchor students' understanding to the physical world and help add meaning to abstract concepts.

CHARACTERISTICS OF STUDENT LEARNING AND INSTRUCTIONAL STRATEGIES

Students who understand and can solve problems, such as the one in the sample problem on page 20, are able to do the following:

- They can describe the relationship between a question asked and the data collection method used.
- They can describe alternative data collection methods and explain their appropriateness in relation to the question asked.
- They can ask alternative questions and explain their appropriateness in relation to the data collection method used.
- They can demonstrate their familiarity with a variety of data collection methods.
- They can identify appropriate methods for organizing the data to be collected.

For students to develop such an understanding, the teacher should use instructional strategies that help them become aware of:

- ways to change the data collection method to better match the question asked;
- ways to change the question to better match the data collection method used;
- the range of methods that can be used to organize the information collected;
- the intended audience for the information collected.

Method 2: Extending the Problem

It is the questions that drive mathematics. Solving problems and making up new ones is the essence of mathematical activity.

(Hersh, 1997, p. 18)

POSING PROBLEMS

Students need a variety of focused activities that will allow them to practise the skills identified in Method 1: Matching Questions With Methods of Data Collection and Organization. However, they also need opportunities to pose their own data management questions and design their own methods of data collection and organization. This goal is reflected in the mathematical process expectations listed for junior students in *The Ontario Curriculum, Grades 1–8: Mathematics, 2005* (pp. 65, 77, 87), which state that students will “pose and solve problems and conduct investigations, to help deepen their mathematical understanding”. *Teaching and Learning Mathematics: The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario, 2004* (p. 7) states in addition that “a variety of problem-solving experiences and a balanced array of pedagogical approaches” are necessary for effective mathematics instruction in the junior grades. An essential aspect of an effective mathematics program is balance (Kilpatrick, Swafford, & Findell, 2001).

Engaging in posing problems and solving problems makes the mathematics that students are learning more interesting to them and helps them become better mathematical thinkers.

One way to engage students in posing problems is to ask them to work in small groups to:

- design their own questions along the lines of those in Figure 4;
- suggest ways of changing and improving the questions or the methods of data collection and organization;
- share and discuss their ideas with others in the class.

By being personally involved in generating questions and in collecting and organizing data, students have a stake in the learning activity and are more likely to have a meaningful learning experience. The skills they develop will form a solid foundation for their future learning and for real-life experiences involving data management.

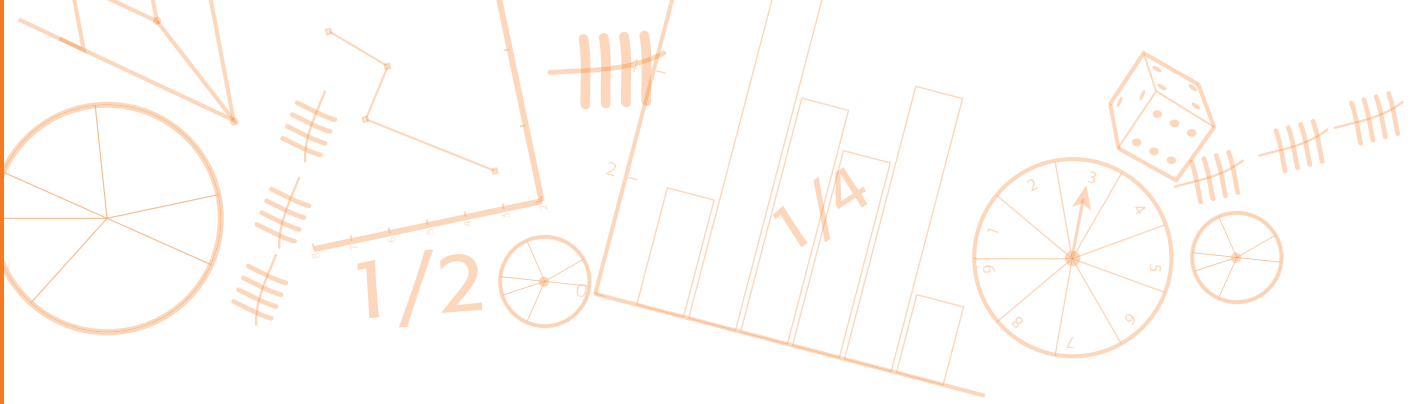
CHARACTERISTICS OF STUDENT LEARNING AND INSTRUCTIONAL STRATEGIES

Extending a problem in more mathematically sophisticated ways requires more in-depth mathematical knowledge from the teacher, who must be flexible and responsive to the (possibly unanticipated) mathematical exploration directions that emerge in a classroom setting. In addition, the teacher needs to use instructional strategies in which:

- students are encouraged to pose and explore “what if” questions related to the concepts and problems being studied;
- students are encouraged to organize their data using a variety of representations and to explore and discuss which representations are most effective;
- students are encouraged to make connections with problems they previously explored and with concepts learned in other strands.

Students who develop problem-solving skills that enable them to explore extensions to data management problems have the following learning characteristics:

- They can pose “what if” questions to extend problems in new mathematical directions.
- They are willing to persevere in their mathematical thinking and solve mathematical problems.
- They can work cooperatively and constructively with others.
- They use a variety of empirical methods to test hypotheses.



DATA RELATIONSHIPS

Overview

Students enter the junior grades with experience of and knowledge in reading, describing, and interpreting “primary data presented in charts and graphs, including vertical and horizontal bar graphs” (Ontario Ministry of Education, 2005, p. 63). As they move through the junior grades, they consolidate the following skills:

- dealing with primary and secondary data;
- reading and interpreting data and drawing conclusions;
- using descriptive statistics (range, mean, median, mode) to describe the shape of the data;
- comparing two related sets of data using a variety of strategies (tally, stem-and-leaf plot, double bar graph, broken-line graph, measures of central tendency);
- making inferences and convincing arguments.

Example: The Masses of Backpacks

Let’s see how the big idea of “data relationships” could be developed with a variation of the masses of backpacks problem from the Data Relationships expectations for Grade 6 in *The Ontario Curriculum, Grades 1–8: Mathematics, 2005* (p. 96).

Sample problem: Use the mean to compare the masses of backpacks of students from two or more Grade 6 classes.

The problem shown above presents students with the opportunity to review and apply descriptive statistics to help them understand a set of data about the masses of backpacks.

This problem can be used as the focus for two classes of students who are generating and sharing data to analyse using descriptive statistics and appropriate forms of representation (as in Method 1 on the next page). The problem can also be extended to allow students to ask their own questions and collect and analyse their own data (see Method 2, p. 30).

Method 1: Understanding Data Relationships

LEARNING TO INVESTIGATE AND UNDERSTAND DATA RELATIONSHIPS

Table 1 shows the masses of student backpacks in two classes. Students can collect their own data on the masses of student backpacks with the help of a common bathroom scale, either by placing each backpack on the scale and noting its mass, or by having a student first step on the scale with a backpack and again without it, then subtracting the second mass from the first.

Masses of Backpacks (kg)	
Grade 4 Class	Grade 6 Class
1.2, 2.4, 3.4, 1.4, 1.5, 2.3, 0.6, 2.6, 1.8, 1.8, 2.1, 2.6, 2.4, 1.6, 1.2, 3.6, 2.7, 1.4, 2.3, 1.4, 3.8, 2.5, 2.7, 1.4, 1.8	1.5, 2.3, 4.1, 3.5, 0.8, 1.8, 2.2, 3.8, 2.5, 1.6, 3.8, 2.1, 4.9, 3.9, 2.8, 1.5, 0.7, 2.6, 4.5, 2.3, 3.4, 2.3, 3.6, 1.4, 1.2, 2.6, 1.9, 2.5

Table 1. Data: the masses of backpacks

Students can then sort the data in a table or enter them into a spreadsheet and sort them using a built-in sort function, as was done in Table 2. Sorting the data is useful for finding the range, the median, and the mode of the data, and it also gives a sense of the shape of the data.

Using the plotting tools in a spreadsheet, students can create various graphs to display the data, as was done in Figure 7 with a line graph. Early in students' data-graphing experiences, it is helpful to have them graph data using various forms and then discuss which graphs are most appropriate.

Grade 4 Class	Grade 6 Class
0.6	0.7
1.2	0.8
1.2	1.2
1.4	1.4
1.4	1.5
1.4	1.5
1.4	1.6
1.5	1.8
1.6	1.9
1.8	2.1
1.8	2.2
1.8	2.3
2.1	2.3
2.3	2.3
2.3	2.5
2.4	2.5
2.4	2.6
2.5	2.6
2.6	2.8
2.6	3.4
2.7	3.5
2.7	3.6
3.4	3.8
3.6	3.8
3.8	3.9
	4.1
	4.5
	4.9

Table 2. The masses of backpacks: sorted data

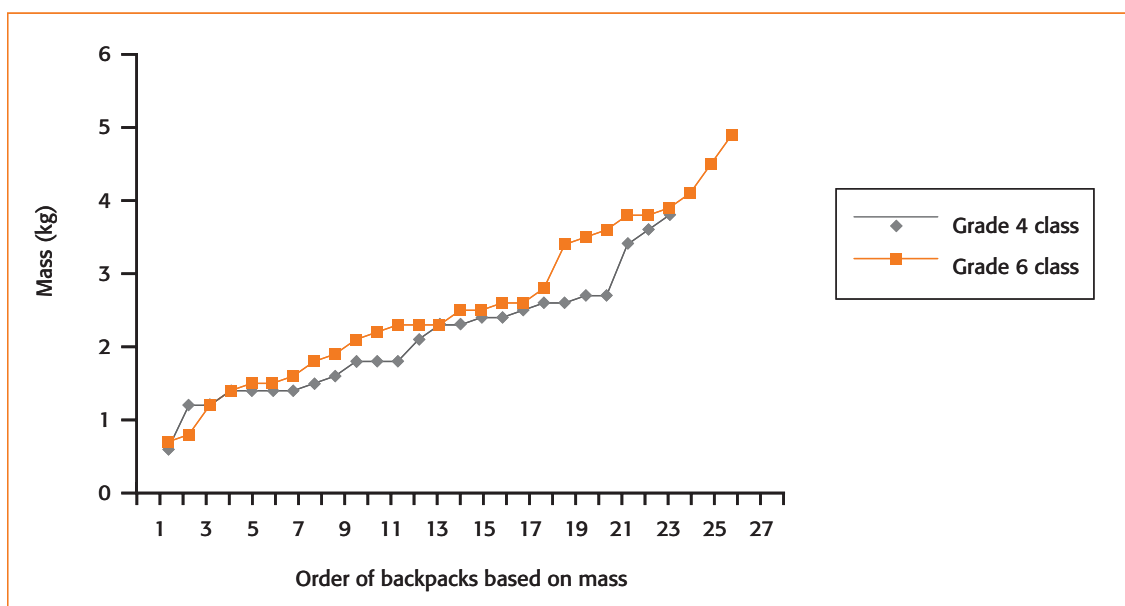


Figure 7. The masses of backpacks: sorted data shown as a line graph

Although double line or double bar graphs help students compare the data, a stem-and-leaf plot (as shown in Figure 8) can give a better sense of the shape of the data.

Grade 4		Grade 6
6	0.	7 8
8 8 8 6 5 4 4 4 2 2	1.	2 4 5 5 6 8 9
7 7 6 6 5 4 4 3 3 1	2.	1 2 3 3 3 5 5 6 6 8
8 6 4	3.	4 5 6 8 8 9
	4.	1 5 9

Figure 8. The masses of backpacks: data shown as a stem-and-leaf plot

Descriptive statistics (as in Table 3) help students to better understand the data. Students need opportunities to calculate such statistics and to think about and discuss which descriptive statistic might be useful for the situation. For example, the mode is useful for describing discrete data (such as shoes sizes) but it has little meaning in the case of the masses of backpacks. However, the other measures of central tendency – mean and median – do help to give a sense of the data.

	Grade 4	Grade 6
Range	3.2	4.2
Mean	2.1	2.6
Median	2.1	2.4
Mode	1.4	2.3

Table 3. The masses of backpacks: descriptive statistics

Situations such as the one described in the backpacks problem offer scope for fruitful explorations that help students understand the nature of the descriptive statistics used in Table 3. For example, the teacher could ask students: “Suppose we changed the data for the Grade 4 class so that all the values below the median became 0.6 (as in Table 4). Would this affect the range? Would it affect the median? Would it affect the mean?”

Considering and discussing such questions helps students understand the meaning and the limitations of the measures of central tendency when used to describe the data they collect.

CHARACTERISTICS OF STUDENT LEARNING AND INSTRUCTIONAL STRATEGIES

Junior students who understand data relationships can:

- read and interpret data;
- use descriptive statistics to describe and analyse data;
- compare related sets of data using a variety of strategies;
- draw conclusions based on data and justify their reasoning.

To help students develop an understanding of data relationships, the teacher should provide many opportunities for them to read and interpret data and compare related data sets. In the junior grades, it is important for the teacher to help students experience and investigate data relationships in contexts that relate to the world around them, some of which will be embedded in contexts provided by other subject areas.

Grade 4 Class	Grade 6 Class
0.6	0.7
0.6	0.8
0.6	1.2
0.6	1.4
0.6	1.5
0.6	1.5
0.6	1.6
0.6	1.8
0.6	1.9
0.6	2.1
0.6	2.2
0.6	2.3
2.1	2.3
2.3	2.3
2.3	2.5
2.4	2.5
2.4	2.6
2.5	2.6
2.6	2.8
2.6	3.4
2.7	3.5
2.7	3.6
3.4	3.8
3.6	3.8
3.8	3.9
	4.1
	4.5
	4.9

Table 4. The masses of backpacks: the effect of changing the data on measures of central tendency

Method 2: Exploring Data Relationships Through Project-Based Learning

One way to engage students in problem posing is to have them work in pairs or in small groups to design their own questions for a data collection project of their own choosing. A real-life activity, such as this one, gives students the opportunity to design data management projects that are personally meaningful. Furthermore, exploring their chosen topic will help them to better understand the world around them.

This is also an opportunity for the teacher to introduce data management contexts from other subject areas, such as social studies, science, and health and physical education. Here are some examples of integrated projects:

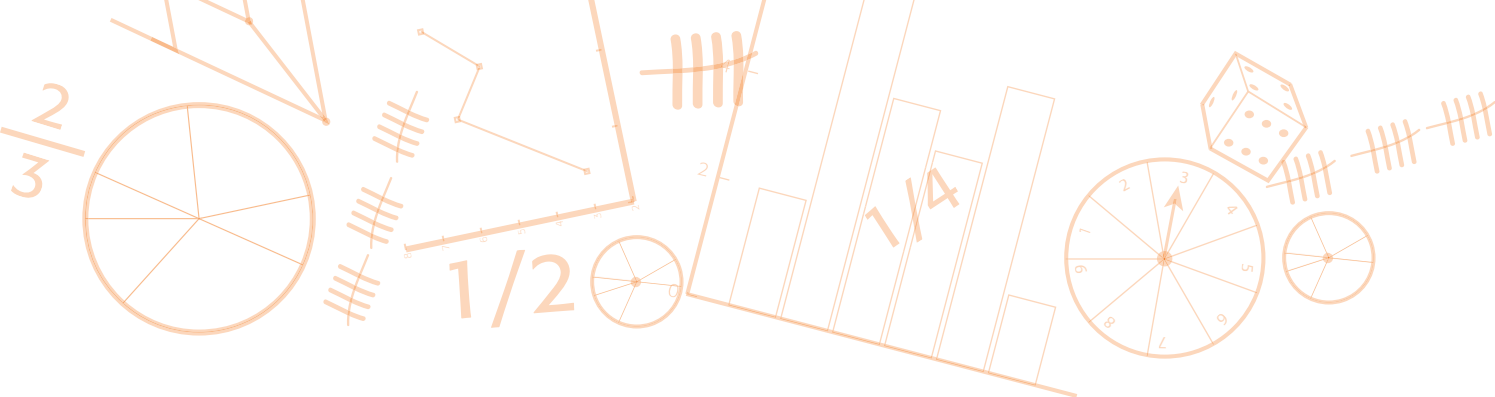
- In social studies, Grade 4 students study Canada's provinces, territories, and regions. Using print and electronic resources (such as those of Statistics Canada), students could access a wide variety of data (economic, physical, etc.) on the basis of which they would compare selected provinces.
- In social studies, Grade 6 students study Canada's links to the world. The Internet gives students access to a vast assortment of data (economic, cultural, etc.), the exploration of which will help them to better understand Canada's relationships with other countries.
- In health and physical education, junior students develop an understanding of physical fitness, health, and well-being and the factors that contribute to them. Students could design surveys around exercise or eating habits or use the Internet to access statistics about smoking (on, for instance, the Canadian Lung Association website) or about drinking and driving (on, for instance, the Canada Safety Council website).

Students need project-based experiences that allow them to design questions and methods of data collection and organization that relate to their personal interests. When students are personally involved in generating questions and in collecting and organizing data, they have a stake in the learning activity and are more likely to have a meaningful learning experience.

These experiences will help students develop the ability to:

- describe and explain the questions and the methods of data collection and organization they plan to use;
- collect and represent data using appropriate charts and graphs;
- use descriptive statistics to describe and analyse data;
- compare related sets of data using a variety of strategies;
- draw conclusions based on data and justify their reasoning.

These skills will form a solid foundation for their future learning and for real-life experiences involving data management.



PROBABILITY

Overview

Students enter the junior grades with experience in and knowledge of predicting and investigating “the frequency of a specific outcome in a simple probability experiment” (Ontario Ministry of Education, 2005, p. 63). As they move through the junior grades, they consolidate the following skills:

- determining and representing all possible outcomes (i.e., the “sample space”) of a probability experiment;
- describing the probability of an event using fractions, area models, and decimals between 0 and 1;
- posing and solving probability questions.

Example: The Sum of Two Number Cubes

Let’s see how the big idea of “probability” could be developed using a variation of the sums of two number cubes problem from the Probability expectations for Grade 4 in *The Ontario Curriculum, Grades 1–8: Mathematics, 2005* (p. 74).

Sample problem: If you toss a pair of number cubes 20 times and calculate the sum for each toss, how many times would you expect to get 12? 7? 1? Explain your thinking. Then conduct the experiment and compare the results with your predictions.

Students are asked to predict the outcomes of a probability situation and are then given opportunities to test their predictions through an experiment.

This problem can be presented to students by way of experimental probability activities (as in Method 1 on the next page). The problem can also be extended to allow students to consider all possible outcomes and explore theoretical probability concepts (see Method 2, p. 34).

Method 1: Experimental Probability

Experimental probability is calculated using experiments. Using experimental data, students can make reasonable conclusions about the probability of various outcomes, such as:

- Which sum is most likely to occur?
- What is the approximate probability of rolling a sum of 2 with two dice?

To develop an understanding of probability, students should have opportunities to experience experimental probability through a variety of hands-on, project-based, and problem-based activities and games. Such investigations enrich students' understanding of probability and help to anchor this understanding in personal experiences. Furthermore, such activities and games help to dispel probability misconceptions held by junior students. Some junior students (and some adults too) believe, for instance, that if a 6 is rolled with a number cube, then it is less likely that the next roll will produce a 6. This thinking is incorrect, because number cubes do not “remember” the number previously rolled, and every number from 1 to 6 has the same chance of occurring regardless of previous outcomes. Similarly, when picking numbers for a lottery ticket, an adult might laugh at the suggestion of playing the winning numbers from the previous week's draw. However, the previous week's winning numbers have absolutely no effect on the numbers that are randomly chosen a week later. They have the same chance of winning as any other set of numbers.

PLAYING A PROBABILITY GAME³

The sample problem shown on the previous page can be explored through a game.

Students working in pairs are given a pair of number cubes, a 12 by 12 cell grid (see Figure 9), and 12 bingo chips (small tiles or pennies will also do).

Students are instructed to place the 12 chips wherever they like on the grid and then to roll the pair of number cubes 12 times, keeping in mind that each time they roll a number cube:

- they must calculate the sum;
- they must check to see if there are any bingo chips in the column above the sum and, if so, they must remove 1 bingo chip (any chip) from that column.

3. This activity is adapted from Gadanidis, G. (1999). Probability experiments. *Mathmania*, 4(1), 10–11.

The object of the game is to end up with as few bingo chips on the grid as possible

1	2	3	4	5	6	7	8	9	10	11	12

Figure 9. A probability game

After students play the game once, they should discuss their strategy for winning and then adapt it, basing their decisions on their observations. After playing the game a few times, students will realize that:

- the sum of 1 never occurs and therefore its probability is 0;
- sums in the middle numbers are more likely to occur than sums in the lower or higher numbers.

This activity can be followed up with a related one: Students roll the number cubes 36 times and record each sum by shading an appropriate space in the column above the sum, thus forming a bar graph (as in Figure 10).

The grids are posted on the walls around the classroom so that students can see the patterns that have emerged.

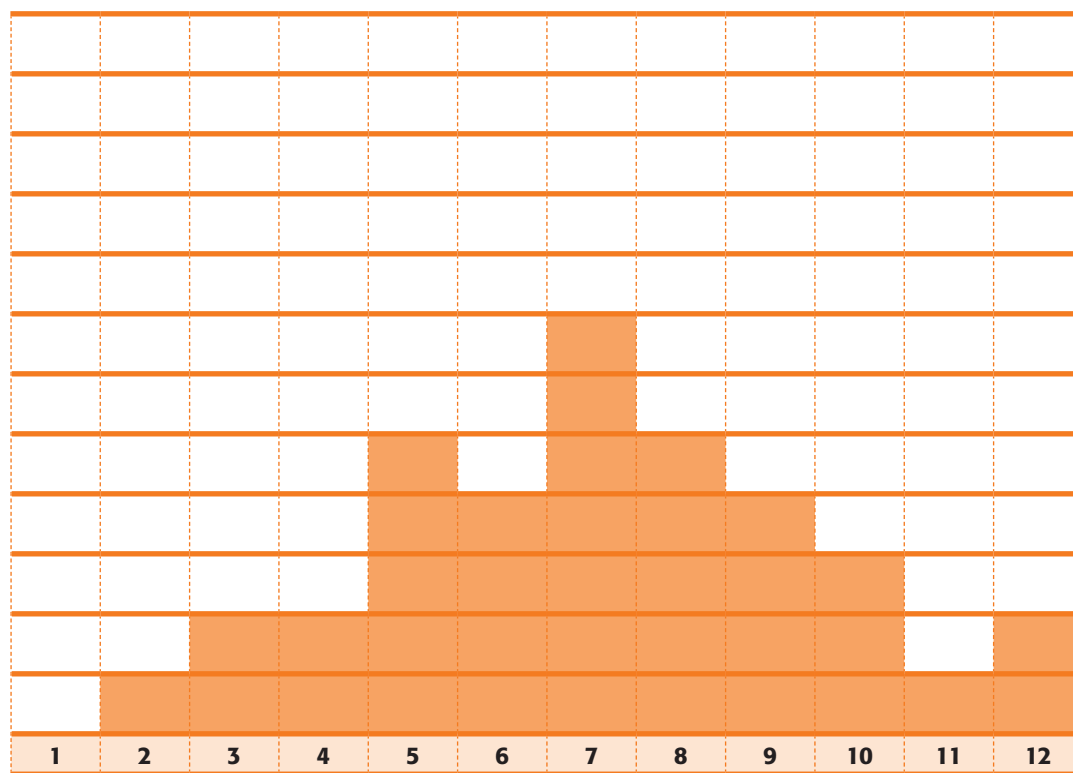


Figure 10. Recording the outcomes as a bar graph (a sample)

Students now have a meaningful context (the data collected by all students) in which to use their data management skills. For example, they can now calculate the following for each sum:

- the mean
- the median
- the mode

They can decide which descriptive statistic best represents the central tendency of the data and then use that statistic as the basis for redrawing Figure 10.

Students will notice that although the bar graphs posted around the classroom vary, they do fit into a pattern or shape, and that this pattern is well represented by the bar graph they generate using their chosen descriptive statistic.

So why does this pattern occur? The question can be answered using theoretical probability.

Method 2: Theoretical Probability

Although experimental probability is calculated using experiments, theoretical probability is calculated by determining all possible outcomes. For example, when asked why the sum of 7 occurs more often than the sum of 3, students will say that there are more ways to make 7 with two number cubes than there are to make 3. To calculate the *exact* probabilities, they need to determine all the different ways of getting each sum using two number cubes.

It is helpful for students to have number cubes of two different colours when they try to determine all the possible outcomes. The colours help them to see the difference between the sum of 7 with a red 3 and a blue 4, and the sum of 7 with a blue 3 and a red 4, and show them that $3 + 4$ and $4 + 3$ are counted as separate outcomes.

						6 + 1					
					5 + 1	5 + 2	6 + 2				
				4 + 1	4 + 2	4 + 3	5 + 3	6 + 3			
			3 + 1	3 + 2	3 + 3	3 + 4	4 + 4	5 + 4	6 + 4		
		2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	3 + 5	4 + 5	5 + 5	6 + 5	
	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	2 + 6	3 + 6	4 + 6	5 + 6	6 + 6
1	2	3	4	5	6	7	8	9	10	11	12

Figure 11. Listing all possible outcomes

The activity can be extended in a variety of ways:

- by using 3 dice instead of 2;
- by changing the numbers on the number cubes (for example, altering a number cube to show the numbers 1, 1, 2, 2, 3, 3 instead of 1, 2, 3, 4, 5, 6);
- by using two octahedra (8 sides each) or two dodecahedra (12 sides each) instead of two cubes.

For an interactive exploration of such extensions, see the Dice Explorations activity at <http://publish.edu.uwo.ca/george.gadanidis/dice.htm>.

CHARACTERISTICS OF STUDENT LEARNING AND INSTRUCTIONAL STRATEGIES

Junior students who are developing an understanding of experimental probability should have hands-on experiences in which they learn that:

- the number of trials affects the accuracy of the results of a probability experiment;
- the probability of an event can be represented by 0 or 1 or a decimal or fraction between 0 and 1;
- carrying out a probability experiment involves asking a clear question and carefully selecting appropriate methods of data collection and organization.

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Learning Activities

Introduction to the Learning Activities

The following learning activities for Grades 4, 5, and 6 provide teachers with instructional ideas that help students achieve some of the curriculum expectations related to data management and probability. The learning activities also support students in developing their understanding of the big ideas outlined in the first part of this guide.

The learning activities do not address all concepts and skills outlined in the curriculum document, nor do they address all the big ideas – one activity cannot fully address all concepts, skills, and big ideas. The learning activities demonstrate how teachers can introduce or extend mathematical concepts; however, students need multiple experiences with these concepts to develop a strong understanding.

Each learning activity is organized as follows:

OVERVIEW: A brief summary of the learning activity is provided.

BIG IDEAS: The big ideas that are addressed in the learning activity are identified. The ways in which the learning activity addresses these big ideas are explained.

CURRICULUM EXPECTATIONS: The curriculum expectations are indicated for each learning activity.

ABOUT THE LEARNING ACTIVITY: This section provides guidance to teachers about the approximate time required for the main part of the learning activity, as well as the materials, math language, and instructional groupings for the learning activity.

ABOUT THE MATH: Background information is provided about the mathematical concepts and skills addressed in the learning activity.

GETTING STARTED: This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or task.

WORKING ON IT: In this part, students work on the mathematical task, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.

REFLECTING AND CONNECTING: This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.

TIERED INSTRUCTION: These are suggestions for ways to meet the needs of all learners in the classroom.

HOME CONNECTION: This section is addressed to parents or guardians and includes a task for students to do at home that is connected to the mathematical focus of the learning activity.

ASSESSMENT: This section provides guidance for teachers on assessing students' understanding of mathematical concepts.

BLACKLINE MASTERS: These pages are referred to and used throughout the activities and learning connections.

Grade 4 Learning Activity

Too Much TV

OVERVIEW

In this learning activity, students conduct an investigation by collecting and organizing data on the number of hours each classmate watches television in a week. After creating a stem-and-leaf plot of the information gathered, they find the median. They also collect data from students in other classes and compare their results on a double bar graph. They go on to develop an advertising campaign to encourage their schoolmates to watch less television and to be more active. Their arguments will be based on the data they have collected. This investigation demonstrates how graphs are used in the real world and shows how mathematics can be applied to very practical situations.

In Grade 3, students learned to sort information into categories, conduct a simple survey, and display data in tables, charts, and horizontal and vertical bar graphs. They already know how to identify the mode or modes within a set of data. Later, they will learn how to determine the mean within a set of data and how to expand the collection and organization of data to include continuous data. This is students' first exploration of the wider concept of central tendency and the effect upon it of the various values in a data set.

BIG IDEAS

Collection and organization of data; data relationships

CURRICULUM EXPECTATIONS

COLLECTION AND ORGANIZATION OF DATA

This learning activity addresses the following **specific expectations**.

Students will:

- collect data by conducting a survey or an experiment to do with themselves, their environment, issues in their school or the community, or content from another subject, and record observations or measurements;
- collect and organize discrete primary data and display the data in charts, tables, and graphs (including stem-and-leaf plots and double bar graphs) that have appropriate titles, labels, and scales that suit the range and distribution of the data, using a variety of tools.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- collect and organize discrete primary data and display the data using charts and graphs, including stem-and-leaf plots and double bar graphs.

DATA RELATIONSHIPS

This learning activity addresses the following **specific expectations**.

Students will:

- read, interpret, and draw conclusions from primary data and from secondary data, presented in charts, tables, and graphs (including stem-and-leaf plots and double bar graphs);
- demonstrate, through investigation, an understanding of median, and determine the median of a set of data;
- describe the shape of a set of data across its range of values, using charts, tables, and graphs;
- compare similarities and differences between two related sets of data, using a variety of strategies.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- read, describe, and interpret primary data and secondary data presented in charts and graphs, including stem-and-leaf plots and double bar graphs.

TIME:

- 1 week for students to track their television viewing time
- 5 one-hour sessions

ABOUT THE LEARNING ACTIVITY

MATERIALS

- **DMP.BLM4a.1: Weekly Television Viewing Averages** (1 per student)
- data collection materials, such as clipboards, paper, and so on (per pair and then per group)
- sheets of chart-size square grid paper (1 per student)
- **DMP.BLM4a.2: Anticipation Guide** (1 per student)
- **DMP.BLM4a.3: Facts–Question–Response Chart** (1 per student)
- **DMP.BLM4a.4: Television Viewing Habits** (1 per student)

MATH LANGUAGE

- | | |
|------------|----------------------|
| • collect | • primary data |
| • organize | • secondary data |
| • graph | • median |
| • data | • stem-and-leaf plot |
| • survey | • bar graph |
| • table | • mode |

ABOUT THE MATH

INSTRUCTIONAL GROUPING:

pairs, small groups of 4–6 students, whole class, and individual work

TYPES OF DATA

- *Categorical data* are data that can be sorted by type or quality rather than by measured or counted values.
- *Discrete data* are data that can include only certain numerical values (often whole numbers) within the range of the data.
- *Primary data* are pieces of information collected directly or first-hand.
- *Secondary data* are pieces of information not collected first-hand.

THE MEDIAN

The concept of *median* is introduced in Grade 4. Students can determine the median by ordering the data from lowest to highest value. The median is then the middle number of this ordered set of values. For an even number of data, the midway point (average) between the two numbers in the middle is the median.

STEM-AND-LEAF PLOT

The stem-and-leaf plot is used to show the frequency with which certain ranges of values occur. For example, the plot on the right shows the following ages: 25, 34, 21, 37, 42, 48, 39, 34, 51. Notice that the stem-and-leaf plot shows the “shape” of the data.

Stem	Leaf
2	1 5
3	4 4 7 9
4	2 8
5	1

Stem-and-leaf plot of ages

GETTING STARTED

INTRODUCING THE PROBLEM

Tell students that you have just read an article that discusses the positive and negative effects that watching television has on children. You are wondering how many hours students in your school spend watching television each week. Ask students to predict the following:

- “How many hours do you watch TV in a week?”
- “What is the average for our class?”
- “Will our class average be different from the averages for students in lower grades or higher grades? How? Why?”

Note: Canadians watch television for more than 20 hours per week on average.

Facilitate a sharing of ideas and reasons.

Provide students with **DMP.BLM4a.1: Weekly Television Viewing Averages**, which shows the average hours of television viewing by province and by age range. Tell students that they will be conducting a survey of their schoolmates to determine how much TV they watch and whether there are differences between grades. Before they conduct this survey, students will collect data from their own class.

WORKING ON IT

Pose the problem:

“Does the amount of time students spend watching television weekly increase, decrease, or stay the same as they move up through the grades in our school?”

DESIGNING THE SURVEY

Ask students to design the survey they will use to collect the data they need to answer this question.

Students may encounter the following difficulties when designing their survey:

- Times may be reported in different and incompatible formats, such as 10.5 hours, 10 1/2 hours, 630 minutes.
- If they pose a question such as “How much television do you watch in a week?” answers might be qualitative (a lot, too much, I don’t know).
- If times are unspecified, some responses might cover only weekdays while others might reflect viewing time over a seven-day week.

Students need to experience these difficulties first-hand and work through the “messiness” of fine-tuning questions in order to collect the data they need. Guiding them through the experience, rather than giving them prepared questions that will avoid these pitfalls, will result in more enduring learning. Have students survey their classmates first. This experience will help them prepare and hone the question that they will use to survey the rest of the school.

COLLECTING THE DATA

Start by having students track the amount of time they themselves watch television in a week. Then ask students to work with a partner and design a survey to collect data on the amount of time each of their classmates watched television in that week. In addition to the survey question or questions, have the partners decide on the instrument they will use to gather the data. They could consider a class list or a table. At the end of the week, set aside 20 to 30 minutes to let students survey their classmates.

After each pair of students has gathered information on all classmates, including themselves, lead a discussion on the process of collecting the data, on the difficulties encountered, and on lessons learned. Ask partners to refine their question.

Lead a class discussion to decide upon the question and data collection tool to be used to survey other students in the school. To keep the data manageable, divide the class into groups of 4 to 6 students and have each group survey students in one other class.

Arrange to have the class do a survey of one class per grade between Kindergarten and Grade 6 (including their own class) or of six classes if the school has many combined classes.

Introduce the Stem-and-Leaf Plot. Teach students how to construct a new type of graph: the *stem-and-leaf plot*. The example on the right illustrates how this organizer can be used to display the data that students have collected.

Stem	Leaf
12	1/4, 1/4, 1/2, 3/4
13	0, 1/4, 1/2, 3/4, 3/4
14	1/2, 1/2
15	0, 0, 3/4

Introduce the Median. The median is one of three measures of central tendency, the mode and mean being the other two. Used in combination, the three values give the most accurate picture of average for a set of data. The median is the physical centre of a set of data. To illustrate this point, ask students to record on chart-size square grid paper, in ascending order, the number of hours of television viewing collected from their classmates. The square grid paper will help ensure that the values are equally spaced. By folding the piece of paper in half, students will find the central value, or median. If the set has an even number of values and there two values in the middle, the median is calculated by finding the halfway point between the two. In the following example, the median is 18 in both cases.

CASE 1: ODD NUMBER OF VALUES

2	4.5	7	11	18	19	20	21	25
---	-----	---	----	----	----	----	----	----

CASE 2: EVEN NUMBER OF VALUES

2	4.5	7	11	17	19	19.5	20	21	25
---	-----	---	----	----	----	------	----	----	----

Once students have explored the concept of median, divide the class into groups of 4 to 6 for the next task.

ANALYSING THE DATA

Ask students to work in their groups and use the stem-and-leaf plot graphs they have created to record their observations about the data they have collected. Use the following guiding questions to help focus the discussion and promote higher-level thinking:

- "What is the general shape of the data?"
- "How are the data spread out? Are there many clusters? Are there unusually high or low values?"
- "What is the mode in this set of data? How did you find it? What does the mode tell us?"
- "What is the median in this set of data? How did you find it? What does the median tell us?"
- "What is the range of values your group thinks is typical?"
- "What can you say about the number of hours you yourself spend watching television in a week? Are the hours similar to those of most of your classmates?"
- "Will the number of hours that students spend watching television decrease, increase, or stay the same as they get older? What is your group's prediction?"

ANSWERING THE QUESTION

To determine whether students watch more television as they grow older, it is necessary for students to compare two sets of data. The mode and median can be used as comparators. Have groups organize the collected information on a separate bar graph for each grade. They can compare Kindergarten data with Grade 6 data, Grade 1 data with Grade 2 data, and so on, by creating double bar graphs in which two sets of data are shown side by side. Use the following starter questions to help them explore the relationships in the data:

- “Which graph, the stem-and-leaf plot or the bar graph, is most useful in representing and analysing the data you collected?”
- “According to your group’s predictions, which age group watches the most television?”
- “Which age group actually watches the most television per week? Which watches the least?”
- “Does the number of hours significantly change from one grade to the next?”
- “If there is an increase or a decrease, is it steady and constant or does the number of hours move up and down across the grades?”
- “From the data we have collected, can you predict how many hours a typical Grade 8 student would spend watching television in a week?”

During this time, circulate among the groups and listen to their discussions. Ask more probing questions or lead students back to simpler ideas where necessary.

REFLECTING AND CONNECTING

Reconvene the class. Ask the groups to share their findings with one another. Discuss the relative usefulness of the measures of central tendency for this situation (for example, although the mode might be useful if the data are limited to whole hours, it is not useful if fractions or minutes are used).

Pose these questions to students:

- “Did the data support your predictions?”
- “What trends did your group observe when you compared the various class data?”
- “Which was more useful to you in determining trends: the mode or the median? Why?”
- “What reasons can you give to explain the shape of the data?”
- “Would you expect the data you have collected to be representative of other schools in our community? Across Ontario? Across Canada? Around the world?”

Ask students to use the collected data to answer the following non-mathematical questions:

- “What factors may affect the amount of time various age groups watch television in a week?”
- “What possible positive and negative effects might watching television have on a student’s health and fitness? General knowledge? Contact with family members? Involvement in other activities? Homework completion? Consumer habits? Contact with friends?”
- “Considering the data you collected, the research you did, and the answers you gave to the previous question, do you feel that you, your classmates, and/or your schoolmates watch too much television? If so, what steps would you like to take to communicate that message? If you don’t think you watch too much television, how could you encourage those who do to change their habits?”

Have the class design an advertising campaign to encourage classmates and schoolmates to watch less television and to be more physically active in their leisure time. Ask students to present arguments in their campaign that will make sense and be convincing, and to use the data they have collected to support their arguments.

TIERED INSTRUCTION

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

SUPPORTS FOR STUDENT LEARNING

- Teach vocabulary related to data management before the learning activity starts, and structure the activity to activate students’ background knowledge.
- See **DMP.BLM4a.2: Anticipation Guide**. To help activate students’ prior knowledge, create an anticipation guide containing three to five general statements to which students can respond, indicating whether they agree or disagree.
- Provide prompts or sentence starters to students who have difficulty with language and who may also have difficulty using mathematical language to share their ideas.
- For students who have not mastered previous skills (for instance, demonstrating an understanding of mode), refer them to the math strategy wall to reinforce previous teaching. The math strategy wall should consist of student-generated strategies.
- For students who have memory difficulties, provide instructions one at a time and check for understanding.

- For students who have difficulty persevering with a task, provide a visual checklist of each step in the activity.
- To help students decide what is important when they observe data, have them complete the **DMP.BLM4a.3: Facts–Question–Response Chart**. This strategy requires students to ask questions, determine the importance of the data, and respond using their own opinions and thoughts. The facts come from **DMP.BLM4a.1: Weekly Television Viewing Averages**.

EXTENSIONS

Surveying Adults. As a follow-up activity, students can survey adults in their own home and adults among family and friends to find out how many hours they spend watching television in a week. Students can organize and graph the results and ultimately compare them with the data collected from students of various age groups. Focus in-class discussions on whether the trends observed in the original data still hold true when survey results from adults are examined and compared with student survey results.

Exploring the Median. To extend the lesson, give students a number and identify it as the median from a set of data. Explain that their task is to describe what the values in the set might look like. Tell them that the set must have at least 10 values and that no value can be repeated. Students can choose to have an odd or an even number of values in the set. The same task can be set using the mode.

Change Over Time? Ask students to collect data again after one month (using the same question) and compare the original set of data with the new set. In a follow-up discussion, ask students to use the data to determine whether their advertising campaign has been successful.

HOME CONNECTION

Send home **DMP.BLM4a.4: Television Viewing Habits** with students. The initial task requires students to gather data related to their television viewing habits. The process by which they gather and record their data is intentionally open-ended to allow for student-generated ideas that can prompt meaningful discussion by the class when the data are being reviewed. Students will have opportunities to explain, justify, and compare their methods of data collection with those presented by their classmates. Consider a subsequent home connection in which students survey their parents on their present television viewing habits or ask them about their television viewing habits when they were younger.

ASSESSMENT

OBSERVATION

Observation and discussion guided by key questions can provide a good sense of student understanding. Consider the following:

- Can students collect, organize, and display primary data using a variety of graphs, including stem-and-leaf plots and double bar graphs?

- Can students distinguish among methods of data organization?
- Can students describe similarities and differences between two sets of data?
- Can students choose appropriate scales to display the range of data?
- Can students describe the shape of a set of data, using mathematical vocabulary?
- Can students read, interpret, and draw conclusions from primary and secondary data?
- Can students use a variety of tools to present data graphically?
- Can students find the median in a set of data?
- Can students compare the median and mode and explain the information these measures provide?
- Can students explain the *stability* of the median? In other words, what effect do extreme values in a set of data have on the median?

PERFORMANCE TASKS

The following are some possible assessment tasks for students:

- “Using the set of data provided, represent the data graphically, using a stem-and-leaf plot, a double bar graph, and one other graph of your choice. Explain which graphic representation would be most appropriate and why.”
- “Using multiple sets of data, calculate the median and mode and explain what information these measures give us.”
- “Using one set of data, manipulate the data to change the median, and describe the effect this change has on the mode and the median.”

RUBRIC

Assessment Category	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding				
– designs appropriate survey questions	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– collects and organizes data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– represents data using stem-and-leaf plots and double bar graphs	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– identifies similarities/differences in two sets of data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– determines the median in a set of data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
Thinking				
– creates a plan of action for conducting a survey	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– identifies and uses patterns when analysing data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– makes predictions based on data gathered	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– explores alternative methods of data collection and representation	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
Communication				
– explains mathematical thinking	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– communicates using a variety of modes (short answers, lengthy explanations, verbal and written reports)	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– uses appropriate vocabulary and terminology	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
Application				
– applies data management skills in familiar contexts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– transfers knowledge and skills to new contexts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– makes connections among concepts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree

WEEKLY TELEVISION VIEWING AVERAGES

	Canada	NL	PE	NS	NB	QC			ON	MB	SK	AB	BC
						Language							
						English	French	Total					
Total Population													
	21.4	22.7	20.0	22.7	23.7	20.6	23.8	23.3	20.6	22.1	21.2	19.4	20.7
Men													
18+	20.9	21.3	19.8	22.4	23.2	19.8	22.9	22.4	20.1	22.0	20.5	18.2	21.5
18–24	12.3	11.1	10.0	11.7	14.8	9.5	12.0	11.6	13.6	12.9	9.7	9.1	13.4
25–34	16.3	15.7	15.6	18.3	19.7	15.7	17.0	16.8	15.2	18.2	16.6	15.0	18.9
35–49	18.3	21.4	20.3	20.6	20.8	16.5	20.0	19.4	17.5	20.5	19.7	16.8	18.1
50–59	23.4	23.6	20.8	24.1	24.4	22.0	25.4	24.7	22.6	23.1	21.7	21.3	24.4
60+	31.1	27.5	26.5	31.2	32.1	29.7	37.0	35.4	29.7	31.5	28.9	28.5	30.3
Women													
18+	25.6	26.8	23.5	27.2	28.4	24.2	29.2	28.5	24.7	26.4	25.7	23.9	23.4
18–24	14.9	17.6	11.7	17.3	15.7	9.9	16.1	15.4	14.6	16.1	15.2	15.3	13.1
25–34	20.8	26.6	20.6	21.2	26.8	18.5	22.4	21.6	20.2	22.0	21.7	20.9	19.1
35–49	22.6	26.6	23.3	25.9	25.4	20.2	24.9	24.2	21.3	24.6	23.6	21.6	22.1
50–59	28.3	25.0	26.0	30.3	29.3	27.7	32.3	31.6	28.3	27.2	26.2	26.9	24.0
60+	35.6	32.2	28.9	33.8	36.9	34.1	42.0	40.7	34.4	35.1	34.2	32.8	32.0
Teens													
12–17	12.9	12.3	12.3	13.8	12.6	13.4	13.7	13.5	13.2	13.0	12.7	12.4	11.7
Children													
2–11	14.1	18.9	14.5	12.9	14.7	14.2	14.3	14.3	13.5	15.5	15.2	14.1	14.4

Source: Statistics Canada. 2006. "Average hours per week of television viewing, by province, and age/sex groups: Fall 2004" (table). *Television Viewing: data table*. Statistics Canada Catalogue no. 87-F0006-XIE. www.statcan.ca/bsolc/english/bsolc?catno=87F0006XIE (accessed October 25, 2007).

Note: For Quebec the language classification is based on the language spoken at home. The Total column includes those respondents who did not reply to this question or who indicated a language other than English or French.

ANTICIPATION GUIDE

Before		Statement	After	
Agree	Disagree		Agree	Disagree
		Kindergarten students watch more television than Grade 1 students.		
		Grade 1 students watch less television than Grade 6 students.		
		Some students don't watch television.		
		There is a difference between the amount of television that boys watch and that girls watch.		
		Watching a video is the same as watching television.		
		The amount of television that I watch is about average for my age.		

FACTS–QUESTION–RESPONSE CHART

Facts	Question	Response
Children age 2 to 11 years living in Ontario watch 13.5 hours of television per week.		
Children age 2 to 11 years living in Newfoundland and Labrador watch 18.9 hours of television per week.		
Teens living in British Columbia watch less television than teens living in Ontario.		

TELEVISION VIEWING HABITS

Dear Parent/Guardian:

In math, we are learning about the collection and organization of data. Over the next few weeks your child will be working on a number of tasks to do with gathering data, organizing data, and presenting the data graphically, using a variety of tools.

To begin this unit, we are asking students to keep track of how much television they watch over a seven-day period. It is important that your child think about how to collect and organize the data, as the method used will be discussed in class. Please take the time to discuss with your child why he or she chose certain ways to report the data and to discuss why the information varies on certain days. Having had the chance to explain this thinking to you at home will definitely make your child more confident and enthusiastic about the discussion at school.

After the class discussion, students will have opportunities to present their data graphically and to compare their data with the information collected by their classmates.

If you have any questions regarding this task, please do not hesitate to contact me.

Thank you for your continued support.

Sincerely,

Grade 4 Learning Activity

Heads or Tails?

OVERVIEW

In this learning activity, students participate in a discussion related to the “Lucky Loonie” buried at centre ice in the hockey arena in Salt Lake City before the 2002 Winter Olympics. Following the discussion, students make predictions and conduct a simple experiment involving the tossing of 1 and 2 coins.

Students will have had previous experience with the mathematical language of probability. They are able to predict the frequency of an outcome in a simple probability experiment or game, and then perform the experiment and compare the results with their predictions. They have an understanding of fair games and relate this to the occurrence of equally likely outcomes.

BIG IDEA

Probability

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- predict the frequency of an outcome in a simple probability experiment, explaining their reasoning; conduct the experiment; and compare the result with the prediction.
- determine, through investigation, how the number of repetitions of a probability experiment can affect the conclusions drawn.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- predict the results of a simple probability experiment, then conduct the experiment and compare the prediction to the results.

Note: There is a strong relationship between probability and data management. Students develop an understanding of probability by collecting and analysing data from experiments and investigations and by exploring patterns in the frequency of outcomes.

TIME:

2 to 3 days

ABOUT THE LEARNING ACTIVITY

MATERIALS

- chart paper
- *A Loonie for Luck*, by Roy MacGregor (1 copy)
- **DMP.BLM4b.1: Anticipation Chart for Coin Toss** (1 per student)
- nickels, dimes, quarters, and loonies (1 of each coin per pair of students)
- squares of felt to minimize the sound of coins being tossed on desks (1 per pair of students)
- **DMP.BLM4b.2: Heads or Tails?** (1 per student)
- **DMP.BLM4b.3: Recording the Coin Toss Outcomes** (as an overhead transparency or reproduced on the board or on chart paper)
- **DMP.BLM4b.4: Two Coins Toss Race** (1 per student)
- **DMP.BLM4b.5: Will It or Won't It?** (1 per student)

MATH LANGUAGE

- | | |
|--------------|---------------------|
| • probable | • certain |
| • improbable | • prediction |
| • impossible | • experiment |
| • possible | • outcome |
| • likely | • equally probable |
| • unlikely | • predicted results |

INSTRUCTIONAL
GROUPING:

pairs, whole class

ABOUT THE MATH

Students explore the outcomes when tossing two coins. They will discover that the following outcomes have different probabilities: 2 heads ($\frac{1}{4}$ or 25 percent); 2 tails ($\frac{1}{4}$ or 25 percent), and 1 tail and 1 head ($\frac{1}{2}$ or 50 percent).

GETTING STARTED

A LOONIE FOR LUCK

On chart paper, start a KWL chart and have students brainstorm what they know about the loonie that was secretly buried at centre ice in the hockey arena in Salt Lake City before the 2002 Winter Olympics. Record this information in the K column of the chart. Next, ask students what they wonder or want to know about this topic. Record their answers in the W column. Read the foreword (by Wayne Gretzky) to *A Loonie for Luck* by Roy MacGregor (Toronto: McClelland and Stewart: 2003, pp. 5–11) to students or have students take turns reading sections aloud. After the reading (which will take about 20 minutes), discuss the foreword with the whole class (for about another 20 minutes). After the discussion, record in the L column the information students learned. Discuss the completed KWL chart with the class.

If you do not have access to the book, use this summary:

The story *A Loonie for Luck* is all about the lucky loonie secretly buried at centre ice by Edmonton ice technician Trent Evans during the 2002 Winter Olympic Games in Salt Lake City, Utah. Hockey legend Wayne Gretzky writes in the book's foreword about how this loonie has become "folklore" in the world of hockey. He believes that this particular loonie is a symbol of the love that Canadians have for the game of hockey. He states that hockey is a game for all Canadians, not just the famous hockey players. He writes, "Here was Trent Evans, a perfectly average guy, a Zamboni driver from Edmonton – and how much more Canadian can you get than that? – who had just become a true hero in his own right" (p. 5).

Many Canadians felt that the loonie buried at centre ice brought the Canadian men's and women's hockey teams good luck during that amazing series. Before the public was let in on the secret of the buried loonie, players from the women's team started kissing the ice after they'd won their gold. Gretzky states, "It wasn't so much about good luck – though we were glad to have any help we could get – as it was about the country as a whole being a part of this wonderful Olympic experience, thanks to Trent Evans" (pp. 5–6).

The men's team also went on to win the gold. Canadians were very proud of their athletes!

The "lucky loonie" eventually became a permanent exhibit at the Hockey Hall of Fame in Toronto and has been viewed by thousands of proud Canadians and visitors from other countries.

CAN A COIN BRING LUCK?

Ask students: "Can a coin really bring you luck? What do you think?" Discuss the word *lucky*.

Explain that hockey players often use a "lucky jersey" or a "lucky towel" when on the road or playing a home game. Some players have to skate for luck around the ice surface in a particular direction when warming up before a game. During playoff games some hockey players refuse to shave and grow a beard for luck. Other players wear their practice jerseys inside out for luck.

Have students talk about their own experiences with "lucky" objects or routines. Ask: "Are there particular objects that you feel have helped you through difficult experiences?"

Discuss coins and how people have often labelled a particular coin "lucky" because something good occurred when the coin was present or in their possession.

Note: The timing and sequencing of this activity will depend on students' prior knowledge of probability. Students have a natural curiosity about how the trials will turn out. They understand that when using a fair coin, two outcomes are equally likely: heads or tails. Students will ask, "What is the chance that the coin will land on heads?" As they gather data from more and more trials, they will begin to use numbers in their discussion of results.

WORKING ON IT

SINGLE COIN TOSS EXPERIMENT

Before students start collecting data from the coin tosses, have them complete the “Before” side of **DMP.BLM4b.1: Anticipation Chart for Coin Toss**. They will complete the “After” side once they have finished the activity.

Ask students if they feel there are any variables that would affect the outcome:

- “If you flip different coins (i.e., nickel, dime, quarter, loonie), will the outcomes be different? Why?”
- “Suppose a quarter is tossed three times and each time the result is heads. Which result is more likely on the fourth toss – heads or tails?”

Give each pair of students a nickel, a dime, a quarter, and a loonie. Ask them to toss each coin 10 times and record their results, heads (H) or tails (T). They can use **DMP.BLM4b.2: Heads or Tails?** for this purpose. Once they have finished collecting data for each coin, ask them to add their data to the class chart (**DMP.BLM4b.3: Recording the Coin Toss Outcomes**). You may find it helpful to have this chart on an overhead transparency, so that the results can be easily seen by the whole class. Alternatively, the chart can be reproduced on the board or on chart paper.

DISCUSSION OF RESULTS

Note: To understand probability, students need to be able to identify all the events (also called “the sample space”) that might occur – a task that is often a challenge for them. The sample space for this activity is (H, T) because when a coin is tossed, it will land on either heads or tails.

Reconvene the class after students have completed their data collection. Facilitate a discussion to explore whether regular coins (such as Canadian currency) can really be lucky. Do lucky coins really exist?

Students will most likely conclude that Canadian currency coins are fair, as opposed to lucky, coins. In a fair coin toss it is equally probable that the coin could land on either heads or tails; it is therefore impossible to predict the result of any single coin toss.

Ask students if the number of trials can affect the overall outcome.

Have students take some time to review and discuss their results. Ask them to talk about the number of heads and tails they tossed for each coin. Did they notice any surprises in their results? Ask them to comment on the variance in data recorded in **DMP.BLM4b.1: Anticipation Chart for Coin Toss**.

Ask students to use the information they have gathered from their data to make predictions of what might happen with an increased amount of data. Post student responses to the following questions:

- “If you toss a quarter 20 times, will it land on heads 10 times and tails 10 times?”
- “If you toss a nickel 200 times, how often do you think it will land on heads?”
- “If you toss a dime 1000 times, how often will it land on tails?”

Ask students if they think that it is equally likely that they will toss heads or tails if they complete a large number of trials. Draw their attention to the totals in **DMP.BLM4b.3: Recording the Coin Toss Outcomes** and ask how these results are different from or similar to their own results. Look at the whole-class results for each coin. A class of 26 students (13 pairs) will conduct 130 trials in total for each coin. Students should notice that although the results for the pairs vary (some are close to 5 heads and 5 tails, and some are not), the totals, which represent a greater number of trials, will be closer, on average, to a result of 50 percent heads and 50 percent tails.

TWO COINS TOSS RACE

Have students work with a partner to complete **DMP.BLM4b.4: Two Coins Toss Race**, tossing two coins and recording the result (2 heads, 1 head and 1 tail, or 2 tails) of each toss by shading appropriate squares in the horizontal bar graph. Frame this activity as a “race”. Ask students to predict which result or bar will win the race: Will it be 2 heads, 1 head and 1 tail, or 2 tails? Have students complete five races, and then work with their partners to answer the following questions:

- “What is the probability of tossing two heads?”
- “What is the probability of tossing a head and a tail?”
- “What is the probability of tossing two tails?”

Invite pairs to present and justify their results. Discuss the various results with the class.

Some students may already have totalled the results from their five races and may have used this total to justify their reasoning in the class discussion. Ask all students to total their results and consider the implications. They will see that, based on the totals, the probability of tossing 2 heads is approximately 25 percent or $\frac{1}{4}$, the probability of tossing 1 head and 1 tail is approximately 50 percent or $\frac{1}{2}$, and the probability of tossing 2 tails is approximately 25 percent or $\frac{1}{4}$.

REFLECTING AND CONNECTING

Explore this question as a class: “With a single coin, the outcomes were equally likely. Why were they not equally likely with two coins?” Guide the discussion to focus on the following facts:

- For a single coin:
 - there are 2 possible outcomes of a toss: H or T;
 - each outcome is equally likely.

- For two coins:
 - there are 4 possible outcomes of a toss: HH, HT, TH, TT;
 - each outcome is equally likely.
- In students' experiment, the order of the coins was not important, so the category of "1 head and 1 tail" was used for both HT and TH, which explains the probability of $\frac{1}{2}$ for 1 head and 1 tail.

TIERED INSTRUCTION

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

SUPPORTS FOR STUDENT LEARNING

- Teach vocabulary related to probability before starting the learning activity.
- Help students activate their prior knowledge by having them complete **DMP.BLM4b.1: Anticipation Chart for Coin Toss**.
- Use sentence starters when students have difficulty applying knowledge to new situations:
 - I think ...
 - It could be ...
 - It's because ...
 - It means that ...
 - I'm guessing ...
 - I think ... is going to happen because ...

Provide prompts or sentence starters to students who have difficulty with language and who may also have difficulty using mathematical language to share their ideas. Students who have difficulty with organizational skills may need guidance in choosing strategies to help them solve the problem.

- All students can benefit from the think-aloud strategy but particularly those who have difficulty processing information. Think-alouds allow these students to verbalize their thinking, demonstrate problem-solving strategies, and model mathematical language.

EXTENSIONS

Software. Have students explore probability by conducting multiple trials using software. For examples, see http://nlvm.usu.edu/en/nav/frames_asid_305_g_3_t_5.html.

Tossing Three Coins. Have students toss three coins and explore the probability of possible outcomes: 3 heads, 2 heads and 1 tail, 1 head and 2 tails, and 3 tails.

Sports. Ask students to use a variety of resources to investigate the use of coin tosses in different sports. Which sports use coin tosses? If one team wins the toss, what does that win allow the team to do? After students have completed their research on the topic, ask them to reflect on the significance of their findings and report back to their classmates.

A Fair Game. Ask students to create a fair board game that uses coin tosses. For example: The game will have four players; each player will toss two like coins; if the two coins land heads/heads, the player gets 3 points; if the coins land tails/tails the player gets 2 points; if the coins land heads/tails the player gets 1 point; the player who gets 30 points first wins the game.

HOME CONNECTION

See **DMP.BLM4b.5: Will It or Won't It?**

ASSESSMENT

OBSERVATION

Collect observational data using class lists or sticky notes during the coin toss activities. Focus on these areas:

- *Problem solving:* Are students able to solve the following problems? (1) Can regular coins (e.g., Canadian currency) really be lucky? (2) Do lucky coins really exist?
- *Reasoning and proving:* Are students able to make sense of the mathematics they are doing? Are they able to use the evidence they are gathering from their coin tosses to support or disprove their responses on **DMP.BLM4b.1: Anticipation Chart for Coin Toss?**
- *Selecting tools and strategies:* Are students able to organize their ideas using any of the strategies discussed earlier (i.e., guess and check, draw a picture, make a list)? Are they able to complete computational strategies in order to organize data?
- *Connecting:* Are students able to make connections to other math strands or other subject areas? Do they say, "This reminds me of ..." or "I wonder what would happen if ..."?
- *Representing:* Are students able to represent their thinking in a variety of ways (for instance, by graphing the results of their coin tosses, making a table of their results, drawing a picture of their results)? Are they able to write a paragraph on the following: "If you tossed a loonie 75 times, how many times would you expect it to land on tails?" Are they able to organize their results in a way that is conducive to analysis, or are their results simply recorded in the order in which they occurred?
- *Communicating:* Are students using a think-aloud strategy to share their ideas? Do they correctly use the language of probability? Do they ask questions of themselves and others during the actual coin toss and during the reflection discussion? Are they respectful of others when they are sharing their ideas?

ASSESSMENT QUESTIONS

- "Can regular coins (e.g., Canadian currency) really be lucky? Do lucky coins really exist? Explain why you think they do or do not."
- "Abby tossed a coin 20 times, getting 14 tails and 6 heads. What conclusions can you draw from these results? What are your observations regarding Abby's results?"
- "If you tossed a coin 300 times, how often do you think it would land on tails? Why?"

RUBRIC

Assessment Category	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding				
– collects and organizes data from probability experiments	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– determines the probability of outcomes based on probability experiments	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– identifies similarities/differences in conditions and results of different probability experiments	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
Thinking				
– creates a plan of action for conducting an experiment	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– identifies and uses patterns in analysing data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– makes predictions based on data gathered	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– explores alternative methods of conducting probability experiments	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
Communication				
– explains mathematical thinking	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– communicates using a variety of modes (short answers, lengthy explanations, verbal and written reports)	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– uses appropriate vocabulary and terminology	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
Application				
– applies probability skills in familiar contexts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– transfers knowledge and skills to new contexts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– makes connections among concepts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree

ANTICIPATION CHART FOR COIN TOSS

Before		Statement	After	
Agree	Disagree		Agree	Disagree
		A dime is luckier than a quarter.		
		A loonie is the luckiest coin of all.		
		In 10 tosses it is likely that heads will come up 5 times.		
		In 100 tosses it is likely that 50 tails will come up.		
		In 1000 tosses it is likely that 500 heads will come up.		
		In 5000 tosses it is likely that more heads will come up.		

HEADS OR TAILS?

Trial	Nickel	Dime	Quarter	Loonie
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

RECORDING THE COIN TOSS OUTCOMES

	Nickel		Dime		Quarter		Loonie	
	H	T	H	T	H	T	H	T
Pair 1								
Pair 2								
Pair 3								
Pair 4								
Pair 5								
Pair 6								
Pair 7								
Pair 8								
Pair 9								
Pair 10								
Pair 11								
Pair 12								
Pair 13								
Pair 14								
Pair 15								
Pair 16								
Totals								

TWO COINS TOSS RACE

2 heads											
1 head, 1 tail											
2 tails											

2 heads											
1 head, 1 tail											
2 tails											

2 heads											
1 head, 1 tail											
2 tails											

2 heads											
1 head, 1 tail											
2 tails											

WILL IT OR WON'T IT?

Dear Parent/Guardian:

We are just beginning a unit on probability, which is a very important part of our lives. Every day we consider the weather and wonder about the chances of rain, snow, or sunshine. We wonder if our favourite hockey team is likely to win the Stanley Cup – and *how likely*. We wonder about our chances of winning a lottery prize.

We will be starting the unit with a discussion of the 2002 Winter Olympics hockey series in Salt Lake City. Canada's men's and women's teams both won the gold medal during those Games. Canadians were very proud of their athletes! One of the wonderful stories that came out of the Games was about the loonie buried at centre ice by Edmonton ice technician Trent Evans. Did that coin bring the Canadian teams luck? Is there such a thing as a lucky coin?

We will be discussing the "lucky loonie" and carrying out simple probability experiments based on tossing one or more coins. Students will explore the question of how likely it is that a coin will land on either heads or tails in a varying number of trials. They will complete a small number of trials and compare their collected data with larger sets of results.

You can help us by encouraging your child to use some of the mathematical language of probability at home. In our classroom investigations we will be using this language: *probable, improbable, equally probable, possible, impossible, likely, unlikely, likelihood, prediction, experiment, outcome, prediction, predicted results, and fair game*.

You may also want to play the following probability game with your child:

- Name two events that will likely happen at home tomorrow.
- Name two events that will likely *not* happen at home tomorrow.
- Name two events that will likely happen in your community this week.
- Name two events that will likely *not* happen in your community this week.

Ask other such questions and have your child record the responses. Check the accuracy of the predictions after the day or the week has gone by. Ask him or her to tell you more about probability.

Please let me know if you have any questions about our unit on probability. Thank you very much for your continued support.

Sincerely,

Grade 5 Learning Activity

Daily Physical Activity

OVERVIEW

In this learning activity, students conduct an investigation to help a school principal justify the purchase of new outdoor equipment for the school's daily physical activity initiative. Students will collect primary data on their own class's preferred outdoor activities and organize the data for presentation. The idea of a sample is introduced as an effective way of gathering information from a larger population – in this case, the school. Students then collect primary data from a sample of the school population. The *broken-line graph* is introduced and students are given opportunities to distinguish between *discrete* and *continuous data*. Multiple sets of data are compared for similarities and differences. Students explore the term *average* and learn to calculate the *mean* for a small set of data. They develop an awareness of the relevance of *mean*, *median*, and *mode* to the numbers in the set as they prepare their presentation.

This activity builds on existing skills in, and knowledge of, the creation of surveys and the organizing of data using tables and graphs, including horizontal and vertical bar graphs and stem-and-leaf plots. Students will already have had some experience in reading and interpreting graphs and in describing the shape of the data.

BIG IDEAS

Collection and organization of data; data relationships

CURRICULUM EXPECTATIONS

COLLECTION AND ORGANIZATION OF DATA

This learning activity addresses the following **specific expectations**.

Students will:

- distinguish between discrete data (i.e., data organized using numbers that have gaps between them, such as whole numbers, and often used to represent a count, such as the number of times a word is used) and continuous data (i.e., data organized using all numbers on a number line that fall within the range of the data, and used to represent measurements such as heights or ages of trees);
- collect data by conducting a survey or an experiment to do with themselves, their environment, issues in their school or community, or content from another subject, and record observations or measurements;
- collect and organize discrete or continuous primary data and secondary data and display the data in charts, tables, and graphs (including broken-line graphs) that have appropriate titles, labels, and scales that suit the range and the distribution of the data, using a variety of tools;

- demonstrate an understanding that sets of data can be samples of larger populations;
- describe, through investigation, how a set of data is collected and explain whether the collection method is appropriate.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- collect and organize discrete or continuous primary data and secondary data and display the data using charts and graphs, including broken-line graphs.

DATA RELATIONSHIPS

This learning activity addresses the following **specific expectations**.

Students will:

- read, interpret, and draw conclusions from primary data and from secondary data, presented in charts, tables, and graphs (including broken-line graphs);
- calculate the mean for a small set of data and use it to describe the shape of the data set across its range of values, using charts, tables, and graphs;
- compare similarities and differences between two related sets of data, using a variety of strategies.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- read, describe, and interpret primary data and secondary data presented in charts and graphs, including broken-line graphs.

ABOUT THE LEARNING ACTIVITY

MATERIALS

- computers and programs such as SmartIdeas or Inspiration (optional)
- sheets of large chart paper and square grid paper (several sheets of both types per group of students)
- large markers (several per group of students)
- data collection materials, such as clipboards, paper, and so on (per group)
- interlocking cubes (40 per student)
- sticky notes (5 per student)
- **DMP.BLM5a.1: A Picture Is Worth a Thousand Words** (1 per student)

TIME:

- 1 week to gather physical activity data throughout the school
- 4 one-hour lessons

MATH LANGUAGE

- median
- mode
- double bar graph
- stem-and-leaf plot
- primary data
- secondary data
- mean
- continuous data
- broken-line graph
- sample

INSTRUCTIONAL GROUPING:

pairs, small groups of 4–6 students, whole class, and individual work

ABOUT THE MATH

POPULATION

A population is the total number of individuals or items that fit a particular description (e.g., salmon in Lake Ontario, students in a school).

SAMPLE

A sample is a representative group chosen from a population and examined in order to make predictions about the population as a whole.

CONTINUOUS DATA

Continuous data are typically measurements of one kind or another: length, mass, volume, time, temperature, and so on. Continuous data can take on any numerical value, including decimals and fractions, represented on a number line between a minimum and a maximum value. Line graphs, as opposed to scatter plots or bar graphs, are typically used to represent continuous data.

DISCRETE DATA

Discrete data usually deal with things that can be counted using whole numbers, such as the number of students in a class, the number of pencils in a pencil case, or the number of words in a sentence. Unlike continuous data, there is a finite set of choices for the numbers used. It is possible to use a finite set of decimals or fractions as discrete data. For example, when recording the number of hours of TV watching, the options might include whole hours as well as half hours.

THE MEAN

Often referred to as the “average”, the mean is one of three ways to express average or central tendency. The other ways are the median and the mode. Used in combination, these three values provide a picture of a set of data.

GETTING STARTED

THE CONTEXT

Tell students that a school (maybe even your school) wants to purchase some outdoor equipment to supplement the resources already available for the school’s daily physical activity program. The school principal, who will be presenting the case at a budget meeting, needs information that will help determine which equipment to buy and will also provide justification

for the choice. The principal has asked this class to obtain the vital data and has provided two key questions to investigate:

- What type of equipment should be purchased?
- How much time do students spend on daily physical activity in each grade?

In preparation for an opening discussion on this activity, you might have students gather information from the following sources:

- *Canada's Physical Activity Guide for Children, Canada's Physical Activity Guide for Youth, and Eating Well with Canada's Food Guide*, available at www.healthycanadians.gc.ca/pubs_e.html;
- the website of Active Healthy Kids Canada at www.activehealthykids.ca/Ophea/ActiveHealthyKids_v2/index.cfm;
- *Daily Physical Activity in Schools, Grades 4 to 6*, a resource guide available in school or from the Ontario Ministry of Education Healthy Schools website (under Tools) at www.edu.gov.on.ca/eng/healthyschools/dpa.html;
- the website of the Canadian Association for Health, Physical Education, Recreation and Dance at www.cahperd.ca.

LINKING TO PRIOR KNOWLEDGE

Ask students to spend 10 minutes individually completing a concept map of what they know about data management. Consider having students use computer applications, such as SmartIdeas or Inspiration. Ask students to share their concept map with a partner, discuss the content, and compare ideas.

WORKING ON IT

MAKING A PLAN

Facilitate a discussion about how the information should be presented, guiding the talk to draw out what students already know about data management. Some discussion points could be:

- "Who is the target audience?"
- "What does the target audience know about the issue?"
- "What do you want the target audience to know?"
- "How will the target audience use the information?"
- "Is one aspect of the information more important than the rest?"
- "What is the most meaningful way to represent the collected information?"

Have students work in mixed ability groupings to consider the questions and prepare a proposal to be discussed in class. An intentionally open-ended discussion will allow you to assess students' level of understanding of the concepts. Ask them to consider the following questions:

- "How should we collect the data?"
- "How should we present the data?"
- "What should the data look like when we are finished?"
- "What information can we get from the data?"

DISCUSSION

Student discussion will most likely include the different types of graphs they are familiar with. The word *average* may or may not come up. Ideally, students will conclude that they should consider the two key questions separately. If they do not, prompt them to move in this direction. Suggest that they survey their own classmates to find out what their favourite outdoor activities are. These data will provide useful information on the types of equipment needed and will give them an idea of what the final survey question might look like.

SURVEY OF THE CLASS

Have students work in groups to formulate a question and to collect class data based on that question. Provide large sheets of chart paper, square grid paper, and markers so that the work can be easily organized and seen. Have the groups graphically present the data in a manner they choose. While this work is going on, ask each group about the choices they are making and about any difficulties encountered, such as the need to refine their question or whether their thinking regarding how to represent the graph has changed and why.

PRESENTATIONS

As the groups present their survey question and data and justify their choice of data presentation, guide the discussion towards their choices and encourage them to relate any difficulties they encountered. Following the presentations, post the large sheets on the board or wall to allow students to compare and contrast the effectiveness of their choices. The activity allows them to see each question's effects on the data. If the question was "What is your favourite outdoor activity?", students might have replied "swimming" or "skiing", neither of which is a relevant response. Also, very open-ended questions, such as "Which outdoor activity do you like to do at school?", might offer too much choice, resulting in too many categories of data. Analysing the questions allows students to fine-tune their wording to answer the key question.

Looking at all the data presentations will allow students to see how the same data can be presented in different ways. If the presentations do not vary, suggest options for representing the data and explore these options with students.

SURVEY OF A SAMPLE OF THE SCHOOL POPULATION

Once the question has been fine-tuned, ask students to consider in their groups how they should go about gathering data from the rest of the school. Introduce the terms *sample* and *population*, and discuss why a sample is sometimes used. Advertisers, political parties, and businesses use the data from a sample to make judgements and predictions about the thinking of large populations. In just this way, data from a sample can be used to represent the thinking of the large population of the school. Ask students to consider the following questions:

- "If the marketing department of a large cosmetics company wants to find out whether the company's new shade of lipstick is successful, whom should it survey?"
- "A new chocolate bar is being developed and the company wants to run a taste trial to see if people like the new flavour. Who should be included among the sample taste-testers?"

- “Would the results of a survey asking parents which sports should be offered at a local sports facility be representative of the larger population?”

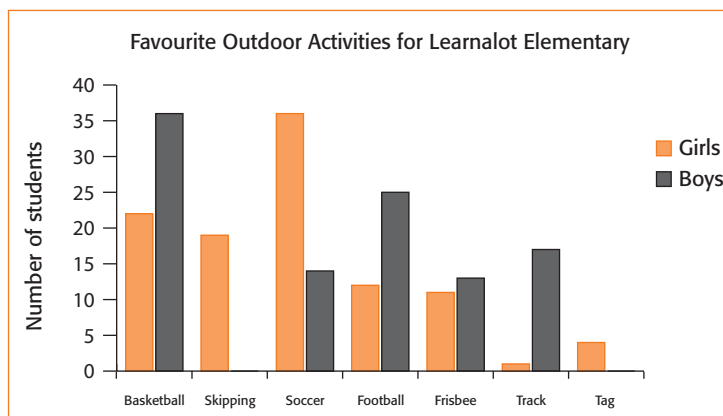
After students have had time to consider why a sample is used and why it must be representative of the population, ask the class to decide who should be included in their own sample to represent the wider population of the school. Suggest that a representative sample should include boys and girls – perhaps five boys and five girls from each class or grade, depending on the size of the school. Emphasize that everyone in the sample must be asked the same question. Then have students work in groups to collect their data.

REPRESENTING THE DATA

A double bar graph would be an accurate and meaningful way to display these data.

Have the groups create their double bar graphs to represent their own data. Then collate the data collected by all the groups and create a larger graph to represent the school population. Ask students to consider whether

they should graph the data separately for each grade or create a single graph for all data, or perhaps do both. Looking again at the question they are trying to answer will give them a better sense of which graph would be most meaningful.



ANALYSING THE DATA

Ask the groups to discuss and analyse the information provided by the data. Guide their discussions by asking:

- “Is the information representative of the school?”
- “What decisions can be made regarding the types of equipment the school should buy?”
- “Do you have enough information to make your decisions?”
- “How do the choices of favourite activities vary from grade to grade? Does this variation affect the outcome (i.e., the types of equipment to be purchased)?”
- “Can you see a pattern in the results?”
- “Can you explain any differences or similarities in the choices made by students in each grade?”
- “Is there a way to prove that your sample is representative?”
- “If we increased the sample to 10 boys and 10 girls from each class, what could we expect the new data to look like?”

To answer the second key question ("How much time do students spend on daily physical activity in each grade?"), students need to collect primary data from each class for a week. This information could be collected by surveying the teachers at the school. The table on the right shows the result of one such survey.

Mrs. Day's Grade 3 Class	
Day	Minutes Spent on Daily Physical Activity
Monday	16
Tuesday	25
Wednesday	22
Thursday	24
Friday	18

The Mean. Students now need to decide how best to present the data to the principal. Introduce the mean. The mean is one of three ways to express average. Finding the average amount of time spent on daily physical activity by each class in each grade will allow students to plot class averages on one graph. Explain that, to help them understand the concept, they will first practise finding the mean with the help of manipulatives. Methods 1 and 2 that follow describe two concrete ways to find the mean.

- **Method 1: Levelling Out**

Note: Before teaching the traditional algorithm of "dividing the sum of the numbers by the number of numbers in the set", be sure to provide students with many concrete opportunities to find the mean. Hands-on activities allow them to develop a deeper understanding of the value of the mean and its relationship to the other numbers in the set.

Provide students with a set of data (6, 10, 11, 4, 9) and interlocking cubes that they can use to make towers to represent each of the values.

Ask: "Can you rearrange the cubes so that all the towers have the same number of cubes?" Allow students some time to work on this problem, then, in a whole-class setting, ask them to share and discuss the strategies they used. Some students might take all the cubes apart and begin again, or might take cubes from the top to "level out" their towers. Others might put all the cubes together and then "divide" the result into equal groups. Students' work should show that each tower would have 8 cubes.

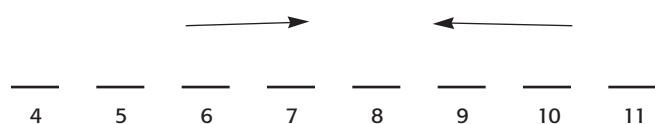
Provide students with more opportunities to practise "levelling out" so that they can develop their own efficient strategies. It is particularly important for them to have opportunities to work with sets whose towers *cannot* be levelled, leaving some cubes left over. Ask students: "What do these leftovers represent? How can we include them in our answer? For instance, if there are five towers of nine and there are three cubes left over, what should we do?"

Method 1 most closely matches the traditional algorithm for finding the mean; however, when the original towers are taken apart, students lose awareness of the values of the original data. Experience with both Method 1 and Method 2 is advisable.

- **Method 2: The Balance Point**

The mean is sometimes referred to as the *balance point*. Ask students to use the same set of values (6, 10, 11, 4, 9), and to place sticky notes on the corresponding number on a number line (one sticky note per number). The range of the numbers on the number line should correspond with the range of the data. Students must then “balance” the values by moving the numbers on the ends of the set towards the centre, with one move balancing the other. For example, if the sticky note on 10 is moved two spaces to the left, towards the centre, this move must be balanced by moving the 6 two spaces to the right, towards the centre, and so on, until all the sticky notes are on the same number. That number represents the mean.

Method 2 is not as closely linked with the algorithm but it demonstrates the concept of the mean as being in the centre of the data, and visually represents all the data. Method 2 also illustrates the effect that extreme values have on the mean.



Comparing the Mean, Median, and Mode. After students have had opportunities to explore the mean and its relationship to the rest of the data, ask them to compare the mean, median, and mode of a set of data. Through investigation, they will begin to see the unique information each “average” gives us. Observe their work and consider the following:

- Do students understand that the mean is somewhere in the middle of the set?
- Can students explain how extreme values in the set affect the mean but not the median or mode?
- Are students beginning to see how measuring each of these “averages” provides different information?

Using the Mean, Median, and Mode With the Survey Data. Have students find the mean, median, and mode for each set of class data collected. The nature of these class data allows students to easily find the mean using concrete methods. Each value is not far from the centre and each set will contain only five numbers. Once students have calculated the mean, median, and mode, ask them to write statements about how these “averages” can be used to describe the data. Discussing the validity of the data will provide students with opportunities to apply the conceptual understanding they have gained from the concrete activities explored earlier. Ask them to consider the following questions:

- “If a class had no physical activity on a certain day, should such data be included?”
- “How will such data affect the mean?”
- “Is the mean the most effective measure of ‘average’ in this case?”

Broken-line graph: A graph formed by line segments that join points representing the data. The horizontal axis represents discrete quantities, such as months or years, whereas the vertical axis can represent continuous quantities.

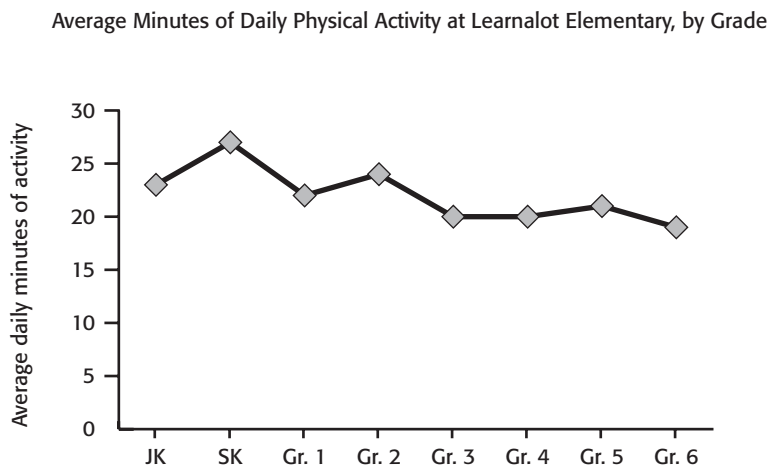
Graphing the Mean for Each Class.

The mean for each class can now be displayed on a graph. Introduce the *broken-line graph*. This type of graph allows the display of continuous data and can make apparent the trends and relationships between data.

Discuss the difference between the data collected on students' favourite

activities (discrete data) and the data collected on the time spent on daily physical activity.

Ask: "What is the best way to present our data?" Have students experiment with at least two types of graphs (e.g., a bar graph, a stem-and-leaf plot, a broken-line graph) and analyse the representations to see which graph best addresses the key question.



REFLECTING AND CONNECTING

Students should now have sufficient information to answer the key questions posed by the principal. Ask students to work in pairs, using the think-pair-share technique, to consider the information they have gathered. Tell them that their thoughts will later be shared during a whole-class discussion.

Encourage student discussion by asking:

- "Can you see a pattern in the averages?"
- "Can you use this information to predict responses for Grade 7?"
- "Can you explain the variations between grades?"
- "Which grade would you say is most typical of the school?"
- "Will the information provide sufficient justification for the purchase of more equipment?"
- "If you changed the scale on the graph, in what way would the graph look different?"
- "How does using a broken-line graph differ from presenting the information on a bar chart or a stem-and-leaf plot?"
- "If you used your sample technique in another school, would you expect to see the same results? Why or why not?"

WRITING A REPORT

Students' findings can now be presented to the principal. The format of this presentation should be determined by students themselves, as they have a personal stake in the outcome of the presentation. The Home Connection activity (see p. 78) will show them some of the many ways in which data are presented in the real world.

TIERED INSTRUCTION

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

SUPPORTS FOR STUDENT LEARNING

- All students will benefit from completing a concept map to help them organize their thoughts on what they already know about data management. They will make more connections if you allow them to decide how many categories, bubbles, titles, groups, and so on, they will need for their map. However, some students may find the open nature of this task too daunting and may benefit from having a template, a set of key words, or a partly completed template to help them get started.
- Create mixed ability groupings, which allow flexibility and provide support for those who need it. Vary group members on the basis of ability, interests, age, and participation and behavioural patterns, as the task allows. Such groupings give students the chance to hear the perspectives of a variety of students. Ensure maximum participation by all members by assigning clearly defined but changing roles within the group. You might hand out role cards, such as *scribe*, *presenter*, *discussion leader*, *group secretary* (in charge of gathering materials), *timekeeper*, and so on, using individual student strengths as your criteria. Occasionally reassign roles to give students experience in all of them.
- When introducing new subject-specific vocabulary, add it to a math word wall. Make sure that the words are easily visible to all students in the class. Give preferential seating close to the word wall to students who require it. Colour coding of words helps students to discriminate among words. Words can also be taken from the wall and placed on the board to highlight the vocabulary needed for a particular lesson. Individual copies of word wall lists can be given to students who would benefit from having a copy at their workspace.
- By modelling the strategies used for the two methods of finding the mean, you help students visualize the concepts and you also verbalize the math thinking. Students with auditory, kinaesthetic, or visual learning preferences benefit when concepts are presented with an auditory and a visual component or with manipulatives. Explicit modelling of the strategies enables all students to make connections between the strategy and their own understanding of the concept.
- The think-pair-share technique provides support for all students and allows all students to participate fully in the activity. Start by having students independently consider a task, then pair them with another student, and ask them to share their ideas. Have two pairs join to discuss the topic further.

EXTENSIONS

- Have students research prices of sports equipment linked to their survey results and prepare cost information for the principal. For example, students could use retail websites or sports catalogues to find five prices for each item and find the average or typical price of each item. They could present their primary data in a stem-and-leaf plot, calculate the average price, and then present prices for all the pieces of equipment in a graph of their choice. They could use presentation software for this purpose.
- Provide students with the mean and mode, and ask them to find the possible values in the set. For example, ask: "If the mean of a set of numbers is 12, what could the values be? Give three possibilities." Students who have had opportunities to explore the "levelling out" and "balancing" methods will be able to approach this task in a concrete manner. Solving the problem will allow them to deepen their conceptual understanding of mean and mode.

HOME CONNECTION

A Home Connection task might involve having students cut out examples of graphs from newspapers, magazines, and so on, and write notes about the kinds of information the graphs give them (see **DMP.BLM5a.1: A Picture Is Worth a Thousand Words**). Later, when students are determining the best way to present their collected information to the principal, the ideas gleaned from this Home Connection activity will allow them to make connections between the real world and what they are doing in their class.

ASSESSMENT

OBSERVATION

Regular observation and discussion will provide you with a good sense of student understanding. Consider the following questions:

- Can students explain why using a sample is an appropriate way of determining the thoughts of a larger population?
- Can students collect, organize, and display primary and secondary data using a variety of graphs, including stem-and-leaf plots, double bar graphs, and broken-line graphs?
- Can students interpret a broken-line graph and describe any trends, patterns, or relationships that are apparent?
- Can students choose scales appropriate to the range of data?
- Can students distinguish between discrete and continuous data?
- Can students describe the shape of a set of data using mathematical vocabulary?
- Can students read, interpret, and draw conclusions from primary and secondary data?
- Can students graph data using a variety of tools (e.g., square grid paper, software)?
- Can students find the mode, median, and mean in a set of data and describe the information these measures give us?
- Can students explain the *stability* of the median compared with the mean? In other words, can they explain what effect extreme values in a set of data have on the median?

RUBRIC

Assessment Category	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding				
– designs appropriate survey questions	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– collects and organizes data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– represents data using stem-and-leaf plots, double bar graphs, and broken-line graphs	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– identifies similarities/differences in two sets of data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– determines the mean, median, and mode in a set of data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
Thinking				
– creates a plan of action for conducting a survey	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– identifies and uses patterns when analysing data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– makes predictions based on data gathered	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– explores alternative methods of data collection and representation	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
Communication				
– explains mathematical thinking	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– communicates using a variety of modes (short answers, lengthy explanations, verbal and written reports)	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– uses appropriate vocabulary and terminology	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
Application				
– applies data management skills in familiar contexts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– transfers knowledge and skills to new contexts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– makes connections among concepts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree

A PICTURE IS WORTH A THOUSAND WORDS

Dear Parent/Guardian:

We are working on a unit on data management, which is a very important part of our lives. One of the skills that your child is developing is the ability to read and interpret data presented in graphs.

Please help your child find two graphs in a newspaper or magazine. Then, discuss the graphs with your child and ask for his or her thoughts on the data presented.

Have your child attach one of the graphs to the back of this sheet by using tape or glue. Then ask your child to write a description of (1) what data the graph is representing and (2) what conclusions he or she can draw from the data.

Please let me know if you have any questions about our unit on data management. Thank you very much for your continued support.

Sincerely,

Grade 5 Learning Activity

Lost Socks

OVERVIEW

In this activity, students explore the following problem: “Suppose you have two pairs of socks in a drawer, one blue pair and one green pair. You reach in and, without looking, pick two socks. Which outcome is more likely: that the two socks will match or that the two socks will not match?” They play a probability game that models the problem, pulling socks from a bag, looking for a match, and keeping track of the outcomes using systematic lists and area models. They draw up a list of all probable outcomes, analyse their results, and then use a common fraction to represent the probability of occurrence of each event.



In Grades 3 and 4, students learned to predict the frequency of outcomes through games, and in Grade 4 they learned to explain their thinking. In Grade 4 they also solved such problems as “If you toss a pair of number cubes 20 times and calculate the sum for each toss, how many times would you expect to get 12? 7? 1? Explain your thinking.” In Grade 5, students are now required to record their results using systematic lists and area models. They also need to become familiar with comparing fractions, because in Grade 6 they will learn to determine the theoretical probability of the occurrence of each event. For example, if the probability of occurrence of the first event is $\frac{1}{6}$ and the probability of occurrence of the second event is $\frac{1}{5}$, they will be required to determine which event is less likely to occur.

BIG IDEA

Probability

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- determine and represent all the possible outcomes in a simple probability experiment, using systematic lists and area models;
- represent, using a common fraction, the probability that an event will occur in simple games and probability experiments;
- pose and solve simple probability problems, and solve them by conducting probability experiments and selecting appropriate methods of recording results.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

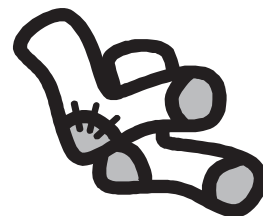
- represent as a fraction the probability that a specific outcome will occur in a simple probability experiment, using systematic lists and area models.

TIME:**2 one-hour sessions**

ABOUT THE LEARNING ACTIVITY

MATERIALS

- 2 pairs of socks, in two different colours (for demonstration)
- paper bags (1 per pair of students; 1 for demonstration)
- 3 colours of colour tiles (2 of each colour per pair or group of students)
- bingo dabbers (1 blue, 1 red, 1 green per student; optional)
- **DMP.BLM5b.1: Laundry Links** (1 per student)



MATH LANGUAGE

- outcome
- frequency
- probability
- equally likely
- sample space
- random

INSTRUCTIONAL GROUPING:**pairs, groups of 3 students, whole class, and individual work**

ABOUT THE MATH

PROBLEM 1

Consider the first problem students will be asked to solve:

Suppose you have two pairs of socks lying loose in a drawer, one blue pair and one green pair. You reach in and, without looking, pick two socks. Which outcome is more likely: the two socks will match, or the two socks will not match?

This is a neat activity because it offers a surprise for students. It is also realistic. On a winter morning when it's quite dark, we might well reach into our sock drawer and just pick out two socks at random.

With regard to the outcomes, most students will say that the two outcomes are equally likely. However, after they have done 20 trials they will discover that it's more likely that the two socks will not match. Why is this the case? Let's label the 4 socks using the numbers 1 for one blue sock, 2 for the other blue sock, 3 for one green sock, and 4 for the other green sock. Now we can list the possible pairings: 1-2, 1-3, 1-4, 2-3, 2-4, 3-4. We immediately see that out of the 6 possible outcomes, only 2 are matching pairs: 1-2 and 3-4.

If you list the outcomes in the format shown below, you will notice that the 6 outcomes can be organized to form a triangular pattern, which means that 6 is a triangular number. The numbers in the list can be coloured to match the socks they represent.

1-2 1-3 1-4

2-3 2-4

3-4

Fractions can be used to express the probability of getting a pair ($2/6$ or $1/3$) and the probability of not getting a pair ($4/6$ or $2/3$).

Students might wonder why we are not counting 1-2 and 2-1 as two different outcomes. The reason is that with socks, there typically isn't a left sock and a right sock – the two are interchangeable.

PROBLEM 2

Now suppose we have three pairs of socks in a drawer, one blue pair, one green pair, and one red pair. What is the probability of getting a matching pair if we reach into the drawer and, without looking, pick two socks?

When students carry out a further 20 trials, they discover that only 3 of the possible 15 outcomes result in a pair (and if you want to make a patterning/algebra/number connection, you will notice that the 15 outcomes form a triangular pattern, which means that 15 is a triangular number).

1-2 1-3 1-4 1-5 1-6

2-3 2-4 2-5 2-6

3-4 3-5 3-6

4-5 4-6

5-6

LOST SOCKS IN THE LAUNDRY

Problems 1 and 2 help to explain a puzzling situation that many of us have encountered when doing laundry; that is, we often end up with mismatched socks.

Look at the problem from this point of view:

Suppose you have three pairs of socks in the laundry, one blue pair, one green pair, and one red pair. Now suppose that somehow four socks disappear. Which is more likely: that the two socks that are left will match or that they will not match?

It turns out (mathematically) that life is not fair, and four times out of five the two socks that are left will not match. For a story-based, interactive exploration of this problem with 10 pairs of socks, see the *Lost Socks* e-story at <http://publish.edu.uwo.ca/george.gadanidis/socks.htm>.

GETTING STARTED

REAL-LIFE CONNECTIONS

Ask students to talk about real-life experiences they have had with probability. When have they found themselves predicting that something would happen? Have they, for instance, pulled clothes out of the dryer and predicted they would pull out a sock or, instead, a shirt? Have they wondered what it means when they hear that today there will be a 40 percent probability of rain? Ask students to work in pairs to make a list of all the times we use probability to predict events in our daily lives. Events could include weather, sporting events, games, and so on.

INTRODUCING THE PROBLEM

Place two pairs of socks of two different colours in a bag (show students that you are doing this). Pose the following problem:

“If I close my eyes, reach into the bag, and pull out two socks, what is the probability that the two socks will match? Is it more likely that the two socks will match? Is it more likely that the two socks will not match? Or is it equally likely that the socks will match or not match?”

Ask students to think about the problem, make a prediction, and explain their reasoning to their elbow partner. Then ask for a show of hands for each possible outcome and record students’ predictions on the board, using a chart, such as the one shown on the right.

Prediction	Tally
Match more likely	
Mismatch more likely	
Both equally likely	

Note: Allow students to design their own method of keeping track of results and organizing them. At the end of the experiment you will initiate a reflection on the different ways in which students chose to record and organize the data from their experiment (see Reflecting and Connecting – Problem 1).

WORKING ON IT – PROBLEM 1

INVESTIGATING PROBLEM 1

Explain to students that they will now work in pairs to carry out a probability experiment to test their predictions. Provide each pair with a paper bag and four colour tiles, two of each colour. Ask students to shake the tiles in the bag, then reach inside and, with eyes closed, pull out two tiles. Explain that each pair of students will carry out 30 trials and will record the results of each trial.

Once the trials have been completed, have students examine the results and reconsider their predictions.

Reconvene the class and discuss all the results.
The average result is shown on the right.

Outcome	Frequency
Matching pair	10
Mismatched pair	20

The results of experimental probability become more accurate (more consistent) as the number of trials increases. Ask five pairs of

Outcome	Frequencies					Total
Matching pair	9	12	8	10	12	51
Mismatched pair	21	18	22	20	18	99

students to volunteer their results, recording them on the board in a table like the one above. Have students estimate the probability of getting a matching pair and express the probability as a fraction. Their responses might include the following equivalent fractions: $10/30$, or $50/150$, or $1/3$.

Note: Students in Grade 5 are required to use organized lists to “determine and represent all the possible outcomes in a simple probability experiment” and to “represent as a fraction the probability that a specific outcome will occur in a simple probability experiment”. The socks problems are “simple probability experiments” even though they involve a case in which all outcomes do not have the same probability. Students will already have had such experiences in Grade 4, having solved problems similar to the simple probability experiment shown in the Grade 4 curriculum expectations: “If you toss a pair of number cubes 20 times and calculate the sum for each toss, how many times would you expect to get 12? 7? 1?”

MAKING SENSE OF THE EXPERIMENTAL RESULTS

The experiment shows that it is more likely that two socks chosen without looking will be mismatched. Ask: Why is this the case? Students can get a sense of the reasoning by considering all the possible outcomes when pulling two socks from two pairs.

Sketch four socks on the board, two in one colour and two in another colour. Identify each sock by writing the numbers 1, 2, 3, or 4 beneath it. If the socks are blue and red, then 1 will stand for the first blue sock, 2 for the second blue sock, 3 for the first red sock, and 4 for the second red sock.

Ask students to use pairs of numbers to list all the possible combinations when two socks are picked without looking. The 6 possible combinations are listed below. Of these, only 2 (1-2 and 3-4) are matching pairs. If necessary, model the process for two of the combinations, and then ask students to find all the other combinations.

Note: Using the list of all outcomes to determine the probability as a fraction prepares students for the formal introduction of “theoretical probability” in Grade 6.

They may find it helpful to colour the numbers to match the socks they represent.

①-2 1-3 1-4

2-3 2-4

③-4

List on the board the combinations determined by students. Discuss which strategies they used or might have used to help organize the search for all the combinations. For instance, the first row of the combinations listed above shows all the combinations that start with 1 (1-2, 1-3, 1-4),

the second row shows the combinations that start with 2 (2-3, 2-4), and the last row shows the combinations that start with 3 (3-4). A fourth row is not needed as there are no combinations that start with a 4.

Students might wonder why we are not counting 1-2 and 2-1 as two different outcomes. The reason is that with socks, there typically isn't a left sock and a right sock – the two socks are interchangeable. The same reasoning applies for 3-1, 4-1, 3-2, 4-2, and 4-3. Ask students to write as a fraction the probability of getting a matching pair ($\frac{2}{6}$) and the probability of getting a mismatched pair ($\frac{4}{6}$).

REFLECTING AND CONNECTING – PROBLEM 1

Engage students in sharing and discussing the different methods they used for recording and organizing the data from their experiment. Such reflection will help to consolidate students' learning and help them understand the methods they used, thus preparing them for Problem 2.

Pose these questions during the discussion:

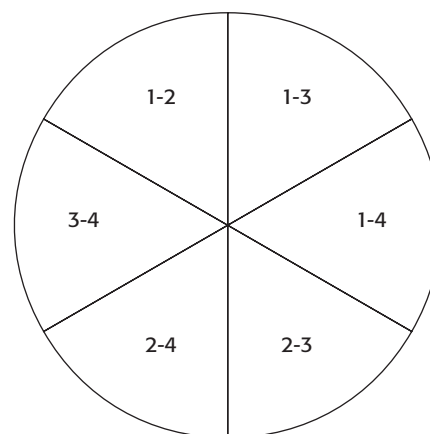
- "How did you record your data?"
 - "If you used a table, how many categories did you use? Why? What did you name the categories? What headings did you use?" (See sample table on the right.)
 - "If you used a bar graph and shaded squares to represent each of the trials, how many bars did you use? Why? How did you label the bars? What did you call your graph?"
 - "Did you use concrete materials, such as colour tiles, to form a physical bar graph?"
- "From your data, how did you determine the probability as a fraction?"
- "What method did you use to count all the possible outcomes?"
- "How did you ensure that you had counted all the possible outcomes?"
- "How would you explain to a friend that when pulling out two socks without looking, it is likely that the socks will be mismatched?"

Outcome	Frequency
Matching pair	
Mismatched pair	

AREA MODEL – DESIGNING A SPINNER

Have students work in pairs to design a spinner that will simulate the socks experiment. The spinner on the right is one way to represent the socks experiment. The six equal spaces in the spinner match the six equal outcomes in the experiment.

Another option is to design a spinner with three equal spaces, since the probability is that one time out of three the socks will match.



WORKING ON IT – PROBLEM 2

Present students with the following problem:

“ Now suppose we have three pairs of socks jumbled up in a drawer, one blue pair, one green pair, and one white pair. What is the probability of getting a matching pair if we reach into the drawer and, without looking, pick out two socks?”

Have students work in groups of three to do the following (which repeats the process they used for Problem 1):

- predict the probability of getting a matching pair
- represent the problem with colour tiles
- carry out a probability experiment to test their prediction
- record the results of the trials in a table
- determine the probability based on their experiment
- list all possible outcomes
- make sense of the experimental probability result by considering all possible outcomes (keeping in mind that the socks in pairs are interchangeable)

THINK, TALK, WRITE

Have students work in pairs to discuss their experiment outcomes. After they have shared their ideas with each other, ask them to write a response in their individual math journals. You might provide them with sentence starters, such as:

- I solved this problem by ...
- The math words that I used were ...
- The steps that I followed were ...
- My strategy was successful because ...
- What I found challenging was ...
- This problem reminds me of another problem ...

The 15 possible combinations for Problem 2 are listed below. Only three of the combinations are matching pairs (shown in colour).

1-2	1-3	1-4	1-5	1-6
2-3	2-4	2-5	2-6	
3-4	3-5	3-6		
4-5	4-6			
5-6				

REFLECTING AND CONNECTING – PROBLEM 2

Invite two or three pairs of students to share their findings with the class. Each presentation will provide an opportunity for other students to ask questions and to share their own ideas and experiences. See Reflecting and Connecting – Problem 1 for things that might be discussed in this forum. Allow students opportunities to express their thinking, to make mistakes, to build on the ideas of others, and to celebrate their mathematical insights.

CULMINATING TASK

As a culminating task have students work in groups of three to create another probability problem. Explain that each group will be responsible for determining and representing all the possible outcomes in their problem, using systematic lists. Ask them to use common fractions to represent the probability of occurrence for a certain event in their problem. Have them record their results using an appropriate method (e.g., a tally chart, an organized list, a picture of the outcomes). At the end of the task, have students present their results to the class.

TRUE OR FALSE?

Have students work in pairs to decide whether the following statements are true or false, and ask them to explain their reasoning:

- The more socks that there are in my drawer, the greater is the probability that the socks will match when I pick two of them without looking.
- There are 6 socks in my drawer and I select 2 without looking. If the 2 socks do not match, it means that I am unlucky.
- I did the laundry and lost 2 of my 6 socks. The probability that the 2 lost socks were a matching pair is $\frac{1}{6}$.

TIERED INSTRUCTION

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

SUPPORTS FOR STUDENT LEARNING

- Use the think-pair-share strategy to encourage students to describe their outcomes.
- Check with students regularly to ensure that they are on track and understand what they are doing. For the extensions tasks, some students may need to work with a smaller number of socks.
- Teach vocabulary related to probability before starting the learning activity.
- Use sentence starters when students have difficulty applying knowledge to new situations:
 - I think ...
 - It could be ...
 - It's because...

- It means that ...
- I'm guessing ...
- I think ... is going to happen because ...
- Remember that students who have difficulty with language may also have difficulty with mathematical language. Provide them with prompts or sentence starters to help them share their math solutions or strategies.
- Provide guidance to students who have difficulty organizing or have trouble choosing strategies that will help them solve the problem. They might:
 - guess and check (make general predictions regarding the colours of socks that could be drawn in the game);
 - make a table (create a simple table that allows them to record their data from their sock draws);
 - draw a picture (use bingo dabbers to record their data, if they need to see a visual representation);
 - make an organized list;
 - keep a tally.
- Use a think-aloud. All students can benefit from this strategy, but particularly those who have difficulty processing information. Think-alouds allow students to verbalize their thinking, demonstrate problem-solving strategies, and model mathematical language (e.g., "If I have two red socks and two green socks in the bag, the probability that I will pick a matching pair of socks is one out of three").

EXTENSIONS

- "Suppose we have 5 pairs of socks jumbled up in a drawer: 1 blue pair, 1 green pair, 1 yellow pair, 1 red pair, and 1 white pair. What is the probability of getting a matching pair if we reach into the drawer and, without looking, pick out two socks?"
- "Suppose we have 10 pairs of socks jumbled up in a drawer, each pair a different colour. What will happen if we pick out 6 socks without looking? How many matching pairs will we get? What is the probability of getting 0 matches, 1 match, 2 matches, and 3 matches?"

HOME CONNECTION

See **DMP.BLM5b.1: Laundry Links**.

ASSESSMENT

OBSERVATION

During the probability experiments, collect observational data using class lists or sticky notes. Focus on the following areas:

- *Problem solving*: Are students able to solve these problems? Can they discuss the sock combinations in terms of probability?
- *Reasoning and proving*: Are students able to make sense of the mathematics they are doing? Do they understand why they are gathering information during the sock experiment? Are they able to represent the outcomes using a common fraction?
- *Selecting tools and strategies*: Are students able to organize their ideas using any of the strategies discussed earlier (e.g., guess and check, draw a picture, make a list). Are they able to complete computational strategies to organize data?
- *Connecting*: Are students able to make connections to other math strands or other probability experiments? Are they able to make connections to other subject areas? Do they say, "This reminds me of ..." or "I wonder what would happen if ..."?
- *Representing*: Are students able to represent their thinking in a variety of ways? Are they able to organize their results in a way that is conducive to analysis, or are their results simply recorded in the order in which they occur?
- *Communicating*: Are students using a think-aloud strategy to share their ideas? Do they use the language of probability correctly? Do they ask questions of themselves and others during the sock experiment and during the reflection discussion? Are students respectful of others when they are sharing their ideas?

Make use, as well, of the information gathered during the Reflecting and Connecting part of the activity.

RUBRIC

Assessment Category	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding				
– collects and organizes data from probability experiments	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– determines the probability of outcomes based on probability experiments	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– identifies similarities/differences in conditions and results of different probability experiments	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– lists all possible outcomes of a simple probability experiment	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– represents the results of a probability experiment using an area model (such as a spinner)	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
Thinking				
– creates a plan of action for conducting an experiment	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– identifies and uses patterns when analysing data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– makes predictions based on data gathered	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– explores alternative methods of conducting probability experiments	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
Communication				
– explains mathematical thinking	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– communicates using a variety of modes (short answers, lengthy explanations, verbal and written reports)	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– uses appropriate vocabulary and terminology	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
Application				
– applies probability skills in familiar contexts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– transfers knowledge and skills to new contexts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– makes connections among concepts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree

LAUNDRY LINKS

Dear Parent/Guardian:

We are beginning an interesting activity designed to study the concept of probability. By the end of this year, it is expected that Grade 5 students will be able to represent as a fraction the probability that a specific outcome will occur in a simple probability experiment.

To help students better understand the concept of probability, we will be asking them to work on a situation that is familiar to most people: sorting socks after they come out of the dryer and finding that some have gone missing.

We are going to be asking students to think about two problems:

- Suppose you have three pairs of socks in the laundry: one blue pair, one green pair, and one white pair. Now suppose that somehow four of the socks get lost. Which is more likely: the two socks left behind will match, or they will not match?
- Now suppose we have three pairs of socks jumbled up in a drawer: one blue pair, one green pair, and one white pair. What is the probability of getting a matching pair if we reach into the drawer and, without looking, pick out two socks?

The purpose of these problems is to encourage students to determine and represent all the possible outcomes, using lists and models. We will then ask students to use a fraction to represent the probability of drawing the matched or mismatched socks.

While working at solving these problems, students will be using the mathematical language of probability, words such as *certain*, *likely*, *outcome*, *likelihood*, *unlikely*, *impossible*, *equally likely*, *experiment*, and *combination*. You can help your child by using some of the same language at home to describe the likelihood of the occurrence of certain events.

You might want to reproduce the probability problems at home, thus giving them a real-life context. Have your child help you with the laundry and give him or her math support at the same time! Such connections with daily life further strengthen your child's understanding of probability and help to demonstrate that mathematics is important in all parts of our lives.

If you have any questions about our work on probability, please feel free to call. Thank you very much for your continued support.

Sincerely,

Grade 6 Learning Activity

Paper Airplane Contest

OVERVIEW

In this learning activity, students design a paper airplane and conduct an investigation to measure the effectiveness of their design. The context for the lesson comes from the science curriculum, where the Matter and Materials strand explores the properties of air and the characteristics of flight. Students collect and organize primary continuous data from their own trials and have an opportunity to use the scientific process. Students are introduced to a *continuous line graph* and provided with opportunities to build upon their knowledge of *discrete data* and *continuous data*. Students make choices as to the most effective way to display the data. They use the collected information to write a report promoting the design and capability of their paper airplane. The effect of changing the *scale* of a graph is explored.

By Grade 6, students will have had extensive experience in reading and interpreting graphs and in describing the shape of the data. They are now asked to consider how information from charts and graphs can be used to make inferences and build convincing arguments. This activity reviews the concepts of *mean*, *median*, and *mode* that were introduced in earlier grades.

BIG IDEAS

Collection and organization of data; data relationships

CURRICULUM EXPECTATIONS

COLLECTION AND ORGANIZATION OF DATA

This learning activity addresses the following **specific expectations**.

Students will:

- collect data by conducting a survey or an experiment to do with themselves, their environment, issues in their school or community, or content from another subject, and record observations or measurements;
- collect and organize discrete or continuous primary data and secondary data and display the data in charts, tables, and graphs (including continuous line graphs) that have appropriate titles, labels, and scales that suit the range and the distribution of the data, using a variety of tools;
- select an appropriate type of graph to represent a set of data, graph the data using technology, and justify the choice of graph (i.e., from types of graphs already studied, such as pictographs, horizontal or vertical bar graphs, stem-and-leaf plots, double bar graphs, broken-line graphs, and continuous line graphs).

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- collect and organize discrete or continuous primary data and secondary data and display the data using charts and graphs, including continuous line graphs.

DATA RELATIONSHIPS

This learning activity addresses the following **specific expectations**.

Students will:

- read, interpret, and draw conclusions from primary data and from secondary data, presented in charts, tables, and graphs (including continuous line graphs);
- compare through investigation, different graphical representations of the same data;
- explain how different scales used on graphs can influence conclusions drawn from the data;
- demonstrate an understanding of mean, and use the mean to compare two sets of related data, with and without the use of technology;
- demonstrate, through investigation, an understanding of how data from charts, tables, and graphs can be used to make inferences and convincing arguments.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- read, describe, and interpret data, and explain relationships between sets of data.

TIME:

- 5 one-hour sessions (depending on students' prior knowledge)
- time to collect data related to airplane trials

ABOUT THE LEARNING ACTIVITY

MATERIALS

- square grid paper (chart size preferable) and large markers for the placemat activity (1 each per group of students)
- paper suitable for making paper airplanes (1 sheet per pair or small group of students)
- measuring tapes or metre sticks (1 per pair or group of students)
- timers or stopwatches (1 per pair or group of students)
- **DMP.BLM6a.1: Share the News** (1 per student)

MATH LANGUAGE

Review

- | | |
|----------------------|-------------------|
| • median | • primary data |
| • mode | • secondary data |
| • mean | • discrete data |
| • stem-and-leaf plot | • continuous data |
| • broken-line graph | |

Introduce

- continuous line graph
- the notion of making inferences and building convincing arguments based on data analysis

ABOUT THE MATH

BROKEN-LINE GRAPH

A broken-line graph is a graph formed by line segments that join points representing the data.

CONTINUOUS LINE GRAPH

A continuous line graph is a graph that consists of an unbroken line and in which both axes represent continuous quantities, such as distance and time.

THE MEAN

The mean of a set of numbers is found by dividing the sum of the numbers by the number of numbers in the set. For example, the mean of 10, 20, and 60 is $(10 + 20 + 60) \div 3 = 30$. Students consider the mean as the “fulcrum” or “balancing point” that balances the set of data.

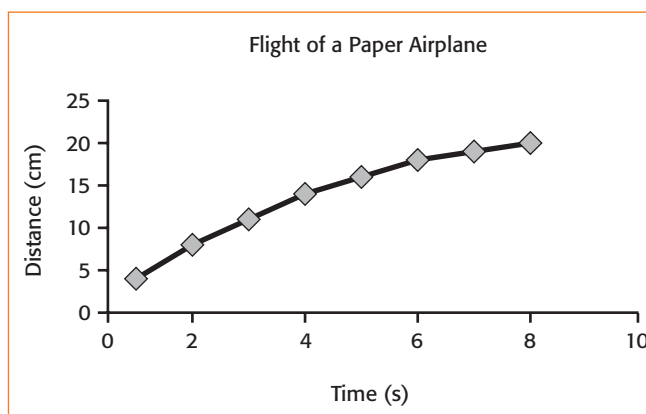
PLOTTING THE DISTANCE-TIME GRAPH

The appropriate graph for this exercise is a continuous line graph with a smooth rather than a broken line, and with one axis to indicate time aloft and the other to indicate distance travelled.

As students test their paper airplanes, they will measure the distance travelled and the time their airplane stayed in the air. These two measures together will represent the point where the graph

stops. Although some students might draw a straight line for the distance-time graph, it is important to explain that in real life the line would not be straight. A straight line means that the airplane was travelling at a constant speed during the entire time it was in the air. However, when the airplane is thrown, it typically has a higher *horizontal* speed (the speed at which it is moving away from its starting point) at the beginning than it does at the end of its flight (it slows down), so the steepness (slope) of the graph changes over time, as shown in the example here.

Graphing the flight of their airplane can uncover another surprise for students. It is possible for the graph to also turn downward, meaning that the distance from the starting point is decreasing rather than increasing: it’s easy to imagine a paper airplane turning and heading back towards its starting point.



INSTRUCTIONAL GROUPING:

pairs, small groups of 4–6 students, whole class, and individual work

DESIGNING PAPER AIRPLANES

The following online resources might be helpful:

- For scientific concepts, see www.exploratorium.edu/exploring/paper/airplanes2.html.
- For designing paper airplanes see www.exploratorium.edu/exploring/paper/airplanes.html and www.paperairplanes.co.uk.
- For information on world records, the history and rules of paper airplane throwing, and so on, visit www.geocities.com/netitall/PAAEnter/PAAMain/paamain.html.
- For information on flight technology see www.education.gov.ab.ca/lrb/resource/.
- See also the Canadian Aviation Museum at www.aviation.technomuses.ca.

GETTING STARTED

INTRODUCING THE TASK TO STUDENTS

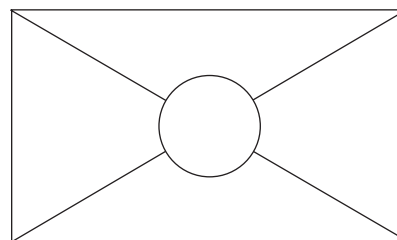
Explain that the lesson connects to the science curriculum and specifically to the Matter and Materials strand, which explores the properties of air and the characteristics of flight. Tell students that, in this activity, the challenge will be to build the best paper airplane – the one that travels the farthest and stays airborne the longest. Explain that, after designing and testing their airplane, they will use the data collected to write a report promoting its design and capability.

UNDERSTANDING THE TASK

Although this is essentially a data collection and analysis activity, those tasks can't be properly carried out unless the airplanes have been designed to give a measurable performance. Explain to students that they will have time to research, design, and test the efficacy of their designs before the start of the formal time-and-distance trials. They will then have to determine how best to collect the data from the trials – the distance flown and the length of time the planes were airborne.

Divide students into groups of four and ask them to use the placemat technique to record and discuss their initial thoughts on the best method for gathering the data they need. Each student in the group uses a quadrant (or section) of the placemat (see diagram on the right) in which to write his or her thoughts and ideas. Advise students that they can use words, pictures, graphs, or diagrams to explain their thinking. Once individual contributions have been made, ask students to discuss their thinking with the other members of their group and to write all common elements of their reasoning in the middle section of the placemat.

Reconvene the class and ask the groups to present their ideas.



DECIDING ON A DATA COLLECTION METHOD

Ask students to discuss the following questions:

- “How are you going to measure the distance and time?”
- “What units of measurement are most appropriate?”
- “When launching the airplanes, how can you ensure that everyone is starting from the same place?”
- “How can you minimize factors, such as wind, obstacles, distractions, and so on?”
- “Do we need to do more than one trial? If so, how many would be a good number and why?”
- “Do we count the results of all the trials? For example, if a plane crashes, should that trial be counted?”

This discussion will provide rich opportunities for scientific enquiry and will prompt students to examine the “commitment to precision and integrity in observation, experimentation, and reporting; ... [and] adherence to safety procedures” (Ontario Ministry of Education. [2007]. *The Ontario Curriculum, Grades 1-8: Science and Technology*) that will help them successfully complete this activity.

WORKING ON IT

DESIGNING AND TESTING THE PAPER AIRPLANES

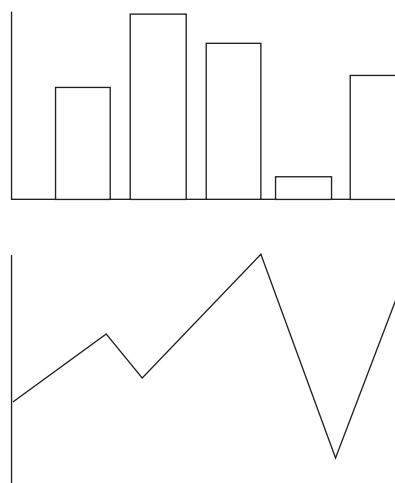
Give students time to research, design, and briefly test their paper airplanes for “flyability”. After this, have students work in pairs or in small groups, determine who will be launching the airplanes and who will be measuring, and then carry out their flight trials, noting down the data for each trial. After the data have been collected, ask each pair or group to organize the data onto a stem-and-leaf plot, one graph for distance, and one graph for time.

GRAPHING THE DATA

Graphing continuous data can be done on a *broken-line graph*, but such a graph typically has one axis for fixed data (e.g., grade, year) and one for continuous data. In this activity you will introduce students to the *continuous line graph*, in which both axes represent continuous data (such as time and distance).

Sketch on the board the two graphs shown on the right. Ask the following questions about the two graphs and give students a few minutes to consider their answers:

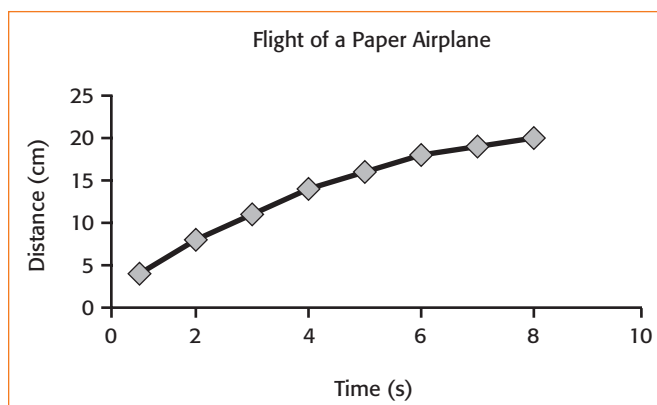
- “What is the same about these two graphs? What is different?”
- “Have you seen these types of graphs before? Where?”



Discussing the difference between discrete and continuous data will allow students to consider the appropriateness of a bar chart or a continuous line graph for this activity.

Have students examine the data they have collected. Ask: "What is the best way to graph our results?"

Explain that, typically, the graph has time on the horizontal axis and distance on the vertical axis. Also, the shape of the graph line is as shown on the right – a smooth line rather than a broken one. The line is not straight. As the airplane slows down, its slope becomes less steep (the distance travelled increases at a lower rate). It is also possible for the graph to turn downwards, which would mean that the distance from the starting point is decreasing rather than increasing. It's easy to imagine a paper airplane turning and heading back towards its starting point.



Once students have chosen the graph they feel is most appropriate, direct them to change the scales they are using, so that they will see the effect the scale has on the shape of the data. Lead a discussion on whether changing the scales would be appropriate and how changing the shape of the data could influence decisions and the interpretation of the data. Ask:

- "Why would we want to change the shape of the data?"
- "How do we know which scale is the best one to use?"

Let students decide what the shape of their data should look like, but remind them to refer to their original challenge for guidance.

INTERPRETING THE DATA

Once students have their graphs and have decided on a scale, they are ready to interpret their data. Ask students to start analysing their data by making statements about the shape of their data on the stem-and-leaf plot. These statements could take the form of jot notes added to the paper on which they recorded their data. Finding the median and mode using this type of graph is straightforward.

Then ask students to consider how to find the mean. Explain that the mean is sometimes seen as the "balancing point" or "fulcrum" on which all the data are dependent.

EXPLORING THE MEAN

Give students a mean and ask them to provide a set of data to match it. For example, ask: "If the mean is 8 and there are 7 values in the set, what might the other values be? What are other possibilities?" The simplest solution is 7 numbers, all with a value of 8. Encourage students to explore the effect that extreme values have on the mean.

Example 1: 8, 8, 8, 8, 8, 7, 9

Example 2: 5, 7, 13, 8, 3, 9, 11

Prompt students to develop their own strategies for finding such answers. Using a number line and “balancing” the data is one strategy. Using sticky notes is equally effective for small sets of data. One move balances out the other, as shown in Example 3 (which uses the data set from Example 2), with the data on one side of the mean balancing out the data on the other.

Once students have had such experience with modelling the mean, they can explore more difficult distributions of data, including those in which the mean is not a value in the original set. Provide students with the following problem:

“What might the values be if:

- the mean is 5, there are 7 values in the set, and 5 is not one of the values;
- the median is 3;
- the mode is 2.”

(More than one answer is possible. Two correct answers are 1, 2, 2, 3, 4, 10, 13, and 1, 2, 2, 3, 6, 10, 11.)

This problem allows students to explore the effect that all the values have on the mean. Providing students with these challenges and encouraging them to develop their own strategies will give them a deeper understanding of mean.

Offer students many opportunities to play with the sets of data before you introduce the algorithm for finding the mean (see the About the Math section). Some students may discover the algorithm while manipulating the data.

As a final step in their explorations, ask students to find the average time their plane was airborne and the average distance their plane travelled.

Example 3

Data Value	Distance From the Mean
3	–5
5	–3
7	–1
8	0
9	+1
11	+3
13	+5

REFLECTING AND CONNECTING

Having calculated the mean, median, and mode for their sets of data, students are ready to analyse the data and write a report promoting the design and capability of their paper airplane.

Ask them to consider their data and determine which “average” is most representative of their design. Encourage them to develop their ideas independently but be prepared to support those needing help to work out the differences in the three “averages”.

Some ideas for students to consider:

- If there are extreme values in the set – for example, if the airplane had one or more short flights – the median might be a better choice than the other averages, as it is not affected by extreme values in the set.
- If the range of the data is relatively small, students might want to use the mean to show the consistency of the performance of the airplane.
- If the range is relatively large or inconsistent, students might want to present the mode as the average.

Ask students to use their collected data to write their own report. The report must contain their graph and must explain and justify their choices for data representation and statistical analysis, including their choice of average. The use of differing averages to make persuasive arguments, in conjunction with a carefully considered graph and a relevant scale, will allow students to see the connection between the collected data and the presentation.

Ask each student to present his or her report. As students watch all their classmates' presentations, they will find their own understanding and experience reinforced and will see the wide variety of choices and possibilities available for managing data.

TIERED INSTRUCTION

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

SUPPORTS FOR STUDENT LEARNING

- All students can benefit from developing their own procedure or method of organizing their data. However, some students may find the open nature of such a task too daunting. It may be beneficial for these students to have a template to help them get started. Providing headings, columns, grids, or a chart on which to record their trial results will allow them to focus on the data.
- Although student-generated strategies help students develop conceptual understanding, some students may benefit from being provided with a strategy, such as using a number line, to work from. Guided instruction will allow them to try new ideas with confidence and will help them make connections between what they already know and what they are working on.
- Note that some students may benefit from direct teaching to fully understand the effect of changing the scales in a graph.
- Offer a balance of shared, guided, and individual learning opportunities, to provide support for students who need scaffolded learning.
- Creating mixed ability groupings gives students the chance to hear the perspectives of a variety of students. Ensure maximum participation by all members by assigning clearly defined but changing roles within the group.

- Make explicit connections for students, such as “Does this remind you of something we have done before?” Such prompts will help students make connections with what they already know.
- Break larger tasks into more manageable pieces to help students grasp the new information.
- If time constraints prevent a discussion of all the strategies used for the task, observe students while they are working and ask certain groups to present their ideas, thus ensuring that the class will have a variety of effective strategies to consider. Each group could present to a partner group before presenting to the class, thereby allowing all groups the chance to share their ideas orally. Asking students to write down their ideas gives them an opportunity to transfer knowledge into written form. Effective oral and written communication skills are essential in mathematics.

EXTENSIONS

- Ask students to research common practices in the judging of paper airplane contests.

HOME CONNECTION

See **DMP.BLM6a.1: Share the News.**

ASSESSMENT

OBSERVATION

Providing opportunities for students to discuss, compare, contrast, justify, and explain their thinking will allow you to assess their conceptual and procedural knowledge. Through observation and the discussion prompted by key questions, you will gain a good sense of students’ understanding. Throughout this series of activities, you have been provided with questions to promote student communication and help students verbalize their math thinking.

You might also want to consider the following questions:

- Can students collect, organize, and display primary and secondary data using a variety of graphs, including stem-and-leaf plots, double bar graphs, and continuous line graphs?
- Can students interpret a continuous line graph and describe any trends, patterns, or relationships that are apparent?
- Can students manipulate the scale on a graph to influence opinion concerning a set of data?
- Can students distinguish between discrete and continuous data, and consider the appropriateness of using differing graphs for each type of data??
- Can students describe the shape of a set of data using mathematical vocabulary?
- Can students read, interpret, and draw conclusions from primary and secondary data?
- Can students find the mode, median, and mean in a set of data and discuss the information these measures provide?
- Can students explain how the mean acts to “balance out” a set of data?
- Can students explain the *stability* of the median compared with the mean? In other words, can they describe the effect on the median of extreme values in a set of data?

RUBRIC

Assessment Category	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding				
– designs appropriate data collection methods	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– collects and organizes data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– represents data using stem-and-leaf plots	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– represents distance-time data using a line graph	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– identifies similarities/differences in sets of data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– determines the mean, median, and mode in a set of data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
Thinking				
– creates a plan of action for conducting an experiment	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– identifies and uses patterns when analysing data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– makes predictions based on data gathered	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– makes convincing arguments based on data gathered	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– explores alternative methods of data collection and representation	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
Communication				
– explains mathematical thinking	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– communicates using a variety of modes (short answers, lengthy explanations, verbal and written reports)	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– uses appropriate vocabulary and terminology	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
Application				
– applies data management skills in familiar contexts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– transfers knowledge and skills to new contexts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– makes connections among concepts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree

SHARE THE NEWS

Dear Parent/Guardian:

We have just completed an interesting activity in which students conducted an investigation to measure the effectiveness of a paper airplane they had designed. From their own flight trials, they collected and organized primary continuous data. They used that information to write a report promoting the design and capability of their paper airplane.

We have asked students to bring their report home and present it to a family member.

Ask your child to explain the following during the presentation:

- the type of data collected and how the collection was carried out;
- the methods used to organize and display the data;
- what the graph represents and why it has the shape it has;
- what your child has learned about paper airplane design;
- what your child has learned about the management of data.

Some of the concepts used in this project were:

- median, mode, and mean;
- the stem-and-leaf plot, the broken-line graph, the continuous line graph;
- primary data and secondary data, discrete data and continuous data.

If you have any questions about our work on this activity on managing data, please feel free to call. Thank you very much for your continued support.

Sincerely,

Grade 6 Learning Activity

Rock-Paper-Scissors

OVERVIEW

In this learning activity, students investigate probability within the context of the popular game of Rock-Paper-Scissors. They list the “sample space” of the game – that is, all the possible outcomes of the game. By changing one variable, such as the number of players or the number of elements, students experience the way that different factors can affect the outcome of an event. Students determine the theoretical probability of each outcome and express it as a fraction and as a ratio of favourable outcomes to the total number of outcomes. Basing their reasoning on collected data, they will also predict the frequency of an outcome in the game of Rock-Paper-Scissors.

Before this activity students should have had experiences in determining the sample space of a probability experiment or game, and in representing as a fraction the probability that any event in the sample space might occur in a simple game or activity. They will also have carried out investigations to verify the truth of this statement: the larger the sample size, the more consistent the results.

BIG IDEA

Probability

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- express theoretical probability as a ratio of the number of favourable outcomes to the total number of possible outcomes, where all outcomes are equally likely;
- predict the frequency of an outcome of a simple probability experiment or game, by calculating and using the theoretical probability of that outcome.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- determine the theoretical probability of an outcome in a probability experiment, and use it to predict the frequency of the outcome.

ABOUT THE LEARNING ACTIVITY

TIME:

2 one-hour
sessions

MATERIALS

- **DMP.BLM6b.1: Rock-Paper-Scissors** (2 per student)
- chart paper and markers (per pair of students)
- dice (2 per pair of students; optional)
- spinners with four coloured quadrants (1 per pair of students; optional)
- **DMP.BLM6b.2: Take It Home** (1 per student)

MATH LANGUAGE

- outcome
- favourable outcome
- sample space
- likely
- probability
- experimental probability
- theoretical probability
- frequency
- ratio

ABOUT THE MATH

INSTRUCTIONAL
GROUPING:

pairs, small
groups of 4–6
students, whole
class, and
individual work

EXPERIMENTAL PROBABILITY

Experimental probability is determined by doing an experiment. For example, if you roll a die 36 times and record how often each number comes up, you might find that 6 comes up 4 times. So the experimental probability of getting a 6 is $\frac{4}{36}$ or $\frac{1}{9}$. The more trials done in an experiment, the more accurate the experimental probability will be.

THEORETICAL PROBABILITY

Theoretical probability is determined by considering all possible outcomes. For example, when rolling a die, there are 6 possible outcomes and each one of them is equally likely. So the probability of getting a 6 is $\frac{1}{6}$.

PROBABILITY AS A FRACTION AND AS A RATIO

A fraction always compares part to whole. A ratio, however, can compare part to whole or part to part. When rolling a die, the probability of rolling a 3 can be written as the fraction $\frac{1}{6}$. The ratio for this can be written as 1:6, where one outcome is compared with all outcomes (part to whole). However, it is also mathematically correct to write the probability as the ratio 1:5, where one outcome is compared with all *other* outcomes (part to part). In fact, it is most often the case that ratio is used as a part:part comparison and not as a part:whole comparison. For example, on a label describing fertilizer, the ratio 10:20:10 is a part:part:part comparison (nitrogen:phosphorus:potassium). We also use a part:part comparison when calculating odds. So the odds that the number 3 will be rolled with one die are 1:5. "Odds" is a comparison of the probability that an event will occur with the probability that the event will not occur.

GETTING STARTED

ROCK-PAPER-SCISSORS

Review the Rock-Paper-Scissors game with the class. Have students play a few games to practise.

THE PROBLEM

Pose this question: "Is there a winning strategy or is it a fact that no matter what the strategy, each player has the same chance of winning when you play the game of Rock-Paper-Scissors?"

Note: There are 9 possible outcomes when playing the Rock-Paper-Scissors game. For **DMP.BLM6b.1: Rock-Paper-Scissors**, we have chosen 27 trials, as 27 is a multiple of 9. However, it would also be fine to do 30 or 50 trials.

WORKING ON IT: DAY 1 – DESIGNING A WINNING STRATEGY

CARRYING OUT A PROBABILITY EXPERIMENT

Ask students to make a personal prediction about what might be a winning strategy when playing the Rock-Paper-Scissors game. Ask each student to record his or her winning strategy (without sharing it with anyone). Next, ask students to play the Rock-Paper-Scissors game with a partner and to check if their winning strategies hold true. Give each student a copy of **DMP.BLM6b.1: Rock-Paper-Scissors** on which to record results.

Note: Some students may say that there is no winning strategy and that they are just as likely to win as they are to draw or lose. They are correct, as will later become evident when they calculate the theoretical probability. Do not confirm or dispute their ideas. Let them make up their own minds.

After students complete **DMP.BLM6b.1: Rock-Paper-Scissors**, ask them to share their winning strategies with their partners and discuss how they might adapt their strategies, basing their reasoning on the results on their recording sheets. Ask them to use the data to calculate the experimental probability of a win, a loss, or a tie for Player 1 on a single trial of the Rock-Paper-Scissors game.

Have each pair design a new winning strategy and come up with an argument to justify their reasoning. Ask them to use the same sheet but a different colour of pen to record the results of their new winning strategy. Ask students to use these new data to recalculate the experimental probability of a win, a loss, or a tie for Player 1 on a single trial of the Rock-Paper-Scissors game.

REFLECTING AND CONNECTING: DAY 1 – DESIGNING A WINNING STRATEGY

Invite a few pairs to share their winning strategies and to explain their reasoning. This is an opportunity for students to share their own ideas, to ask questions, and to develop their thinking on probability.

WORKING ON IT: DAY 2 – CALCULATING THE THEORETICAL PROBABILITY

DEFINING THEORETICAL PROBABILITY

Introduce the concept of *theoretical probability* and contrast it with *experimental probability*.

Remind students that experimental probability is determined by doing an experiment. The more trials done in an experiment, the more accurate the experimental probability will be. Theoretical probability, conversely, is determined by considering all possible outcomes. (See the About the Math section for examples of experimental and theoretical probability.)

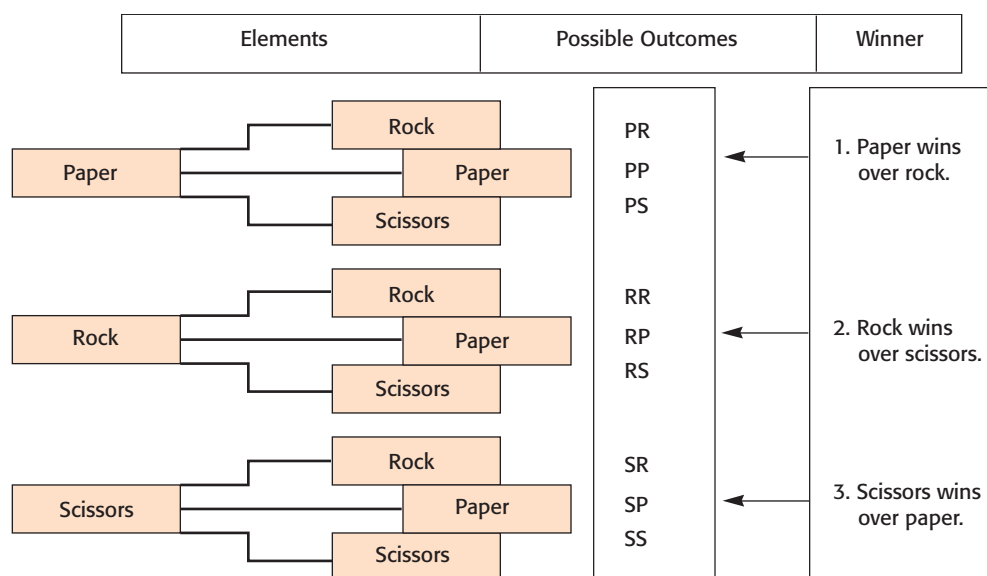
Tell students that in order to calculate the theoretical probability of winning, losing, or tying in the Rock-Paper-Scissors game, they need to determine all possible outcomes of the game.

Identifying outcomes. Rock/Paper and Paper/Rock are two different outcomes because each of the two scenarios represents a different “winner”. In Rock/Paper, Rock represents the choice of the first student, and Paper represents the choice of the second student. The sample space therefore includes Rock/Paper, Rock/Scissors, Paper/Rock, Paper/Scissors, Scissors/Rock, Scissors/Paper, Paper/Paper, Rock/Rock, and Scissors/Scissors.

LISTING ALL OUTCOMES

Ask students to think about all the possible outcomes, such as Rock/Paper or Scissors/Rock. Explain that we call the list of all possible outcomes the *sample space*. Pair up students. Ask each pair to list and organize all the possible outcomes for the game of Rock-Paper-Scissors using chart paper and markers. When students have finished, have all groups post their list.

While students are working, select a few groups whose work represents a range of approaches and strategies. Look for lists that show deliberate organization. Once those groups have shared their work with the class, ask students to discuss which methods of organization are most efficient and why. Encourage students to suggest other ways of organizing the outcomes.



Show the class a tree diagram similar to the one above. Discuss the strengths of this type of diagram and tell students that you want to use this type of diagram to discuss and represent theoretical probability.

Explain that by charting their results using a tree diagram, students can keep track of the possibilities they have generated. Encourage them to list the elements on the left and systematically move through each element on the right (first Paper, then Rock, then Scissors), as a way of making sure that none of the possibilities is missed. This type of organizer creates a visual representation that makes the mathematics more explicit and therefore visible. Later on, students will establish the connection that the number of outcomes is 3×3 or 9.

Ask students to list all the possible outcomes of the game.

REFLECTING AND CONNECTING: DAY 2 – CALCULATING THE THEORETICAL PROBABILITY

Reconvene the class and ask students the following questions:

- "What is the total number of outcomes?"
- "Are they all equally likely to occur? Why or why not?"
- "If you think they are not equally likely, which outcome do you think might be more likely?"
- "What is the theoretical probability of each outcome?"
- "What fraction of the sample space does each outcome represent?"
- "What is the theoretical probability of winning a trial? Losing a trial? Tying in a trial?"
- "How would you write those three results as a fraction? As a ratio?"
- "Is there a winning strategy?"

Ask students to identify the relationships between the number of elements and the number of players (e.g., with 3 elements and 2 players, there are 3×3 outcomes).

TIERED INSTRUCTION

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

SUPPORTS FOR STUDENT LEARNING

- Some students might find the management of the results too challenging. If so, they can play a supporting role within their team by, for instance, contributing to the analysis of the data without being involved in the creation of a table or chart.

EXTENSIONS

Odds. Have students look up the definition of odds. Odds compare the probability that an event will occur with the probability that the event will not occur. Ask students to write ratios to represent the odds of winning, losing, and tying in Rock-Paper-Scissors.

Challenges. Pose the following problems:

- “What would happen if you played Rock-Paper-Scissors with three people? List the combinations and permutations (all the possible outcomes). Choose a way to represent the sample space. In an organized way, record the sample space on chart paper.”
- “What would happen if you added one element to the game? For example, Rock-Paper-Scissors-_____. Choose your own fourth element. Assuming there are two players, list the combinations and permutations (all the possible outcomes). Choose a way to represent the sample space. In an organized way, record the sample space on chart paper.”
- “In the original game, when 2 people are playing the game and the game has 3 elements, there are 9 possible outcomes. Find the relationship between the number of players, the number of elements, and the number of possible outcomes when there are (a) 2 players and 3 elements, (b) 2 players and 4 elements, and (c) 2 players and 5 elements.”

Dice Roll-Out. Have students work as partners, rolling two dice and recording the outcome without adding the values. For example, after a roll they might record (1, 5) or (2, 4). Ask the partners to begin by choosing a way to keep track of the different outcomes and to organize their data. Have them roll the dice and record the outcomes. Once they have finished recording and organizing the possible outcomes, have them reflect and connect with their partners by answering the following questions:

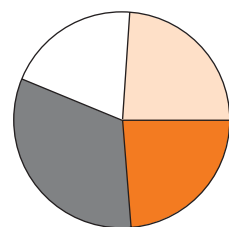
- “Are all outcomes equally likely to happen? Why or why not?”
- “If you think they are not equally likely, which outcome do you think might be more likely?”
- “How many outcomes are possible?”
- “What is the theoretical probability of each outcome?”
- “What fraction of the sample space does each outcome represent?”
- “How many outcomes contain a 5?”

- “If the outcomes that contain a 5 are the favourable outcomes, what is the ratio of favourable outcomes to the total number of outcomes?”
- “How many rolls did it take to come up with all the different outcomes?”

Spinning Fun. Have students work in pairs with a four-quadrant spinner (each quadrant is a different colour). Explain that the partners must discuss the sample space and make a list of the possible outcomes. They are to establish the theoretical probability of each outcome and express it as a fraction and as a ratio. Then they must spin the spinner to test their prediction and record the colour of each spin’s result.

Ask students to answer the following questions:

- “Was your prediction verifiably correct after 4 spins?”
- “Was your prediction verifiably correct after 12 spins?”
- “Was your prediction verifiably correct after 25 spins?”
- “Was your prediction verifiably correct after 50 spins?”
- “Was your prediction verifiably correct after 100 spins?”
- “Did the number of spins make a difference in terms of verifying your predication? If so, why would that be?”



HOME CONNECTION

Provide students with **DMP.BLM6b.2: Take It Home** and a copy of **DMP.BLM6b.1: Rock-Paper-Scissors**, so that they can play several games of Rock-Paper-Scissors with a family member. They can then compare their home results with those gathered in class.

ASSESSMENT

OBSERVATION

Observe and converse with students as they play the game and chart their results. Look for ways in which they track the outcomes and, through a system of their own design, ensure that there is no repetition.

Look also for a demonstration that they understand that the theoretical probability of an event is the relationship between the number of favourable outcomes and the total number of possible outcomes. For example, can they state the theoretical probability of Paper’s being a winner? (It’s 3 out of 9, or $\frac{3}{9}$, or 3:9.)

Ask probing questions to determine whether students can predict how often an outcome is likely to occur if there is a change in the number of games played.

Ask questions, such as these, to assess students’ understanding:

- “Is a particular element (say Paper) more or less likely to be the winner if more people play the game?”
- “Is a particular element (say Paper) more or less likely to be the winner if the number of elements in the game is increased to 4 (e.g., Rock-Paper-Scissors-Hammer)?”

RUBRIC

Assessment Category	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding				
– collects and organizes data from probability experiments	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– determines the probability of outcomes based on probability experiments	<input type="checkbox"/> limited <input type="checkbox"/> limited	<input type="checkbox"/> some <input type="checkbox"/> some	<input type="checkbox"/> considerable <input type="checkbox"/> considerable	<input type="checkbox"/> thorough <input type="checkbox"/> thorough
– lists all possible outcomes of a simple probability experiment	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– represents the results of a probability experiment using a fraction or a ratio	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
– distinguishes between experimental and theoretical probability	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> thorough
Thinking				
– creates a plan of action for conducting an experiment	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– identifies and uses patterns in analysing data	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– makes predictions based on data gathered	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– explores alternative methods of conducting probability experiments	<input type="checkbox"/> limited <input type="checkbox"/> limited	<input type="checkbox"/> some <input type="checkbox"/> some	<input type="checkbox"/> considerable <input type="checkbox"/> considerable	<input type="checkbox"/> high degree <input type="checkbox"/> high degree
Communication				
– explains mathematical thinking	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– communicates using a variety of modes (short answers, lengthy explanations, verbal and written reports)	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– uses appropriate vocabulary and terminology	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
Application				
– applies probability skills in familiar contexts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– transfers knowledge and skills to new contexts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree
– makes connections among concepts	<input type="checkbox"/> limited	<input type="checkbox"/> some	<input type="checkbox"/> considerable	<input type="checkbox"/> high degree

ROCK-PAPER-SCISSORS

● = Rock wins ✂ = Paper wins
 ✂ = Scissors wins ☒ = Tie (play again)

Game	R/R ☒	R/P ✂	R/S ●	P/R ✂	P/P ☒	P/S ✂	S/R ●	S/P ✂	S/S ☒
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
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16									
17									
18									
19									
20									
21									
22									
23									
24									
25									
26									
27									

TAKE IT HOME

Dear Parent/Guardian:

We have just completed an interesting activity in which students conducted an investigation to determine the probabilities of different outcomes in the game Rock-Paper-Scissors.

Please take some time to play this game with your child, and have your child (1) record on the attached handout the results of the games played at home and (2) compare the results of the “home games” with the results that were gathered in class. Ask your child to explain to you the probability of winning when playing Rock-Paper-Scissors and whether it is possible to have a winning strategy.

Sincerely,

APPENDIX: GUIDELINES FOR ASSESSMENT

There are three types of assessment: assessment *for* learning, assessment *as* learning, and assessment *of* learning.

Assessment *for* learning involves teachers observing the knowledge, skills, experience, and interests their students demonstrate, and using those observations to tailor instruction to meet identified student needs and to provide detailed feedback to students to help them improve their learning.

Assessment *as* learning is a process of developing and supporting students' metacognitive skills. Students develop these skills as they monitor their own learning, adapt their thinking, and let the ideas of others (peers and teachers) influence their learning. Assessment *as* learning helps students achieve deeper understanding.

Assessment *of* learning is summative. It includes cumulative observations of learning and involves the use of the achievement chart to make judgements about how the student has done with respect to the standards. Assessment *of* learning confirms what students know and are able to do, and involves reporting on whether and how well they have achieved the curriculum expectations.

Teachers use assessment data, gathered throughout the instruction–assessment–instruction cycle, to monitor students' progress, inform teaching, and provide feedback to improve student learning. Effective teachers view instruction and assessment as integrated and simultaneous processes. Successful assessment strategies – those that help to improve student learning – are thought out and defined ahead of time in an **assessment plan**.

An **assessment profile**, developed by the teacher for each student, can be an effective way of organizing assessment data to track student progress. Students can also maintain their own **portfolios**, in which they collect samples of their work that show growth over time.

Creating an Assessment Plan

To ensure fair and consistent assessment throughout the learning process, teachers should work collaboratively with colleagues to create assessment and instructional plans. Ideally, such planning should start with learning goals and work backwards to identify the assessment and instructional strategies that will help students achieve those goals.

Guiding Questions	Planning Activities
"What do I want students to learn?"	Teachers begin planning by identifying the overall and specific expectations from the Ontario curriculum that will be the focus of learning in a given period. The expectations may need to be broken down into specific, incremental learning goals. These goals need to be shared with students, before and during instruction, in clear, age-appropriate language.
"How will I know they have learned it?"	Teachers determine how students' learning will be assessed and evaluated. Both the methods of assessment and the criteria for judging the level of performance need to be shared with students.
"How will I structure the learning?"	Teachers identify scaffolded instructional strategies that will help students achieve the learning goals and that integrate instruction with ongoing assessment and feedback.

An assessment plan should include:

- clear learning goals and criteria for success;
- ideas for incorporating both assessment *for* learning and assessment *as* learning into each series of lessons, before, during, and after teaching and learning;
- a variety of assessment strategies and tools linked carefully to each instructional activity;
- information about how the assessment profiles will be organized;
- information about how the students' assessment portfolios will be maintained.

Feedback

When conducting assessment *for* learning, teachers continuously provide timely, descriptive, and specific feedback to students to help them improve their learning. At the outset of instruction, the teacher shares and clarifies the learning goals and assessment criteria with the students. Effective feedback focuses the student on his or her progress towards the learning goals. When providing effective feedback, teachers indicate:

- what good work looks like and what the student is doing well;
- what the student needs to do to improve the work;
- what specific strategies the student can use to make those improvements.

Feedback is provided during the learning process in a variety of ways – for example, through written comments, oral feedback, and modelling. A record of such feedback can be maintained in an assessment profile.

Assessment Profile

An assessment profile is a collection of key assessment evidence, gathered by the teacher over time, about a student's progress and levels of achievement. The information contained in the profile helps the teacher plan instruction to meet the student's specific needs. An extensive collection of student work and assessment information helps the teacher document the student's progress and evaluate and report on his or her achievement at a specific point in time.

The assessment profile also informs the teacher's conversations with students and parents about the students' progress. Maintaining an assessment profile facilitates a planned, systematic approach to the management of assessment information.

Assessment profiles may include:

- assessments conducted after teaching, and significant assessments made during teaching;
- samples of student work done in the classroom;
- samples of student work that demonstrates the achievement of expectations;
- teacher observation and assessment notes, conference notes;
- EQAO results;
- results from board-level assessments;
- interest inventories;
- notes on instructional strategies that worked well for the student.

Student Assessment Portfolio

A portfolio is a collection of work selected by the student that represents his or her improvement in learning. It is maintained by the student, with the teacher's support. Assembling the portfolio enables students to engage actively in assessment *as* learning, as they reflect on their progress. At times, the teacher may guide students in the selection of samples that show how well they have accomplished a task, that illustrate their improvement over a period of time, or that provide a rationale for the teacher's assessment decisions. Selections are made on the basis of previously agreed assessment criteria.

Student assessment portfolios can also be useful during student/teacher and parent/teacher conferences. A portfolio may contain:

- work samples that the student feels reflect growth;
- the student's personal reflections;
- self-assessment checklists;
- information from peer assessments;
- tracking sheets of completed tasks.

Students should not be required to assign marks, either to their own work or to the work of their peers. Marking is part of the evaluation of student work (i.e., judging the quality of the work and assigning a mark) and is the responsibility of the teacher.

Assessment Before, During, and After Learning

Teachers assess students' achievement at all stages of the instructional and assessment cycle.

Assessment *before* new instruction identifies students' prior knowledge, skills, strengths, and needs and helps teachers plan instruction.

Effective assessment *during* new instruction determines how well students are progressing and helps teachers plan required additional instruction. The teacher uses a variety of assessment strategies, such as focused observations, student performance tasks, and student self- and peer assessment, all based on shared learning goals and assessment criteria. As noted earlier, the teacher provides students with feedback on an ongoing basis during learning to help them improve.

During an instructional period, the teacher often spends part of the time working with small groups to provide additional support, as needed. The rest of the time can be used to monitor and assess students' work as they practise the strategies being learned. The teacher's notes from his or her observations of students as they practise the new learning can be used to provide timely feedback, to develop students' assessment profiles, and to plan future lessons. Students can also take the opportunity during this time to get feedback from other students.

Assessment *after* new learning has a summative purpose. As assessment *of* learning, it involves collecting evidence on which to base the evaluation of student achievement, develop teaching practice, and report progress to parents and students. After new learning, teachers assess students' understanding, observing whether and how the students incorporate feedback into their performance of an existing task or how they complete a new task related to the same learning goals. The assessment information gathered at this point, based on the identified curriculum expectations and the criteria and descriptors in the achievement chart, contributes to the evaluation that will be shared with students and their parents during conferences and by means of the grade assigned and the comments provided on the report card.

Glossary

area model. A way of modelling probability by shading a part of the area of a shape (like a circle or a square). The diagram above shows an area model for the probability of getting “heads” when tossing a coin.



average. Another word for **mean**.

bar graph. *See graph.*

benchmark. A number or measurement that is internalized and used as a reference to help judge other numbers or measurements. For example, the width of the tip of the little finger is a common benchmark for one centimetre. *Also called* a referent.

bias. An emphasis on characteristics that are not typical of an entire population and that may result in misleading conclusions.

big ideas. In mathematics, the important concepts or major underlying principles.

binomial theorem. A theorem used to determine the expansion of powers of sums. For example, $(H + T)^2 = H^2 + 2HT + T^2$ and $(H + T)^3 = H^3 + 3H^2T + 3HT^2 + T^3$. This is useful in determining some probabilities. If H = “heads” and T = “tails”, then $(H + T)^2 = H^2 + 2HT + T^2$ would mean that when tossing a coin twice in a row, there is 1 way to get HH , 2 ways to get HT , and 1 way to get TT .

broken-line graph. *See graph.*

categorical data. *See discrete data.*

census. The collection of data from an entire population.

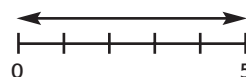
circle graph. *See graph.*

conceptual understanding. A connection of mathematical ideas that provides a deep understanding of mathematics. Students develop their understanding of mathematical concepts through rich problem-solving experiences.

concrete materials. Objects that students handle and use in constructing or demonstrating their understanding of mathematical concepts and skills. Some examples of concrete materials are base ten blocks, connecting cubes, construction kits, number cubes, games, geoboards, geometric solids, measuring tapes, Miras, pattern blocks, spinners, and tiles. *Also called* manipulatives.

connecting cubes. Commercially produced learning tools that help students learn about spatial sense, volume, surface area, patterning, and so on. Some connecting cubes attach on only one face, while others attach on any face.

continuous data. Measurements, such as length, mass, volume, time, temperature, and so forth. Continuous data can take on any numerical value, including decimals and fractions. Line graphs, as opposed to scatter plots or bar graphs, are typically used to represent continuous data.

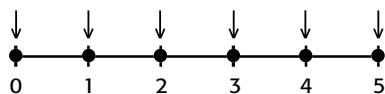


data. Facts or information. *See also* **continuous data** and **discrete data**.

database. An organized and sorted list of facts or information; usually generated by a computer.

descriptive statistics. Statistical measures (like mean, median, and mode) that describe the data but do not make inferences or conclusions.

discrete data. Data that can include only certain numerical values (often whole numbers) within the range of the data. Discrete data usually represent things that can be counted; for example, the number of times a word is used or the number of students absent. There are gaps between the values. For example, if whole numbers represent the data, as shown in the following diagram, fractional values, such as $3\frac{1}{2}$, are not part of the data. Scatter plots or bar graphs, as opposed to line graphs, are typically used to represent discrete data. *Also called categorical data.*



distribution. An arrangement of measurements and related frequencies; for example, a table or graph that shows how many times each score, event, or measurement occurred.

dodecahedron. A 12-sided polygon.



double bar graph. *See graph.*

double line graph. *See graph.*

dynamic geometry software. Computer software that allows the user to explore and analyse geometric properties and relationships through dynamic dragging and animations. Uses of the software include plotting points and making graphs on a coordinate system; measuring line segments and angles; constructing and transforming two-dimensional shapes; and creating

two-dimensional representations of three-dimensional objects. An example of the software is The Geometer's Sketchpad.

dynamic statistical software. Computer software that allows the user to gather, explore, and analyse data through dynamic dragging and animations. Uses of the software include organizing data from existing tables or the Internet, making different types of graphs, and determining measures of central tendency. Examples of the software include TinkerPlots and Fathom.

equation. A mathematical statement that has equivalent expressions on either side of an equal sign.

equilateral triangle. A triangle with three equal sides.

event. A possible outcome, or group of outcomes, of an experiment. For example, rolling an even number on a number cube is an event with three possible outcomes: 2, 4, and 6.

experimental probability. The likelihood of an event occurring, determined from experimental results rather than from theoretical reasoning.

exponent. *See exponential form.*

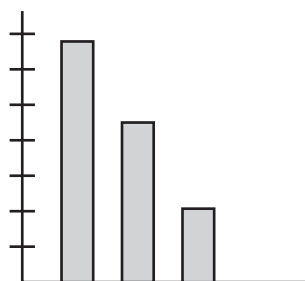
exponential form. A representation of a product in which a number called the *base* is multiplied by itself. The *exponent* is the number of times the base appears in the product. For example, 5^4 is in exponential form, where 5 is the base and 4 is the exponent; 5^4 means $5 \times 5 \times 5 \times 5$.

fair game. In a fair game, all players have the same chance of winning.

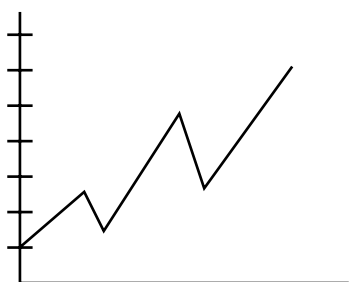
frequency. The number of times an event or outcome occurs.

graph. A visual representation of data. Some types of graphs are:

– **bar graph.** A graph consisting of horizontal or vertical bars that represent the frequency of an event or outcome. There are gaps between the bars to reflect the categorical or discrete nature of the data.



– **broken-line graph.** A graph formed by line segments that join points representing the data. The horizontal axis represents discrete quantities, such as months or years, whereas the vertical axis can represent continuous quantities.

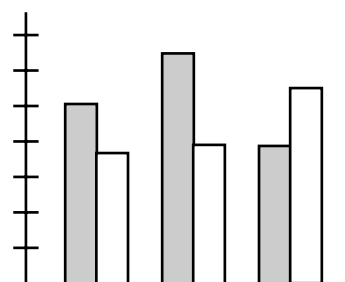


– **circle graph.** A graph in which a circle is used to display categorical data, through the division of the circle proportionally to represent each category.



– **double bar graph.** A graph that combines two bar graphs to compare two aspects of the data in related contexts; for example, comparing the populations of males and

females in a school in different years. *Also called a comparative bar graph.*



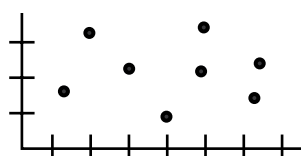
– **double line graph.** A graph that combines two line graphs to compare two aspects of the data in related contexts; for example, comparing the distance travelled by two cars moving at different speeds.

– **line graph.** A graph that consists of an unbroken line and in which both axes represent continuous quantities, such as distance and time. Also called a continuous line graph.

– **pictograph.** A graph that uses pictures or symbols to compare frequencies.



– **scatter plot.** A graph designed to show a relationship between corresponding numbers from two sets of data measurements associated with a single object or event; for example, a graph of data about marks and the corresponding amount of study time. Drawing a scatter plot involves plotting ordered pairs on a coordinate grid. Also called a scatter diagram.



– **stem-and-leaf plot.** An organization of data into categories based on place values. The plot allows easy identification of the greatest, least, and median values in a set of data. The following stem-and-leaf plot represents these test results: 72, 64, 68, 82, 75, 74, 68, 70, 92, 84, 77, 59, 77, 70, 85.

5	9
6	4, 8, 8
7	0, 0, 2, 4, 5, 7, 7
8	2, 4, 5
9	2

line graph. See **graph**.

mathematical communication. The process through which mathematical thinking is shared. Students communicate by talking, drawing pictures, drawing diagrams, writing journals, charting, dramatizing, building with concrete materials, and using symbolic language (e.g., $2 =$).

mathematical language. The conventions, vocabulary, and terminology of mathematics. Mathematical language can be used in oral, visual, or written forms. Some types of mathematical language are:

- terminology (e.g., factor, pictograph, tetrahedron);
- visual representations (e.g., 2×3 array, parallelogram, tree diagram);
- symbols, including numbers (e.g., 2, $\frac{1}{4}$), operations [e.g., $3 \times 8 = (3 \times 4) + (3 \times 4)$], and signs (e.g., $=$).

mean. One measure of central tendency. The mean of a set of numbers is found by dividing the sum of the numbers by the number of numbers in the set. For example, the mean of 10, 20, and 60 is $(10 + 20 + 60) \div 3 = 30$. A change in the data produces a

change in the mean, similar to the way in which changing the load on a lever affects the position of the fulcrum if balance is maintained. See also **measure of central tendency**.

measure of central tendency. A measure of the location of the middle or centre of an ordered set of data. **Mean**, **median**, and **mode** are measures of central tendency.

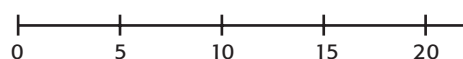
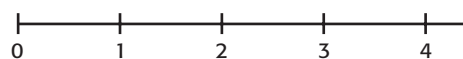
median. The middle value in a set of values arranged in order. For example, 14 is the median for the set of numbers 7, 9, 14, 21, 39. If there is an even number of numbers, the median is the average of the two middle numbers. For example, 11 is the median of 5, 10, 12, and 28. See also **measure of central tendency**.

mode. The value that occurs most often in a set of data. For example, in a set of data with the values 3, 5, 6, 5, 6, 5, 4, 5, the mode is 5. See also **measure of central tendency**.

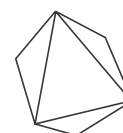
modelling. The process of describing a relationship using mathematical or physical representations.

monomial. An algebraic expression with one term; for example, $2x$ or $5xy$.

number line. A line that represents a set of numbers using a set of points. The increments on the number line reflect the scale.



octahedron. An eight-sided polygon.



ordered pair. Two numbers, in order, that are used to describe the location of a point on a plane, relative to a point of origin (0,0); for example, (2,6). On a coordinate plane, the first number is the horizontal coordinate of a point, and the second is the vertical coordinate of the point. *Also called* coordinates.

percent. A ratio expressed using the percent symbol, %. Percent means “out of a hundred”. For example, 30% means 30 out of 100. A percent can be represented by a fraction with a denominator of 100; for example, $30\% = 30/100$.

pictograph. *See* graph.

population. The total number of individuals or objects that fit a particular description; for example, salmon in Lake Ontario.

primary data. Information that is collected directly or first-hand; for example, observations and measurements collected directly by students through surveys and experiments. *Also called* first-hand data or primary-source data. *See also* secondary data.

probability. A number from 0 to 1 that shows how likely it is that an event will happen.

probability experiment. An experiment used to determine the probability of an outcome. For example, to determine the probability of getting “heads” when tossing a coin, you could toss a coin 100 times and use the result of the experiment to determine the probability.

range. The difference between the highest and lowest numbers in a group of numbers or set of data. For example, in the data set 8, 32, 15, 10, the range is 24, that is, $32 - 8$.

ratio. A comparison of quantities with the same units. A ratio can be expressed in ratio form or in fraction form; for example, 3:4 or $3/4$.

sample. A representative group chosen from a population and examined in order to make predictions about the population.

sample space. A listing of all possible outcomes.

scale (on a graph). A sequence of numbers associated with marks that subdivide an axis. An appropriate scale is chosen to ensure that all data are represented on the graph.

scatter plot. *See* graph.

secondary data. Information that is not collected first-hand; for example, data from a magazine, a newspaper, a government document, or a database. *Also called* second-hand data or secondary-source data. *See also* primary data.

shape of data. The shape of a graph that represents the distribution of a set of data. The shape of data may or may not be symmetrical.

simple probability experiment. An experiment with the same possible outcomes each time it is repeated, but for which no single outcome is predictable; for example, tossing a coin, rolling a number cube.

simulation. A probability experiment with the same number of outcomes and corresponding probabilities as the situation it represents. For example, tossing a coin could be a simulation of whether the next person you meet will be a male or a female.

slope. The steepness of a graph.



Ministry of Education
Printed on recycled paper
ISBN 978-1-4249-5491-9
07-054

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